Homework 3

MATH 201 (Summer 2023, Session A2)

Wednesday 24$^{\text{th}}$ May, 2023

Instructions

- This homework is due on Saturday, May 27th at 11 PM Eastern Time.
- Justify your answers.
- Late submissions are not permitted unless there are extenuating circumstances.
- Please read the honesty policy of the course (available on the course webpage) and make sure you understand the collaboration policy.

Problem 0. [0 points] Copy paste the following text in the beginning of your submission:

> This submission conforms to the honesty policy of the course. In particular, I have not made use of any unauthorized online resources and any collaboration did not violate the expectations outlined in the policy.

After that, list all students you collaborated with, clearly indicating which problems you worked with them on. If you did not collaborate with anyone, clearly state this instead.

Problem 1. [20 points] Recall that a p-biased coin satisfies $P(H) = p$ and $P(T) = 1 - p$ where H is the event that the coin turns up heads when flipped once and T is the event that the coin turns up tails when flipped once. In Gotham City, 70% of all coins are fair (i.e., $p = 1/2$), 20% of all coins are biased towards heads (i.e., $p = 3/4$) and 10% of all coins are biased toward tails (i.e., $p = 1/4$).
Batman has a random coin in his pocket that he gives to Two-Face. Two-Face flips the coin twice.
Suppose H_j is the event that the jth flip is heads, F is the event that Batman's coin is fair, A is the event that it is heads-biased and B is the event that it is tails-biased.
You can reasonably assume that successive flips of a coin whose identity is known are independent. Thus,

$$P(H_1 H_2 | X) = P(H_1 | X) P(H_2 | X),$$

for $X = F, A,$ or B. This is known as conditional independence.

(a) Are the events H_1 and H_2 independent without conditioning?

(b) Give an intuitive explanation for your answer to the previous part.

Problem 2. [15 points] Show that if $X \sim \mathrm{Geom}(p)$ then

$$P(X = n + k | X > n) = P(X = k), \text{ for every } n, k \geq 1.$$

This is called the memoryless property of the geometric distribution. It says that if there are no successes in the first n trials then the probability that the first success at trial $n + k$ is the same as the probability that a freshly started sequence of trials yields the first success at trial k. In particular, the outcome of the first n trials are forgotten.