

Homework 9

MATH 201 (Summer 2023, Session A2)

Sunday 11th June, 2023

Instructions

- This homework is due on Wednesday, June 21st at 11 PM Eastern Time.
- Justify your answers.
- Late submissions are not permitted for this homework. If you are unable to submit this homework on time due to extenuating circumstances, please contact the instructor and ask to set up an incomplete contract.
- Please read the honesty policy of the course (available on the course webpage) and make sure you understand the collaboration policy.

Problem 0. [0 points] Copy paste the following text in the beginning of your submission:

This submission conforms to the honesty policy of the course. In particular, I have not made use of any unauthorized online resources and any collaboration did not violate the expectations outlined in the policy.

After that, list all students you collaborated with, clearly indicating which problems you worked with them on. If you did not collaborate with anyone, clearly state this instead.

Problem 1. [20 points] Let $X \sim \text{Bin}(n, p)$.

- (a) Compute the moment generating function of X .
- (b) Compute $E[X^3]$.

Problem 2. [25 points] Suppose that X is a nonnegative random variable with $E[X] = 10$.

- (a) Give an upper bound on the probability that X is larger than 15.
- (b) Suppose that we also know that $\text{Var}(X) = 3$. Give a better upper bound on $P(X > 15)$ than in part (a).
- (c) Suppose that Y_1, Y_2, \dots, Y_{300} are independent and identically distributed (i.i.d.) random variables with the same distribution as X so that, in particular, $E[Y_i] = 10$ and $\text{Var}(Y_i) = 3$. Estimate the probability that $\sum_{i=1}^{300} Y_i$ is larger than 3030.

Problem 3. [15 points] Nate is a competitive eater specializing in eating hot dogs. The probability distribution for the time it takes Nate to eat a hotdog is unknown, but it need not be normally distributed. What we can say from past experience is that it takes him on average 15 seconds to consume one hot dog, with a standard deviation of 4 seconds. In this year's hot dog eating contest he hopes to consume 64 hot dogs in just 15 minutes. Use the CLT to approximate the probability that he achieves this feat.