MTH 201

Midterm

June 4, 2020

Name: ________________________________

Instructions: Instructions on gradescope....

PLEASE COPY THE HONOR PLEDGE AND SIGN:

I affirm that I will not give or receive any unauthorized help on this exam, and all work will be my own.

YOUR SIGNATURE:________________________________

QUESTION	VALUE	SCORE
1	6	
2	10	
3	8	
4	9	
5	7	
6	10	
TOTAL	50	

1. (6 points) Suppose an urn contains twenty chips labeled $1, \cdots, 20$. Ten of the chips are red, seven are blue, and three are green. The chips are drawn randomly one at a time without replacement until the urn is empty.

(a) What is the probability that the 20th draw is the chip labeled with the number 5?

(b) What is the probability that there are at least two red chips among the first 6 draws?

(c) What is the probability that the first three draws all have different colors?

2. (10 points) Alice, Betty, and Cathy are playing a dice game at the *Pittsford Casino and Pool Hall*. In each round, the host rolls a fair die. If a 1, 2, or 3 is rolled then nobody wins the round. If a four is rolled, Alice wins \$300. If a five is rolled, Betty wins \$300. If a six is rolled, Cathy wins \$300. There are 20 rounds to the game.

(a) Let X be the total amount of money Alice wins in the entire game (including all 20 rounds). What type of variable is X (i.e. what type of distribution) and what are its parameters?

(b) Find $E[X]$.

(c) Find the probability that all players win at least \$300.

Hint: You might need the inclusion-exclusion formula

$$\mathbb{P}(\cup_{i=1}^n A_i) = \sum_{k=1}^n (-1)^{k+1} \sum_{1 \le i_1 < \cdots < i_k \le n} \mathbb{P}(A_{i_1} \cap \cdots \cap A_{i_k}).$$

3. (8 points) You are playing soccer (also called "football") next to a river. Each time you kick the ball there is 30% probability it goes into the goal and you score 1 point, 50% probability you miss the goal (worth 0 points) but do not lose the ball, and a 20% probability the ball falls into the river and is lost forever. You kick the ball repeatedly until it is lost. Let X be the total number of points you score before the ball is lost.

(a) Calculate $P(X > 0)$.

(b) Calculate $E[X]$.

4. (9 points) Let X be a random variable with probability density given by:

$$f(x) = \begin{cases} \frac{C}{x^5}, & x > 1 \\ 0, & \text{otherwise} \end{cases}$$

where C is a constant.

(a) (2pts) Find the constant C.

(b) (1pt) What is the probability of the event "$X = 3$"?

(c) (2pts) Find the cumulative distribution function of X.

(d) (2pts) What is the probability of that X lies between $\frac{1}{2}$ and 4 given that $X > 3$?

(e) (2pts) Find $E[X]$ and $\text{Var}(X)$.

5. (7 points) You are playing a game where you have to guess on which month your friend is born. Suppose all months are equally likely. Your friend tells you that the month is either February, November, or December. Moreover, your friend randomly chooses a number k between 1 and 8 and reveals the k'th letter of the month. If your friend doesn't tell you k but tells you that the k'th letter is "r", what is the probability your friend was born in December?

6. (10 points) Roger Waters is running in an election in a heavily populated state. *Polls 'R Us Inc.* wants to determine the level support for Roger among the population of likely voters. They conduct a poll of 10000 randomly selected people from the population, where 4900 people say that they will vote for Roger. Note: recall that for large n, if $S_n \sim \text{Bin}(n,p)$, then $P(|\frac{S_n}{n} - p| < \epsilon) \geq 2\Phi(2\epsilon\sqrt{n}) - 1$.

(a) Let p be the true proportion of voters in the entire population who support Roger. Express in terms of p the Binomial probability that exactly 4900 people out of the 10000 polled support Roger.

(b) Give a 80% confidence interval for the true proportion of people who support Roger.

(c) Suppose Roger wins the election by winning a negligible amount more than 50% of the vote. Calculate an approximate probability that the poll would produce a result where **at most** 4900 candidates would say they support Roger.

EXTRA PAGE. You may use this page if you run out of space. Be sure to label your problems on this page and also include a note on the original page telling the graders to look for your work here.