Homework 5

MATH 201 (Summer 2023, Session A2)

Monday 29^{th} May, 2023

Instructions

- This homework is due on Saturday, June 3rd at 11 PM Eastern Time.
- Justify your answers.
- Late submissions are not permitted unless there are extenuating circumstances.
- Please read the honesty policy of the course (available on the course webpage) and make sure you understand the collaboration policy.

Problem 0. [0 points] Copy paste the following text in the beginning of your submission:

> This submission conforms to the honesty policy of the course. In particular, I have not made use of any unauthorized online resources and any collaboration did not violate the expectations outlined in the policy.

After that, list all students you collaborated with, clearly indicating which problems you worked with them on. If you did not collaborate with anyone, clearly state this instead.

Problem 1. [15 points] Let X be a discrete random variable that only takes values that are nonnegative integers. That is,

$$P(X = k) = 0,$$

unless $k \in \{0, 1, 2, 3, \cdots\}$. (Note that the converse need not hold, so one may have, for example, $P(X = 0) = 0$.)

(a) Show that X must satisfy the following tail identity:

$$E(X) = \sum_{k=1}^{\infty} P(X \geqslant k).$$

(b) Using this identity, compute $E(X)$ when $X \sim \text{Geom}(p)$.

Problem 2. [20 points] Let $Z \sim \mathcal{N}(0, 1)$ and $X \sim \mathcal{N}(\mu, \sigma^2)$.

(a) Compute $E(Z^3)$. [**Hint:** integration by parts.]

(b) Compute $E(X^3)$. [**Hint:** don't try to compute the integral directly; instead try to relate it to $E(Z^j)$ for different values of j.]

Problem 3. [15 points] The Joker has decided to run a lottery in Gotham. He manages to sell $10,000$ tickets. In this clown lottery, each ticket has a 0.025% chance of winning – at this scale you can assume that the tickets are all independent of each other. The Joker would lose his cool it if 3 or more tickets won the lottery.

(a) Let W be the number of winning tickets. Describe the distribution of W as a binomial random variable. Is it more appropriate to use the normal approximation or the Poission approximation for W?

(b) Compute, using the more appropriate approximation, an estimate for the probability that the Joker does not lose his cool.