MTH 201

Final Exam

June 25, 2020

Name: ____

Instructions:

- The presence of calculators, cell phones, and other electronic devices at this exam is strictly forbidden. Notes or texts of any kind are strictly forbidden.
- You must be in the instructor's zoom room for the duration of the exam with your camera turned on and in view of your camera. The zoom session will be recorded; you can find the zoom link in course materials in blackboard.
- You will need to view the exam on gradescope while you are taking it; other than this and zoom, you may not have any files or windows on any computer/phone during the exam. The instructor may ask you to share your screen at any time during the exam, so be sure to close blackboard and all other files/windows before we start. Your work area must be clear of all items/materials during the exam, including a keyboard, or any electronic devices. The only exception is blank sheets of paper, pens/pencils, and perhaps a computer mouse to view the exam pages.
- When you are done with your exam, please get the attention of the instructor before starting your scanning process. You must get the instructor's permission before you use any device to start scanning.

- Please start every problem on a separate sheet of paper. Subparts must be written in correct order and clearly labeled. You will upload one pdf file and select pages to correspond to problems, similar to how you submit homework assignments.
- Show your work! You may not receive full credit for a correct answer if insufficient work or insufficient justification is given. In your answers, you do not need to simplify arithmetic expressions like $\sqrt{5^2 - 4^2}$. However, known values of functions should be evaluated, for example, $\ln e, \sin \pi, e^0$. Be sure to include units when applicable!
- The exam is 3 hours and is worth 100 points. You must start the scanning and upload process immediately after; no writing will be allowed after the exam time has expired.
- If you have technical issues try not to panic and contact your professor immediately so we can help you through these issues.
- When applicable, you may leave answers in terms of $\Phi(a)$, the cumulative distribution function of the standard normal distribution.

PLEASE COPY THE HONOR PLEDGE AND SIGN:

I affirm that I will not give or receive any unauthorized help on this exam, and all work will be my own.

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YOUR SIGNATURE:______________________________

1. (20 points) This problem is a bit long, tests counting and geometric rv 12 people have gathered together in at the council of Elrond, a fictional meeting. They want to pick a band of 7 people to help throw a ring into a volcano. Since Elrond, the defacto leader of the meeting, studied probability at UR under Professors Herman and Krishnan, they decide that they will pick a group of 7 uniformly at random.

In how many ways can you pick a band of 7 people?

It's known that Boromir and Frodo (2 people present at the meeting) do not trust each other. What is the probability that they will be picked in the band of 7?

Frodo says that he will only go if Sam goes along with him. So the council decides to modify the selection in the following way. If only one of them is picked, then that band of 7 is discarded, and the selection procedure is repeated.

What is the probability that Frodo and Sam are chosen in the band?

On average, how many times will the selection procedure have to be repeated before a successful band of 7 is chosen?

Given that Frodo and Sam are selected, what is the probability that the selection procedure was repeated 5 times.

2. (20 points) random variables, joint density Consider the uniform measure on the semi-circular region centered at the origin: $\{x^2 + y^2 \leq 1, \text{ and } y > 0\}$. We pick a point uniformly at random in it.

Draw a picture of the region in which the point falls.

What is the ranges of the X and Y coordinates?

What is the probability density function $f_{X,Y}(x, y)$. *Hint: it is a piecewise function. Use the area of the semicircular region.*

Draw a picture of the region $X \leq s$ for some $s \in (-1, 1)$.

Compute the pdf f_X by integrating $f_{X,Y}(s, t)$ over t. *Hint:* $\int \sqrt{1 - x^2} = (x\sqrt{1 - x^2} + \sin^{-1}(x))/2$

Compute the pdf f_Y by integrating $f_{X,Y}(s, t)$ over s.

Are X and Y independent? Justify your answer.

3. (20 points) redsimple test of convolution, expectation and variance of indep random variables Let X the outcome of a two-sided die, and let Y represent the number obtained by independently rolling a three-sided die. The ranges of X and Y are $\{1, 2\}$ and $\{1, 2, 3\}$ respectively.

Find the probability mass function of $2Y$?

What is the average of $X + 2Y$?

Find the variance of $X + 2Y$.

Find the probability mass function of $X + 2Y$, $p_{X+2Y}(t)$ for all possible values of t.

4. (20 points) Students in two sections of a probability class (professors H. and K. no relation) get the following two sets of average scores:

Section	Mean	Variance
1	40	15
2	50	25

Suppose picking a student score at random from one of the sections $i = 1, 2$ is well-approximated by a Gaussian random $X_i \sim N(\mu, \sigma)$ with the appropriate mean and variance. We pick one random student from each section. Let their scores be X_1 and X_2.

Express the average score of the two randomly selected students as a function of $X_1 + X_2$. Call this *random variable* Y.

What is the expected average score of the two students?

Suppose the two instructors don't talk to each other, and hence the scores in the two sections are independent.

Find the variance of Y assuming independence.

Find the distribution of Y assuming independence.

We call a student strong if they score one standard deviation above the average score in their respective section. What is the probability that both of the students that we selected are strong?