Homework 7

MATH 201 (Summer 2023, Session A2)

Monday 5th June, 2023

Instructions

- This homework is due on Sunday, June 11th at 11 PM Eastern Time.
- Justify your answers.
- Late submissions are not permitted unless there are extenuating circumstances.
- Please read the honesty policy of the course (available on the course webpage) and make sure you understand the collaboration policy.

Problem 0. [0 points] Copy paste the following text in the beginning of your submission:

> This submission conforms to the honesty policy of the course. In particular, I have not made use of any unauthorized online resources and any collaboration did not violate the expectations outlined in the policy.

After that, list all students you collaborated with, clearly indicating which problems you worked with them on. If you did not collaborate with anyone, clearly state this instead.

Problem 1. [20 points] Let the random variables X, Y have joint density function

$$f(x,y) = \begin{cases} 3(2-x)y, & \text{if } 0 < y < 1 \text{ and } y < x < 2-y \\ 0, & \text{else.} \end{cases}$$

(a) Find the marginal density functions f_X and f_Y.

(b) Compute $E[XY]$.

(c) Calculate the probability $P(X + Y \leq 1)$.

Problem 2. [30 points] In this question, we will look at an example of a trinomial distribution, and more generally, at multinomial distributions which generalize binomial distributions when there are more than two choices[1]. A professor is teaching a course with n lectures.

[1] i.e., there are more possibilities than just success or failure in each trial

For each lecture, she chooses to use white chalk with probability p_1, yellow chalk with probability p_2 and green chalk with probability p_3. You can assume that she makes the choice independently for each lecture. She has no other types of chalk, and hence,

$$p_1 + p_2 + p_3 = 1.$$

Let W, Y, G be the number of white, yellow and green chalk respectively used over the course of the semester. What is the joint p.m.f. of (W, Y, G)? Express it using the trinomial coefficient,

$$\binom{n}{k_1, k_2, k_3} = \binom{n}{k_1}\binom{n-k_1}{k_2} = \frac{n!}{k_1!k_2!k_3!},$$

where $k_1 + k_2 + k_3 = n$.
The subsequent semester, she buys new chalk that is red in color. If the probabilities are now q_1 for white, q_2 for yellow, q_3 for green and q_4 for red, with

$$q_1 + q_2 + q_3 + q_4 = 1,$$

and R is the number of red chalk used, then what is the joint distribution of (W, Y, G, R)? Express it using the multinomial coefficient,

$$\binom{n}{k_1, k_2, \cdots, k_j} = \binom{n}{k_1}\binom{n-k_1}{k_2}\binom{n-k_1-k_2}{k_3}\cdots\binom{n-k_1-\cdots-k_{j-1}}{k_j} = \frac{n!}{k_1!k_2!\cdots k_j!},$$

where $k_1 + k_2 + \cdots + k_j = n$.