MATH 201 HW 4

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a) Let $\Omega = \mathbb{R}$, and X be a random variable with the pdf

$$f(x) = \begin{cases} 1/x^2 & x \geq 1 \\ 0 & x < 1 \end{cases}$$

. Then,

$$EX = \int_{-\infty}^{\infty} xf(x)dx = \int_1^{\infty} xf(x)dx = \int_1^{\infty} \frac{x}{x^2}dx = \int_1^{\infty} \frac{1}{x}dx = \infty$$

This random variable is continuous, as the cdf

$$F(c) = P(X \leq c) = \begin{cases} 0 & c < 1 \\ \int_1^c \frac{1}{x^2}dx = \left(1 - \frac{1}{c}\right) & c \geq 1 \end{cases}$$

Is continuous (for all $c \in \mathbb{R}$).

b) Let $\Omega = \mathbb{R}$, and Y be a random variable with the pdf

$$g(x) = \begin{cases} 1/x^3 & x \geq 1 \\ 0 & x < 1 \end{cases}$$

Then,

$$EY = \int_{-\infty}^{\infty} xg(x)dx = \int_1^{\infty} \frac{1}{x^3}dx = -\frac{1}{2x^2}\Big|_1^{\infty} = \frac{1}{2}$$

The variance is

$$E[Y^2] - (E[Y])^2 = \int_{-\infty}^{\infty} x^2 g(x)dx - \frac{1}{4}$$

This simplifies to

$$\int_1^{\infty} \frac{1}{x}dx - \frac{1}{4} = \infty$$

This random variable is continuous, as the cdf

$$G(c) = P(Y \leq c) = \begin{cases} 0 & c < 1 \\ \int_1^c \frac{1}{x^3}dx = \frac{1}{2}\left(1 - \frac{1}{c^2}\right) & c \geq 1 \end{cases}$$

Is continuous for all $c \in \mathbb{R}$.

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b) Let X be the random variable counting the number of flips. The distribution of X will be the geometric distribution,

$$P(X = k) = p^{k-1}(1 - p)$$

Therefore the expectation of X is

$$EX = \sum_{k=0}^{\infty} kP(X = k) = \sum_{k=1}^{\infty} k(p^{k-1})(1-p) = (1-p)\sum_{k=1}^{\infty} kp^{k-1} = (1-p)\frac{d}{dp}\sum_{k=1}^{\infty} p^k$$

Evaluating this geometric series, we get that

$$EX = (1-p)\frac{d}{dx}\left(\frac{p}{1-p}\right) = (1-p)\frac{1}{(1-p)^2} = \frac{1}{1-p}$$

In our case, there are no issues with dividing by 0 as $p = 1/2$. Therefore $EX = 2$. We expect to see the game stop after two flips, which means that we would expect Batman to give Two-face one dollar.