

Homework 4

MATH 201 (Summer 2023, Session A2)

Friday 26th May, 2023

Instructions

- This homework is due on Tuesday, May 30th at 11 PM Eastern Time.
- Justify your answers.
- Late submissions are not permitted unless there are extenuating circumstances.
- Please read the honesty policy of the course (available on the course webpage) and make sure you understand the collaboration policy.

Problem 0. [0 points] Copy paste the following text in the beginning of your submission:

This submission conforms to the honesty policy of the course. In particular, I have not made use of any unauthorized online resources and any collaboration did not violate the expectations outlined in the policy.

After that, list all students you collaborated with, clearly indicating which problems you worked with them on. If you did not collaborate with anyone, clearly state this instead.

Problem 1. [20 points] Find continuous random variables X and Y with the following properties:

- (a) $E(X) = \infty$.
- (b) $E(X) < \infty$ but $\text{Var}(X) = \infty$.

[**Hint:** Try random variables with p.d.f. $f(x) = Cx^{-k}$ for some constants C and k .]

Problem 2. [20 points] Batman and Two-Face decide to play some games to pass the time.

(a) In game 1, Two-Face keeps flipping a fair coin; every time it turns up heads Batman gives Two-Face \$1, and every time it turns up tails then Two-Face gives Batman \$1. What is the expected value of the amount of money Batman gives Two-Face? If Two-Face ends up giving money to Batman, you should interpret that as a negative number.

[**Hint:** This is a trick question.]

(b) In game 2, Two-Face keeps flipping a fair coin until it turns up tails; when at a tails shows up, Batman gives Two-Face $\$X$ where X is the number of heads that appeared. What is the expected value of the amount of money Batman gives Two-Face?