MATH 201 HW 1

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a) The sample space is the set of all possible outcomes. Let B and R denote blue and red balls respectively. Then,

$$\Omega = \{BB, RB, RR\}$$

Note that $BR = RB$.

b) There are $\binom{2n}{2}$ ways of selecting 2 balls from $2n$ balls. There are n ways of choosing red or blue balls. This then means there are n^2 ways of selecting one red and one blue ball. The probability of selecting one red and one blue ball is

$$P(RB) = \frac{n^2}{\binom{2n}{2}} = \frac{n^2}{\frac{(2n)!}{2!(2n-2)!}} = \frac{n^2}{\frac{2n(2n-1)}{2}} = \frac{n^2}{n(2n-1)} = \frac{n}{2n-1}$$

c) The probability of all of the events in the sample space must add to 1. This requires that

$$p_n = 1 - \frac{n}{2n-1} = \frac{n-1}{2n-1}$$

Then,

$$\lim_{n\to\infty} p_n = \lim_{n\to\infty} \frac{n-1}{2n-1} = \frac{1}{2}$$

2

8 squares in an 8×8 array can be selected in $\binom{64}{8}$ different ways. When selecting a square, we can think about selecting a row and a column. For the first square, there are 8 options for both rows and columns. This produces 64 possible squares. The next square has 7 options for rows and columns. This produces 49 possible squares. The pattern continues, and there are n^2 options for the n^{th} square. Since the order that the squares are placed doesn't matter, we can get the number of combinations by calculating

$$\frac{(8^2)(7^2)\ldots(2^2)(1)}{8!} = 8!$$

Hence, the probability of selecting 8 squares that don't share a column or a row is

$$\frac{8!}{\binom{64}{8}} = \frac{8!}{\frac{64!}{8!(56!)}} = \frac{(8!)^2(56!)}{64!}$$

There is another method. This method uses the fact that since there are 8 squares and 8 rows, each square must lie on a unique row. Then, select a row. There are 8 options for a square on this row. For the next selected row, there are 7 options, since 2 squares can't share a column. For the next row there are 6 options. This process continues, resulting in i possible options for the i^{th} square. Multiplying the number of choices together, we get 8! total configurations for 8 squares placed on the grid (satisfying the condition that any two squares don't share a row and a column). Therefore the probability is

$$\frac{8!}{\binom{64}{8}} = \frac{8!}{\frac{64!}{8!(56!)}} = \frac{(8!)^2(56!)}{64!}$$

3

Suppose it is possible to uniformly choose a positive integer. If $P(A) \propto \#A$, then for all $a \in \mathbb{N}$,

$$P(\{a\}) \propto 1 \implies P(\{a\}) = c$$

Where $0 \leq c \leq 1$. We can decompose the natural numbers into the following countable union:

$$\mathbb{N} = \{1\} \cup \{2\} \cup \{3\} \cup \dots$$

In order to have a probability measure on $\mathbb{N}$, $P(\mathbb{N}) = 1$. However, from the line above we see that

$$P(\mathbb{N}) = P\left(\{1\} \cup \{2\} \cup \{3\} \cup \dots\right) = \sum_{i=1}^{\infty} P(\{i\}) = \sum_{i=1}^{\infty} c$$

If $c = 0$, the sum above is 0, implying that $P(\mathbb{N}) = 0$. This contradicts the assumption that $P(\mathbb{N}) = 1$. If $c > 0$, then the above some must diverge. This would imply that $P(\mathbb{N}) = \infty$. This also contradicts the assumption that $P(\mathbb{N}) = 1$. If one assumes that choosing a positive integer at random is possible, we get contradictions. Therefore it must be the case that it is not possible to uniformly choose a positive integer at random.