MTH 201

Midterm 2

April 9, 2020

Name: ______________________________

UR ID: ______________________________

Circle your Instructor's Name:

Ian Alevy   Mark Herman

Instructions:

• THIS IS NOT TRUE .... The presence of calculators, cell phones, and other electronic devices at this exam is strictly forbidden. Notes or texts of any kind are strictly forbidden.

• For each problem, please put your final answer in the answer box. We will judge your work outside the box as well (unless specified otherwise) so you still need to show work and justify your answers. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.

• In your answers, you do not need to simplify arithmetic expressions like $\sqrt{5^2 - 4^2}$ and you can leave your answers in terms of $\binom{n}{k}$ or $k!$. However, known values of functions should be evaluated, for example, $\ln e, \sin \pi, e^0$. Summations must also be evaluated, in particular, the symbols "$\sum$" or "$\cdots$" should not appear in final answers.

• This exam is out of 50 points. You are responsible for checking that this exam has all 10 pages.

PLEASE COPY THE HONOR PLEDGE AND SIGN:

I affirm that I will not give or receive any unauthorized help on this exam, and all work will be my own.

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YOUR SIGNATURE:______________________________

1. (10 points) Anne and Bob both enjoy swimming. Every day they each decide whether or not to go swimming independently from previous days and independently from each other. On average Anne goes swimming 1% of days and Bob goes swimming 20% of days. Let A be the event that Anne goes swimming less than 7 days in the next year. Let B be the event that Bob goes swimming less than 65 days in the next year. Assume there are 365 days in a year.

(a) Give an exact expression for $P(A)$, without attempting to evaluate it.

(b) Determine whether the normal or the Poisson approximation is appropriate for approximating $P(A)$, justifying your answer.

(c) Use either the normal or the Poisson approximation, whichever is appropriate, to give an approximation of $P(A)$.

(d) Determine whether the normal or the Poisson approximation is appropriate for approximating $P(B)$, justifying your answer.

(e) Use either the normal or the Poisson approximation, whichever is appropriate, to give an approximation of $P(B)$.

2. (8 points) You are tasked with operating a machine in a factory which malfunctions every X hours where X is a random variable with exponential distribution. You are told that on average the machine malfunctions every 8 hours. Your shift lasts 8 hours. If the machine malfunctions you are sent home early and paid \$10 for your day of work. If the machine does not malfunction you earn \$100 for your day of work. If the machine has been functioning for 8 hours when you arrive how much do you expect to earn for the day?

3. (9 points) Let X be a random variable with CDF

$$F(x) = \begin{cases} 0 & \text{if } x < 0, \\ x^3 & \text{if } 0 \leq x < 1/2, \\ x/4 & \text{if } 1/2 \leq x < 4 \\ 1 & \text{if } 4 \leq x, \end{cases}$$

(a) Compute the PDF of X.

(b) Compute $E[X]$.

(c) Compute $\mathrm{Var}(X)$.

4. (8 points) Let $X \sim \mathrm{Bin}(n, p)$.

(a) Compute $E[2X+3]$.

(b) Compute $\mathrm{Var}[2X+3]$.

5. (9 points) Let $Z \sim \mathcal{N}(0,1)$ and $X \sim \mathcal{N}(\mu,\sigma^2)$. This means that Z is a standard normal random variable with mean 0 and variance 1, while X is a normal random variable with mean μ and variance σ^2. Note: for each part, you will not earn points if you do not use the method indicated.

(a) Use integration by parts to show the reduction formula:

$$\int_{-\infty}^{\infty} x^n e^{-x^2/2}\,dx = \int_{-\infty}^{\infty} (n-1)x^{n-2}e^{-x^2/2}\,dx \quad \text{for} \quad n \geq 2.$$

(b) Use the formula from part (a) to show $E(Z^4) = 3$ and $E(Z^3) = 0$.

(c) Use the result in part (b) to show $E(X^4) = 3\sigma^4 + 6\sigma^2\mu^2 + \mu^4$.

6. (6 points) An important engine part is built and shipped from the Buffalo Truck Parts Factory. Before a large shipment goes out, the quality control manager Alice needs to estimate p, the true proportion of parts in the shipment that are mildly defective. Alice takes a sample of $n = 400$ parts and finds that 5% are mildly defective. She then tells her boss that she has 80% confidence that p lies in a confidence interval (a, b), but she has forgotten what a and b are. What is the interval (a, b)?

EXTRA PAGE. You may use this page if you run out of space. Be sure to label your problems on this page and also include a note on the original page telling the graders to look for your work here.