Homework 8

MATH 201 (Summer 2023, Session A2)

Sunday 11^{th} June, 2023

Instructions

- This homework is due on Thursday, June 15th at 11 PM Eastern Time.
- Justify your answers.
- Late submissions are not permitted unless there are extenuating circumstances.
- Please read the honesty policy of the course (available on the course webpage) and make sure you understand the collaboration policy.

Problem 0. [0 points] Copy paste the following text in the beginning of your submission:

> This submission conforms to the honesty policy of the course. In particular, I have not made use of any unauthorized online resources and any collaboration did not violate the expectations outlined in the policy.

After that, list all students you collaborated with, clearly indicating which problems you worked with them on. If you did not collaborate with anyone, clearly state this instead.

Problem 1. [30 points] There are 7 students stepping into the elevator on the ground floor, G of the Hylan building which has floors numbered $\{G, 1, 2, ..., 11\}$. Each one of the students needs to get to a floor which is chosen independently and uniformly at random from $\{1, ..., 11\}$ (more than one student may get off on a given floor).

(a) Let N be the number of floors that the elevator stopped in (not counting G where they got on) until the last student stepped out. Find $\mathrm{E}[N]$.

(b) Let X_j be the number of students that stepped out on floor j. Find $\mathrm{E}[X_j]$ for each j.

(c) Find the covariance between X_1 and X_2.

Problem 2. [20 points] Suppose that a professor randomly picks a student in a class of 50 students (40 Americans and 10 non-Americans) to perform a calculation on the board. The professor does this 20 times, choosing a new student each time (i.e. no student goes twice). Let X be the total number of Americans chosen to do a calculation. Calculate the mean and variance of X.

Problem 3. [10 points]
Let X be a continuous random variable which is symmetric around 0, let $f : \mathbb{R} \to \mathbb{R}$ be any continuous and even function, and let $Y = f(X)$. That is, if $p_X(x)$ is p.d.f. of X, then

$$p_X(x) = p_X(-x),$$

and

$$f(x) = f(-x),$$

for every $x \in \mathbb{R}$. Show that $\mathrm{Cov}(X, Y) = 0$. Are X and Y independent?