

# Homework 6

MATH 201 (Summer 2023, Session A2)

Thursday 1<sup>st</sup> June, 2023

## Instructions

- This homework is due on Tuesday, June 6th at 11 PM Eastern Time.
- Justify your answers.
- Late submissions are not permitted unless there are extenuating circumstances.
- Please read the honesty policy of the course (available on the course webpage) and make sure you understand the collaboration policy.

**Problem 0.** [0 points] Copy paste the following text in the beginning of your submission:

This submission conforms to the honesty policy of the course. In particular, I have not made use of any unauthorized online resources and any collaboration did not violate the expectations outlined in the policy.

After that, list all students you collaborated with, clearly indicating which problems you worked with them on. If you did not collaborate with anyone, clearly state this instead.

**Problem 1.** [30 points] You install 2 new light bulbs: a 60 watt bulb and a 100 watt bulb. Let  $X$  be the lifetime of the 60 watt bulb and  $Y$  the lifetime of the 100 watt bulb. The package says that the expected lifetime of the 60 watt bulb is an exponential random variable with mean 200 days while the lifetime of the 100 watt bulb is an exponential random variable with mean 100 days.

- Write down the PDFs of  $X$  and  $Y$ .
- Compute the CDFs of  $X$  and  $Y$ .
- If  $f_X$  is the PDF of  $X$  and  $F_Y$  is the CDF of  $Y$  then

$$P(X < Y) = \int_0^{\infty} f_X(x)(1 - F_Y(x))dx.$$

Use the above equation and your answers to parts (a) and (b) of this problem to compute  $P(X < Y)$ .

- (d) Now suppose both light bulbs have been running for 10 days straight. How does this change  $P(X < Y)$ ?

**Problem 2.** [20 points] Suppose that  $X \sim \text{Geom}(p)$  and  $Y \sim \text{Geom}(r)$  are independent. Find the probability  $P(X < Y)$ .