Homework 2

MATH 201 (Summer 2023, Session A2)

Saturday 20th May, 2023

Instructions

- This homework is due on Tuesday, May 23rd at 11 PM Eastern Time.
- Justify your answers.
- Late submissions are not permitted unless there are extenuating circumstances.
- Please read the honesty policy of the course (available on the course webpage) and make sure you understand the collaboration policy.

Problem 0. [0 points] Copy paste the following text in the beginning of your submission:

> This submission conforms to the honesty policy of the course. In particular, I have not made use of any unauthorized online resources and any collaboration did not violate the expectations outlined in the policy.

After that, list all students you collaborated with, clearly indicating which problems you worked with them on. If you did not collaborate with anyone, clearly state this instead.

Problem 1. [10 points] Call a coin p-biased if $P(H) = p$ and $P(T) = 1 - p$ where H is the event that the coin turns up heads when flipped once and T is the event that the coin turns up tails when flipped once. Two-Face keeps flipping a p-biased coin until it shows up tails.

(a) Write down a reasonable sample space Ω that encodes the outcomes of Two-Face's activities.

(b) What is the probability that Two-Face never sees a tails and keeps flipping forever? (**Hint:** try mimicing the argument from Lecture 3 where we looked at this problem with $p = 1/2$; be careful to avoid dividing by zero when doing so.)

Problem 2. [10 points] Imagine a game of 3 players where exactly one player wins in the end and all players have equal chances of being the winner. The game is repeated six times. Find the probability that there is at least one person who wins no games.

Problem 3. [20 points] A crime has been committed in a suburb of Gotham City which has 200,000 inhabitants. The police are looking for a single perpetrator, believed to live in town. DNA evidence is found on the crime scene. The Joker's DNA matches the DNA recovered from the crime scene. According to DNA experts, the probability that a random person's DNA matches the crime scene DNA is 1 in 5,000. Before the DNA evidence, the Joker was no more likely to be the guilty person than any other person in town.

(a) What is the probability that the Joker is guilty after the DNA evidence appeared? You may assume that if the perpetrator's DNA is tested then it will match the crime scene DNA 100% of the time.

(b) Suppose Batman is called in, and he develops a new method to test DNA. With this new method the probability that a random person's DNA matches the crime scene DNA is now 1 in 30,000. Suppose that with the new method it is confirmed that the Joker's DNA matches the DNA recovered from the crime scene. What is the probability that the Joker is guilty now?

(c) Suppose that Robin observed an orange car fleeing the scene of the crime but could not give any more details about the car. The town has 2,000 orange cars. In questioning, it is revealed that the Joker's clown car is orange. What is the probability that the Joker is guilty now if the original method for testing DNA is used? What if the new, more accurate method is used?