Math 201: Introduction to Probability

Midterm 1

February 28, 2019

NAME (please print legibly): ______________________________

Your University ID Number: ______________________________

Instructions:

1. Indicate your instructor with a check in the appropriate box:

Krishnan	MW 14:00	
Tucker	MW 10:25	

2. Read the notes below:

   - **The presence of any electronic or calculating device at this exam is strictly forbidden, including (but not limited to) calculators, cell phones, and iPods.**
   - **Notes of any kind are strictly forbidden.**
   - **Show work and justify all answers. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.**
   - **You do not need to simplify complicated numerical expressions such as** $\binom{100}{30}$ **and** $50!$ **to a number.**
   - **You are responsible for checking that this exam has all 12 pages.**

3. Read the following Academic Honesty Statement and sign:

   I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.

   Signature:______________________________

QUESTION	VALUE	SCORE
1	15	
2	15	
3	15	
4	15	
5	10	
6	10	
7	20	
TOTAL	100	

1. (15 points) Three balls are chosen without replacement from an urn that contains 2 red, 2 green, and 2 yellow balls (for a total of six balls).

(a) What is the chance that of the three balls chosen, exactly two are green?

(b) What is the chance that all three balls are the same color?

(c) What is the chance that all three balls are different colors?

2. (15 points) Suppose we have subsets A, B, and C of a sample space such that $P(A) = .5$, $P(B) = .2$, $P(C) = .7$, $P(A \cup B) = .6$, and $P(A \cup C) = .7$.

(a) Calculate $P(A \cap B)$

(b) Are A and B independent?

(c) Are A and C independent? (Recall the data from the previous page: $P(A) = .5$, $P(B) = .2$, $P(C) = .7$, $P(A \cup B) = .6$, and $P(A \cup C) = .7$.)

3. (15 points) Suppose Ram and Laxman play a game: they take turns shooting arrows at a bullseye. Ram goes first, and if he misses, Laxman goes next. If Laxman misses as well, the round ends, and the next round begins where Ram again goes first. Ram hits the bulls eye with probability 1/3, and Laxman hits with probability 1/2. The game ends when one of them hits the bullseye. Let X be the total number of rounds played.

(a) What kind of random variable is X? Name its distribution and give its parameter values. Also write down the pmf (probability mass function) of X.

4. (15 points) There are 30,000 offshore oil rigs in North America, and about 50 rig inspectors in the Minerals Management Service, a government oversight agency. Being severely understaffed, the agency can only survey a randomly selected group of 1,000 rigs annually. Given that a rig is inspected, there is a 1 percent (0.01 probability) that it will have a catastrophic failure the following year. If it's not inspected, there is a 5 percent chance that it will fail. Label the rigs with numbers $\{1, 2, \ldots, 30,000\}$.

(a) Let I_1 be the event the rig number 1 is inspected. What is $P(I_1)$?

(b) Let E_1 be the event that rig number 1 will fail. What is $P(E_1)$?

(c) What is the probability that rig number 1 was inspected given that it failed the following year? *Hint: express the probability first in terms of E_1 and I_1.*

5. (10 points) Consider the square in the plane consisting of all (x, y) such that $-1 \leq x, y \leq 1$, and let Q be a point chosen uniformly at random inside the square. Let X be the distance from Q to $(0, 0)$. Calculate $P(X > 1)$.

6. (10 points) A random sample (without replacement) of 1000 Americans are polled about their voting preferences. They vote either Democrat or Republican. Democrats make up 49% of the total number of voters. The total number of voters is 150 million. Give an expression for the probability that exactly 550 people in the sample of 1000 people are Democrats.

7. (20 points) Jane must get at least three of the four problems on the exam correct to get an A. She has been able to do 80% of the problems on old exams, so she assumes that the probability she gets any problem correct is 0.8. She also assumes that the results on different problems are independent. Let X describe the number of problems she gets correct on the exam.

(a) Write down the pmf (probability mass function) for X.

(b) What is the probability she gets an A?

(c) If she gets the first problem correct, what is the probability she gets an A?