MATH 201 HW 3

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a) First, the law of total probability yields

$$\begin{aligned} P(H_1H_2) &= P(H_1H_2|F)P(F) + P(H_1H_2|A)P(A) + P(H_1H_2|B)P(B) \\ &= P(H_2|F)P(H_1|F)P(F) + P(H_2|A)P(H_1|A)P(A) + P(H_2|B)P(H_1|B)P(B) \\ &= \frac{1}{4}\frac{7}{10} + \frac{9}{16}\frac{1}{5} + \frac{1}{16}\frac{1}{10} = \frac{28+18+1}{160} = \frac{47}{160} \end{aligned}$$

Similarly, for $i = 1, 2$,

$$\begin{aligned} P(H_i) &= P(H_i|F)P(F) + P(H_i|A)P(A) + P(H_i|B)P(B) \\ &= \frac{1}{2}\frac{7}{10} + \frac{3}{4}\frac{2}{10} + \frac{1}{4}\frac{1}{10} = \frac{21}{40} \end{aligned}$$

This means that $P(H_1)P(H_2) = \frac{441}{1600} \neq \frac{47}{160}$. Therefore you don't get independence without conditioning! Another way to see this is to compute

$$P(H_2|H_1) = \frac{P(H_1H_2)}{P(H_1)} = \frac{\frac{47}{160}}{\frac{21}{40}} = \frac{47}{160}\frac{40}{21} = \frac{47}{84} > \frac{21}{40}$$

Since $P(H_2|H_1) \neq P(H_2)$, you don't get independnce.

b) If you see a heads on the first flip, you could reason that there's a better chance that the coin is heads biased. As a result, the probability that you will get a heads on the second flip increases. You can see this above in the math. Given H_1, the probability $P(H_2|H_1) > P(H_2)$.

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Let $n, k \geq 1$. By the geometric distribution,

$$P(X > n) = \sum_{m=n+1}^{\infty} (1-p)^{m-1}p = (1-p)^n p \sum_{m=1}^{\infty} (1-p)^{m-1} = (1-p)^n p \left(\frac{1}{1-(1-p)} \right) = (1-p)^n$$

Then, since $\{X = n+k\} \subset \{X > n\}$,

$$P(\{X = n+k\} \cap \{X > n\}) = P(X = n+k) = (1-p)^{n+k-1}p$$

Then,

$$P(X = n+k|X > n) = \frac{P(\{X = n+k\} \cap \{X > n\})}{P(X > n)} = \frac{(1-p)^{n+k-1}p}{(1-p)^n} = (1-p)^{k-1}p = P(X = k)$$