

MTH 201

Midterm 2

April 9, 2020

Name: _____

UR ID: _____

Circle your Instructor's Name:

Ian Alevy

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Instructions:

- THIS IS NOT TRUE The presence of calculators, cell phones, and other electronic devices at this exam is strictly forbidden. Notes or texts of any kind are strictly forbidden.
- For each problem, please put your final answer in the answer box. We will judge your work outside the box as well (unless specified otherwise) so you still need to show work and justify your answers. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.
- In your answers, you do not need to simplify arithmetic expressions like $\sqrt{5^2 - 4^2}$ and you can leave your answers in terms of $\binom{n}{k}$ or $k!$. However, known values of functions should be evaluated, for example, $\ln e, \sin \pi, e^0$. Summations must also be evaluated, in particular, the symbols “ \sum ” or “ \dots ” should not appear in final answers.
- This exam is out of 50 points. You are responsible for checking that this exam has all 10 pages.

PLEASE COPY THE HONOR PLEDGE AND SIGN:

I affirm that I will not give or receive any unauthorized help on this exam, and all work will be my own.

YOUR SIGNATURE: _____

1. (10 points) Factories in Albany and Buffalo both produce the same type of gadget. On average 15% of the gadgets produced by Albany are mildly defective (they use cheap parts) and 3% of the gadgets produced by Buffalo are mildly defective. The NY State inspectors visit each factory and take a random sample of 100 gadgets. If more than 8 gadgets from the sample are mildly defective, they will shutdown production. Let A be the event that the Albany factory gets shutdown and let B be the event that the Buffalo factory gets shutdown.

(a) Give an exact expression for $P(A)$, without attempting to evaluate it.

(b) Determine whether the normal or the Poisson approximation is appropriate for approximating $P(A)$, justifying your answer.

(c) Use either the normal or the Poisson approximation, whichever is appropriate, to give an approximation of $P(A)$.

(d) Determine whether the normal or the Poisson approximation is appropriate for approximating $P(B)$, justifying your answer.

(e) Use either the normal or the Poisson approximation, whichever is appropriate, to give an approximation of $P(B)$.

2. (6 points) WeLovePolls.com recently conducted a poll of $n = 1000$ likely voters asking whether they intend to vote for candidates Boe Jiden or Sernie Banders in the upcoming election. 53% said they will vote for Jiden, 47% said they will vote for Banders. WeLovePolls.com reports that the confidence interval for p , the true population proportion of voters that intend to vote for Jiden on election day, is $(0.50, 0.56)$. What is the confidence level for the reported confidence interval?

3. (9 points) Let $Z \sim \mathcal{N}(0, 1)$ and $X \sim \mathcal{N}(\mu, \sigma^2)$. This means that Z is a standard normal random variable with mean 0 and variance 1, while X is a normal random variable with mean μ and variance σ^2 . Note: for each part, you will not earn points if you do not use the method indicated.

(a) Use integration by parts to show the reduction formula:

$$\int_{-\infty}^{\infty} x^n e^{-x^2/2} dx = \int_{-\infty}^{\infty} (n-1)x^{n-2} e^{-x^2/2} dx \quad \text{for } n \geq 2.$$

(b) Use the formula from part (a) to show $E(Z^4) = 3$ and $E(Z^3) = 0$.

(c) Use the result in part (b) to show $E(X^4) = 3\sigma^4 + 6\sigma^2\mu^2 + \mu^4$.

4. (8 points) Let $X \sim \text{Exp}(\lambda)$.

(a) Compute $E[3X + 5]$.

(b) Compute $\text{Var}[3X + 5]$.

5. (9 points) Let X be a random variable with CDF

$$F(x) = \begin{cases} 0 & \text{if } x < 0, \\ x^2 & \text{if } 0 \leq x < 1/2, \\ x/2 & \text{if } 1/2 \leq x < 2 \\ 1 & \text{if } 1 \leq x, \end{cases}$$

(a) Compute the PDF of X .

(b) Compute $E[X]$.

(c) Compute $\text{Var}(X)$.

6. (8 points) You are tasked with operating a machine in a factory which malfunctions every X hours where X is a random variable with exponential distribution. You are told that on average the machine malfunctions every 6 hours. Your shift lasts 8 hours. If the machine malfunctions you are sent home early and paid \$20 for your day of work. If the machine does not malfunction you earn \$100 for your day of work. If the machine is functioning when you arrive how much do you expect to earn for the day?

EXTRA PAGE. You may use this page if you run out of space. Be sure to label your problems on this page and also include a note on the original page telling the graders to look for your work here.