

Math 201: Introduction to Probability

Midterm 2

November 21, 2017

NAME (please print legibly): _____

Your University ID Number: _____

Instructions:

1. Indicate your instructor with a check in the appropriate box:

Krishnan	MW 2:00	
Herman	MW 10:25	

2. Read the notes below:

- The presence of any electronic or calculating device at this exam is strictly forbidden, including (but not limited to) calculators, cell phones, and iPods.
- Notes of any kind are strictly forbidden.
- Show work and justify all answers. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.
- You do not need to simplify complicated numerical expressions.
- You are responsible for checking that this exam has all 11 pages.

3. Read the following Academic Honesty Statement and sign:

I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.

Signature: _____

QUESTION	VALUE	SCORE
1	20	
2	20	
3	12	
4	12	
5	20	
6	16	
TOTAL	100	

1. (20 points) Let X be a random variable with probability density given by:

$$f(x) = \begin{cases} k x e^{-4x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

where k is a constant.

(a) Find the constant k .

(b) What is the probability of the event “ $X = 10$ ”?

(c) Find the cumulative distribution function $F(t)$ for the variable X .

(d) What is the probability of the event “ X lies between 3 and 4”?

2. (20 points) A pollster wants to determine the support candidate A has in a population of a very large number of potential voters. She conducts a poll of 10000 randomly selected people from the population, where 4870 people say that they will vote for the candidate.

(a) Let p be the true proportion of voters in the entire population who support candidate A. Express in terms of p the Binomial probability that exactly 4870 people out of the 10000 polled support candidate A.

(b) Give a 95% confidence interval for the true proportion of people who support candidate A.

(c) Suppose candidate A wins the election by winning a negligible amount more than 50% of the vote. Calculate an approximate probability that the poll would produce a result where at most 4870 candidates would say they support candidate A.

3. (12 points) Suppose the number of insurance claims a company receives in a year is modeled by a Poisson distribution. On average the company sees 10 claims a year.

(a) What is the standard deviation of the number of claims that the company receives?

(b) The company has reserves that let it pay 25 claims in a year. Give an expression for the probability that the company goes bankrupt.

4. (12 points) The time you have to wait for a bus once you've arrived at the bus stop is exponentially distributed with a mean waiting time of 10 minutes.

(a) What is the probability that you wait more than 20 minutes for the bus?

(b) Suppose that the bus is 10 minutes late and has still not arrived. What is the probability that you will have to wait 10 more minutes for the bus to arrive?

5. (20 points) Let (X, Y) be random variables with joint probability mass function $p_{X,Y}$ given by

$$\begin{array}{lll} p_{X,Y}(-1, -3) = 0.1 & p_{X,Y}(-1, 2) = 0.1 & p_{X,Y}(-1, 6) = 0 \\ p_{X,Y}(4, -3) = 0.2 & p_{X,Y}(4, 2) = 0.2 & p_{X,Y}(4, 6) = 0.1 \\ p_{X,Y}(10, -3) = 0.1 & p_{X,Y}(10, 2) = 0 & p_{X,Y}(10, 6) = 0.2 \end{array}$$

(a) Find $P(XY < 0)$.

(b) Find $P(Y = -3)$.

(c) Find $E(Y)$.

(d) Find $M_X(t)$, the moment generating function of X .

6. (16 points) Let X be a random variable with moment generating function

$$M_X(t) = \frac{1}{(1 - 2t)^4} \text{ defined for } t < \frac{1}{2}.$$

(a) Find $E(X)$ and $Var(X)$.

(b) Suppose $Y = 5X^{50} + 2$. Find $E(Y)$ and $Var(Y)$.

Table of values for $\Phi(x)$

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998