MTH 201

Midterm 1

March 12, 2020

Name: ____

UR ID: ____

Circle your Instructor's Name:

Ian Alevy   Thomas Tucker

Instructions:

• The presence of calculators, cell phones, and other electronic devices (other than a device for being on Zoom during the exam and uploading your exam afterwards) at this exam is strictly forbidden. Notes or texts of any kind are strictly forbidden.

• In your answers, you do not need to simplify arithmetic expressions like $\sqrt{5^2 - 4^2}$ and you can leave your answers in terms of $\binom{n}{k}$ or $k!$. However, known values of functions should be evaluated, for example, $\ln e, \sin \pi, e^0$. Summations must also be evaluated, in particular, the symbols "$\sum$" or "$\cdots$" should not appear in final answers.

• This exam is out of 50 points. You are responsible for checking that this exam has all 10 pages.

PLEASE COPY THE HONOR PLEDGE AND SIGN:

I affirm that I will not give or receive any unauthorized help on this exam, and all work will be my own.

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YOUR SIGNATURE:____

1. (5 points) For this problem, justification is not required and partial credit will **not** be awarded.

Suppose an urn contains 5 red balls, 7 green balls and 9 orange balls. Five balls are drawn randomly one at a time without replacement from the urn.

(a) 2 pt: What is the probability that the sample contains exactly 3 red balls?

(b) 3 pts: What is the probability that the sample contains at least one red ball **and** one green ball?

2. (8 points) Peter and Mary take turns rolling a fair die. If Peter rolls 1 he wins and the game stops. If Mary rolls 3 or 6, she wins and the game stops. They keep rolling in turn until one of them wins. Suppose Peter rolls first.

(a) 2 pts: What is the probability that Mary wins on her fifth roll?

(b) 6 pts: What is the probability that Mary wins? (To receive full credit, you must evaluate any infinite series in your answer.)

3. (8 points) Consider events A, B, and C which are mutually independent (recall that this means that A and B are independent, A and C are independent, B and C are independent, and that $P(A \cap B \cap C) = P(A)P(B)P(C)$) with $P(A) = 1/2$, $P(B) = 1/4$ and $P(C) = 1/2$.

(a) 3 pts: Compute $P(A \cup B)$.

(b) 5 pts: Are the events $A \cup B$ and C independent? Explain your answer carefully or no credit will be given.

4. (8 points) A fair coin is flipped four times. Let A be the event that tails comes up at least three times. Let B be the event that the first three flips are tails.

(a) 3 pts: Find $P(A)$.

(b) 5 pts: Find the conditional probability $P(B \mid A)$.

5. (8 points)

The continuous random variable X is uniformly distributed on the interval $[-2, 5]$.

(a) 4 pts: Find $P(X < 0)$.

(b) 4 pts: Find $P(X^2 > 1)$.

6. (6 points)

Three numbers are chosen with replacement from the set $\{1, 2, 3\}$.

(a) 2 pts: Find the chance that no number is chosen twice.

(b) 4 pts: Find the chance that at least two different numbers are chosen.

7. (7 points) There are three types of coins in circulation. There are fair coins with $P(H) = 1/2$, moderately biased coins with $P(H) = 1/3$ and heavily biased coins with $P(H) = 1/5$. Suppose $1/2$ of the coins are fair, $1/4$ are moderately biased and $1/4$ are heavily biased. A coin is flipped twice and the outcome is heads followed by tails. What is the probability that the coin is fair? **You may leave your answer as a fraction.**