MATH 201 (SUMMER 2023, SESH A2)

LECTURE 7 : 05/24/23 (RECORDED ON 05/23)

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LECTURES:
9:00 AM – 11:15 AM (ET)
M, T, W, R

Zoom ID:
979-4693-0650

COURSE WEB PAGE https://people.math.rochester.edu/grads/asahay/summer2023/math201/index.html

ALL PHOTOS TAKEN FROM TEXTBOOK

ANNOUNCEMENTS

1. LECTURE 6, WEEK 2 H.W. IS UPLOADED. (PANOPTO/WEBSITE)
2. OFFICE HOURS: THURS – AFTER CLASS. (11:15 – 12:15)
3. DEADLINES:
   a. WW04 – FRI, MAY 26th
   b. HW03 – SAT, MAY 27th
   c. WW05 – TUES, MAY 30th

   } → TO BE UPLOADED
4. ERROR IN HW 2 – SEE ANNOUNCEMENT ON BLACKBOARD

   ↳ ngrand@ur.rochester.edu

§ 3.3 EXPECTATION

RECALL: $X \to$ R.V.
(p.m.f., p.d.f., c.d.f.)

NOTATION : $\underline{\underline{E}}(X)$, $\mathbb{E}(X)$, μ $\in \mathbb{R}$

ALSO CALLED : (MEAN / AVERAGE / 1st MOMENT)
EXPECTED VALUE

DISCRETE RVs

Definition 3.21. The **expectation** or **mean** of a discrete random variable X is defined by

$$E(X) = \sum_k k \underbrace{P(X = k)}_{\text{p.m.f.}} \tag{3.19}$$

$X \in \{k_1, k_2, k_3, \ldots\}$

where the sum ranges over all the possible values k of X. ♣

Example 3.22. Let Z denote the number from the roll of a fair die. Find $E(Z)$

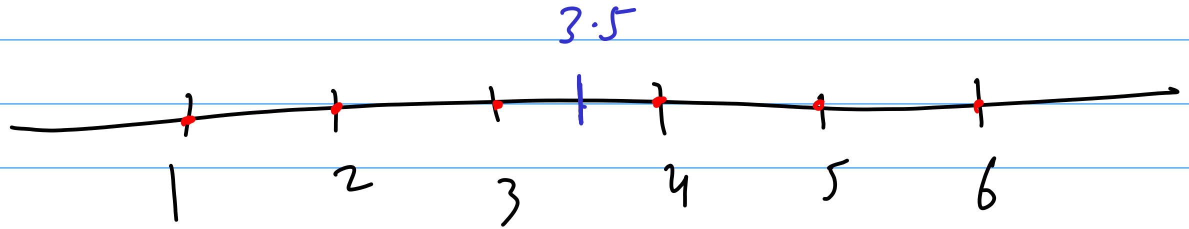
$$Z \in \{1,2,3,4,5,6\}$$

$$P(Z=k) = \begin{cases} 1/6 & \text{IF } k \in \{1,\dots,6\} \\ 0 & \text{o.w.} \end{cases}$$

$$E(Z) = \sum_k k \cdot P(Z=k)$$

$$= 1 \cdot \underbrace{P(Z=1)}_{1/6} + 2 \cdot \underbrace{P(Z=2)}_{1/6} + \cdots + 6 \cdot \underbrace{P(Z=6)}_{1/6}$$

$$= \frac{1}{6}(1+2+\cdots+6) = 21/6 = 7/2 = 3.5$$


3.5
1
2
3
4
5
6

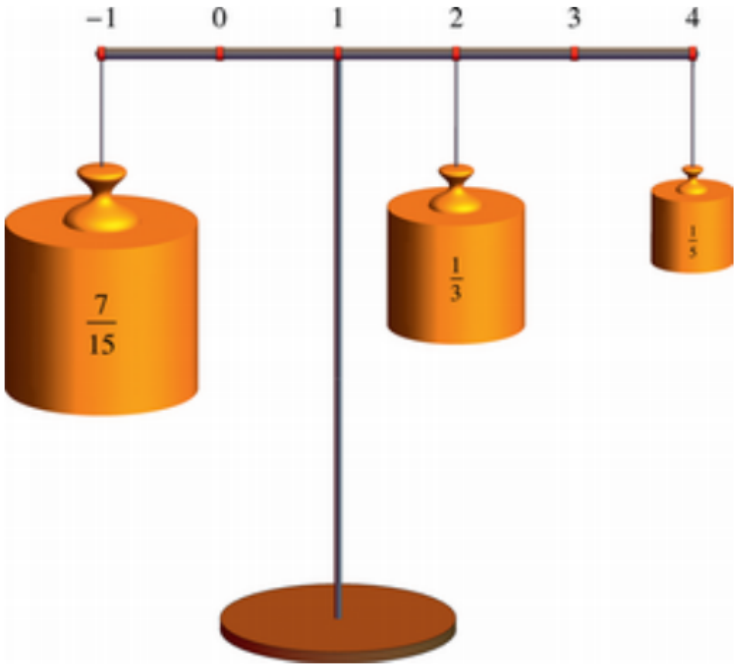


Figure 3.8. The expectation as the center of mass. If X takes the values $-1, 2, 4$ with probabilities $p_X(-1) = \frac{7}{15}$, $p_X(2) = \frac{1}{3}$, $p_X(4) = \frac{1}{5}$ then the expectation is

$$E[X] = -1 \cdot \tfrac{7}{15} + 2 \cdot \tfrac{1}{3} + 4 \cdot \tfrac{1}{5} = 1.$$

Hence the scale in the picture is perfectly balanced.

MEAN OF SOME PREVIOUSLY DEFINED DISTRIBUTIONS

1. $X \sim Ben(p)$ $\qquad X \in \{0, 1\}$

$$P(X=1) = p \quad , \quad P(X=0) = 1-p$$

$$\mathbb{E}(X) = \sum_{k} k \cdot P(X=k) = \cancel{0 \cdot P(X=0)} + 1 \cdot \underbrace{P(X=1)}_{p} = p$$

$$E(X) = p$$

EVENT

SPECIAL CASE : $A \subseteq \Omega$, $I_A \rightarrow$ INDICATOR RANDOM VARIABLE

$$I_A(\omega) = \begin{cases} 1 & \omega \in A \\ 0 & \omega \notin A \end{cases}$$

BERNOULLI w/ $p = P[I_A = 1] = P(\{\omega : \omega \in A\})$

$I_A \in \{0,1\}$ $\qquad = P(A)$

$E(I_A) = p = P(A)$

(2) $X \sim \text{Bin}(n, p)$ $\quad X \in \{0, 1, \dots, n\}$

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$E(X) = \sum_k k \cdot P(X = k) = \sum_{k=0}^{n} k \binom{n}{k} p^k (1-p)^{n-k}$$

$$= \sum_{k=1}^{n} \cancel{k} \cdot \frac{n!}{\cancel{k!}\,(n-k)!} p^k (1-p)^{n-k}$$

$(k-1)!$

$k = 0 \to$ NO CONTRIBUTION

$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!}$$

np

$$= \sum_{k=1}^{n} \frac{n!}{(k-1)\,(n-k)!} p^k (1-p)^{n-k} = np \sum_{k=1}^{n} \frac{(n-1)!}{(k-1)!\,(n-k)!} p^{k-1} (1-p)^{n-k}$$

$$\sum_{k=1}^{n} \frac{(n-1)!}{(k-1)!\,(n-k)!}\, p^{k-1} (1-p)^{n-k} \qquad j = k-1$$

$$= \sum_{j=0}^{n-1} \overbrace{\boxed{\frac{(n-1)!}{j!\,(n-1-j)!}}}^{\binom{n-1}{j}}\, p^{j} (1-p)^{n-1-j}$$

$$= \left[\, p + (1-p) \,\right]^{n-1} = 1$$

$$\boxed{E(X) = np}$$

$$\underbrace{\underset{p}{O}\quad \underset{1}{O}\quad O\quad O\quad O\quad \cdots\quad \underset{n}{O}}_{\text{FORMALIZED}}$$

③ Geom (p)

RECALL : $\dfrac{1}{1-t} = \displaystyle\sum_{k=0}^{\infty} t^k \qquad \left(|t| < 1 \right)$

$$\therefore \sum_{k=1}^{\infty} \underbrace{k \cdot t^{k-1}}_{\frac{d}{dt}(t^k)} = \sum_{k=1}^{\infty} \frac{d}{dt}\left[t^k \right] = \frac{d}{dt}\left(\sum_{k=1}^{\infty} t^k \right)$$

$$= \frac{d}{dt}\left(\frac{1}{1-t} \right) = \frac{1}{(1-t)^2}$$

$$\sum_{k=1}^{\infty} k \cdot t^{k-1} = (1-t)^{-2} \qquad / \; (t = q)$$

$X \sim \text{Geom}(p)$ $\qquad X \in \{1, 2, 3, \ldots\}$

$$P(X = k) = p(1-p)^{k-1} = p\,q^{k-1}$$

$[q := 1 - p]$

$1 - q = p$

$$E(X) = \sum_{k} k \cdot P(X = k)$$

$$= \sum_{k=1}^{\infty} k\,p\,q^{k-1} = p \sum_{k=1}^{\infty} k \cdot q^{k-1} = \frac{p}{(1-q)^2}$$

$$= p/p^2 = \frac{1}{p}$$

RECALL Geom(p)

$$\begin{array}{cccccc} 0 & 0 & 0 & 0 & 0 & \cdots \\ p & p & p & p & p & \end{array}$$

$$E(X) = \frac{1}{p}$$

CONT. R.V.

$$P(X=k)=0 \quad , \; E(X) = \underbrace{\sum_{k}}_{\text{NOT COUNTABLE}} k \cdot \underbrace{P(X=k)}_{0}$$

CONTINUOUS RV

Definition 3.26. The expectation or mean of a continuous random variable X with density function f is

$$E[X] = \int_{-\infty}^{\infty} x \boxed{f(x)\,dx} \tag{3.23}$$

An alternative symbol is $\mu = E[X]$.

$P(X \approx x) = P(X \in (x, x+\epsilon))$

COMPARE

$$\sum_{k} k \cdot P(X=k)$$

HEURISTIC : p.d.f $\simeq$ INFINITESIMAL p.m.f.

(4) $X \sim \text{Unif}[a,b]$

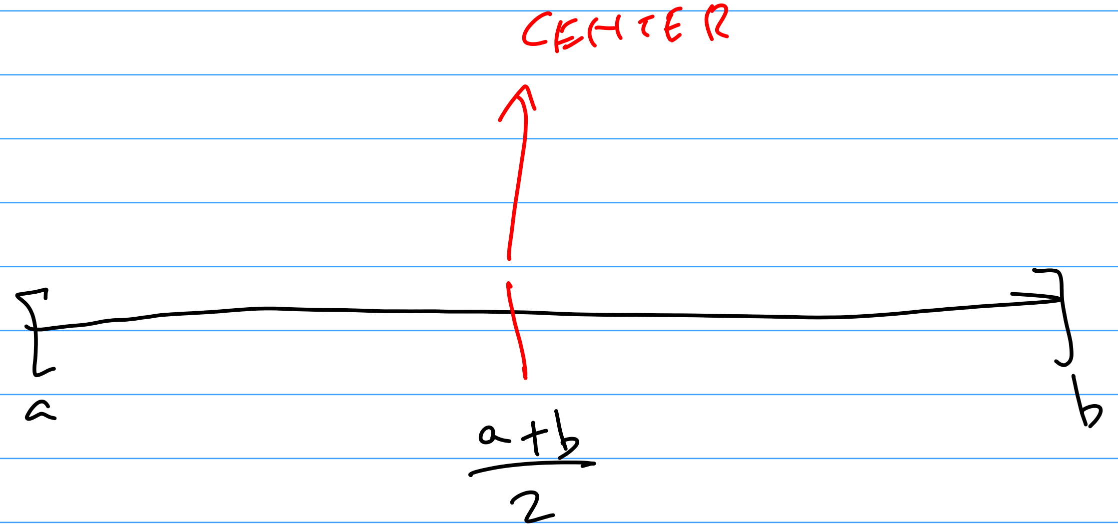
p.d.f. $f(x) = \begin{cases} \dfrac{1}{b-a} & x \in [a,b] \\ 0 & x \notin [a,b] \end{cases}$

$$E(x) = \int_{-\infty}^{\infty} x\, f(x)\, dx = \int_a^b x \left[\frac{1}{b-a}\right] dx$$

$$= \left(\frac{x^2}{2} \cdot \left[\frac{1}{b-a}\right]\right)_{x=a}^{x=b}$$

$$= (b^2 - a^2) / 2(b-a) = \frac{b+a}{2}$$

$X \sim \text{Unif}[a,b]$



Q: DOES $E(X)$ ALWAYS EXIST?

A: NO!

IN FACT, ONE CAN HAVE

$$E(X) = \infty$$

$$E(X) = -\infty$$

OR $E(X) =$ INDETERMINATE

(WILL BE COVERED IN H.W.)

Example 3.32. In Example 1.30 a roll of a fair die determined the winnings (or loss) W of a player as follows:

$$W = \begin{cases} -1, & \text{if the roll is 1, 2, or 3} \\ 1, & \text{if the roll is 4} \\ 3, & \text{if the roll is 5 or 6.} \end{cases}$$

$P(X=-1) = 1/2$
$P(X=1) = 1/6$
$P(X=3) = 1/3$

Let X denote the outcome of the die roll. The connection between X and W can be expressed as $W = g(X)$ where the function g is defined by $g(1) = g(2) = g(3) = -1$, $g(4) = 1$, and $g(5) = g(6) = 3$.

WHAT IS $E(W) = E(g(X))$?

$$E(W) = \sum_{k \in \{-1,1,3\}} k \cdot P(X=3) = (-1)\cdot\frac{1}{2} + 1\cdot\left(\frac{1}{6}\right) + 3\left(\frac{1}{3}\right)$$

$$= \frac{4}{6} = \frac{2}{3}$$

$$W = g(X)$$

$$E(W) = E(g(X))$$

$$= \sum_{k=1}^{6} g(k) \cdot P(X = k)$$

$$= g(1) \cdot \frac{1}{6} + g(2) \cdot \frac{1}{6} + \cdots + g(6) \cdot \frac{1}{6}$$

$$= \frac{-1}{6} - \frac{1}{6} - \frac{1}{6} + \frac{1}{6} + \frac{3}{6} + \frac{3}{6}$$

$$= 4/6 = 2/3$$

Fact 3.33. Let g be a real-valued function defined on the range of a random variable X. If X is a discrete random variable then

$$E[g(X)] = \sum_k g(k)P(X = k) \tag{3.24}$$

while if X is a continuous random variable with density function f then

$$E[g(X)] = \int_{-\infty}^{\infty} g(x)f(x)\,dx. \tag{3.25}$$

Pf: DISCRETE CASE.

$$E(g(x)) = \sum_{\ell} \ell \cdot P(g(X) = \ell)$$

$$g^{-1}(\ell) = \{k : g(k) = \ell\}$$

$$\{g(x) = \ell\} = X \in g^{-1}(\ell)$$

$$P(X \in g^{-1}(l)) = \sum_{k \in g^{-1}(l)} P(X = k)$$

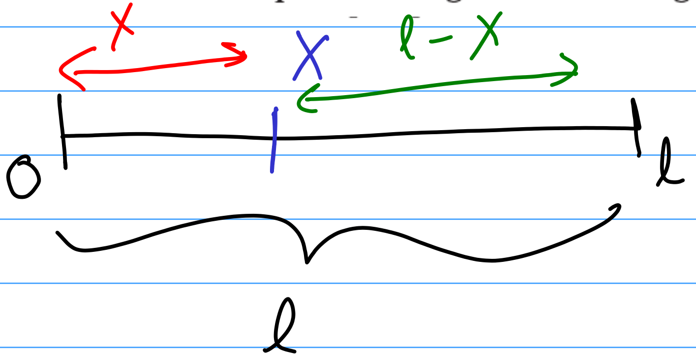
$$= \sum_{k : g(k) = l} P(X = k)$$

$$E(g(X)) = \sum_{l} \sum_{k : g(k) = l} g(k) P(X = k)$$

$$= \sum_{k} g(k) \, P(X = k)$$

$\square$

Example 3.34. A stick of length ℓ is broken at a uniformly chosen random location. What is the expected length of the longer piece?

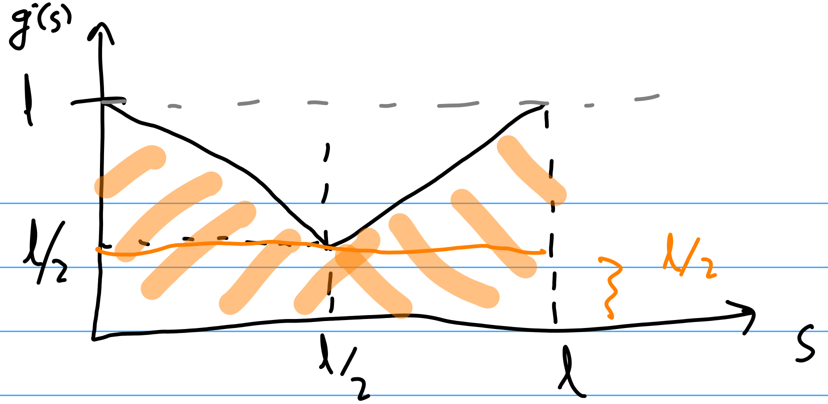


$X \sim \text{Unif}[0, \ell]$

Y = LENGTH OF LONGER PIECE $= \max(X, \ell - X)$

$E(Y) \longrightarrow$ DIRECTLY IS HARD

$Y = g(X) \qquad g(s) = \max(s, \ell - s)$



$$f(x) = \begin{cases} \frac{1}{l} & x \in [0, l] \\ 0 & o.w. \end{cases}$$

$$f = f_X$$

$$E(Y) = E(g(X)) = \int_{-\infty}^{\infty} g(x) \, f(x) \, dx$$

$$= \frac{1}{l} \int_0^l g(x) \, dx \qquad \left(= \int_0^{l/2} + \int_{l/2}^{l} \right)$$

$$= \frac{1}{l} \left[l/2 \cdot l + 2 \left[\frac{1}{2} \cdot \frac{l}{2} \cdot \frac{l}{2} \right] \right)$$

$$= \frac{1}{l}\left[\frac{l^2}{2} + \frac{l^2}{4}\right] = \frac{1}{l}\left[\frac{3l^2}{4}\right] = \frac{3l}{4}$$

c.d.f.

NOT DISCRETE

NOT CONTINUOUS

Example 3.20. Carter has an insurance policy on his car with a \$500 deductible. This means that if he gets into an accident he will personally pay for 100% of the repairs up to $\boxed{\$500}$, with the insurance company paying the rest. For example, if the repairs cost \$215, then Carter pays the whole amount. However, if the repairs cost \$832, then Carter pays \$500 and the remaining \$332 is covered by the insurance company.

Suppose that the cost of repairs for the next accident is uniformly distributed between \$100 and \$1500. Let X denote the amount Carter will have to pay. ~~Find the cumulative distribution function of X.~~

Example 3.38. (Revisiting Example 3.20) Recall from Example 3.20 the case of Carter and the \$500 deductible. Find the expected amount that Carter pays for his next accident.

X = CARTER'S LIABILITY

Y = ACTUAL COST OF ACCIDENT

$$X = \min(Y, 500) = h(Y)$$

$$h(s) = \min(s, 500)$$

$f_Y \to$ p.d.f. of Y.

$$f_Y(x) = \begin{cases} \frac{1}{1400} & x \in [100, 1400] \\ 0 & \text{o.w.} \end{cases}$$

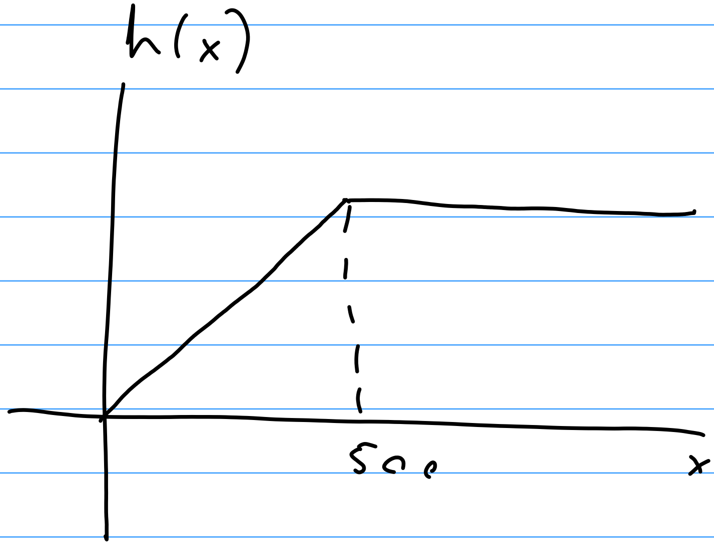
$$\mathbb{E}(X) = \mathbb{E}(h(Y))$$

$$= \int_{-\infty}^{\infty} h(x)\, f_Y(x)\, dx$$

$$= \int_{100}^{1400} \frac{h(x)}{1400}\, dx$$

$$= \int_{100}^{500} \frac{x}{1400}\, dx + \int_{500}^{1400} \frac{500}{1400}\, dx$$

$$= 3100/7 \approx \$\, 442.86$$



$P(X = 500) = 5/7 \to$ BIG

$g(x) = x^n$ X, X^n

$E(X^n)$

nth MOMENT

Fact 3.35. The *n*th **moment** of the random variable X is the expectation $E(X^n)$. In the discrete case the *n*th moment is calculated by

$$E(X^n) = \sum_k k^n P(X = k). \tag{3.27}$$

If X has density function f its *n*th moment is given by

$$E(X^n) = \int_{-\infty}^{\infty} x^n f(x)\, dx. \tag{3.28}$$

The second moment, $E(X^2)$, is also called the *mean square*.

EXERCISE: COMPUTE

1. $E(X^n)$ $X \sim \mathrm{Unif}[a,b]$
2. $E(Y^2)$ $Y \sim \mathrm{Geom}(p)$
3. $E(Z^2)$ $Z \sim \mathrm{Bin}(n,p)$

DO YOURSELF.

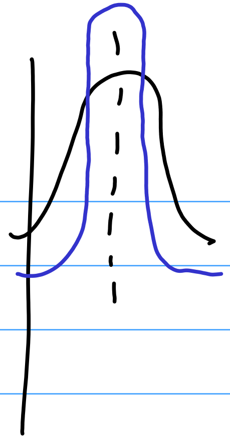
Example 3.36. Let $c > 0$ and let U be a uniform random variable on the interval $[0, c]$. Find the nth moment of U for all positive integers n.

$U \sim \mathrm{Unif}[\underset{a}{0}, \underset{b}{c}]$

$$f(x) = \begin{cases} \frac{1}{c} & x \in [0,c] \\ 0 & \text{o.w.} \end{cases}$$

$$E(U^n) = \int_{-\infty}^{\infty} x^n f(x)\,dx = \int_0^c x^n \cdot \frac{1}{c}\,dx$$

$$= \frac{1}{c}\int_0^c x^n\,dx = \frac{1}{c} \cdot \frac{c^{n+1}}{n+1} = \frac{c^n}{n+1}$$



§ 3.4 VARIANCE

Definition 3.44. Let X be a random variable with mean μ. The **variance** of X is defined by

$$\mathrm{Var}(X) = E\left[(X - \mu)^2\right]. \tag{3.31}$$

E(X)

An alternative symbol is $\boxed{\sigma^2} = \mathrm{Var}(X)$.

$\sigma = S.D(X) = \sqrt{\mathrm{Var}(X)}$

(STANDARD DEVIATION

WHY NOT $E(|X-\mu|)$?

VARIANCE → MEAS. OF SPREAD

(WHEN X → DISC/CONT.)

Fact 3.45. Let X be a random variable with mean μ. Then

$$\operatorname{Var}(X) = \sum_k (k-\mu)^2 P(X=k) \qquad \text{if } X \text{ is discrete} \tag{3.32}$$

and

$$\operatorname{Var}(X) = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx \qquad \text{if } X \text{ has density function } f. \tag{3.33}$$

Example 3.46. Consider two investment opportunities. One yields a profit of \$1 or a loss of \$1, each with probability $\frac{1}{2}$. The other one yields a profit of \$100 or a loss of \$100, also each with probability $\frac{1}{2}$. Let X denote the random outcome of the former and Y the latter. We have the probabilities

$$P(X = 1) = P(X = -1) = \tfrac{1}{2} \quad \text{and} \quad P(Y = 100) = P(Y = -100) = \tfrac{1}{2}.$$

$$E(X) = \sum_k k \cdot P(X=k) = 1 \cdot (1/2) + (-1) \cdot 1/2 = 0$$

$$E(Y) = \sum_k k \cdot P(Y=k) = (100) \cdot \frac{1}{2} + (-100) \cdot \frac{1}{2} = 0$$

$$\operatorname{Var}(X) = \sum_k (k-\mu)^2 P(X=k) = (1)^2 \cdot (1/2) + (-1)^2 \cdot (1/2)$$
$$= 1$$

$$\operatorname{Var}(Y) = \sum_k (k-\mu)^2 P(Y=k) = (100)^2 \cdot \frac{1}{2} + (-100)^2 \cdot \frac{1}{2} = 10000$$

Fact 3.48. (Alternative formula for the variance)

1 st MOMENT SQUARED

$$\mathrm{Var}(X) = \underbrace{E(X^2)}_{\text{2nd MOMENT}} - (E[X])^2. \tag{3.34}$$

Pf DISCRETE :

$$\mathrm{Var}(X) = \sum_k (k-\mu)^2 \, P(X=k)$$

$$= \sum_k \left(k^2 - 2k\mu + \mu^2 \right) P(X=k)$$

$$= \sum_k k^2 P(X=k) - 2\mu \left(\sum_k k \, P(X=k) \right) + \mu^2 \sum_k P(X=k)$$

$$\mathrm{Var}(X) = \sum_k k^2 P(X=k) - 2\mu \underbrace{\left(\sum_k k\, P(X=k)\right)}_{\mu} + \mu^2 \underbrace{\sum_k P(X=k)}_{1}$$

$$\sum_k P(X=k) = 1$$

$$\sum_k k \cdot P(X=k) = \mathbb{E}(X) = \mu$$

$$\sum_k k^2 P(X=k) = \mathbb{E}(X^2)$$

$$\mathrm{Var}(X) = \mathbb{E}(X^2) - 2\mu \cdot \mu + \mu^2$$

$$= \mathbb{E}(X^2) - \mu^2$$

$$= \mathbb{E}(X^2) - [\mathbb{E}(X)]^2$$

$\blacksquare$

VARIANCE OF SOME RVs

$$f(x) = \begin{cases} \frac{1}{b-a} & x \in [a,b] \\ 0 & \text{o.w.} \end{cases}$$

(1) $X \sim \text{Unif}\,[a,b]$

$$\mathbb{E}[X] = \frac{a+b}{2}$$

$$\text{Var}[X] = \mathbb{E}[X^2] - (\mathbb{E}[X])^2 = \frac{b^2+ab+a^2}{3} - \left(\frac{a+b}{2}\right)^2$$

$$\mathbb{E}[X^2] = \int_{-\infty}^{\infty} x^2 f(x)\,dx = \int_a^b \frac{x^2}{b-a}\,dx = \frac{x^3/3\,\Big]_a^b}{b-a} = \frac{b^2+ab+a^2}{3}$$

$$\mathrm{Var}[X] = \mathbb{E}[X^2] - (\mathbb{E}[X])^2 = \frac{b^2+ab+a^2}{3} - \left(\frac{a+b}{2}\right)^2$$

$$= \frac{b^2+ab+a^2}{3} - \left[\frac{a^2+2ab+b^2}{4}\right]$$

$$= \frac{a^2+b^2-2ab}{12}$$

$$\mathrm{Var}[X] = \frac{(b-a)^2}{12}$$

② $\mathrm{Ber}(p)$ $\qquad X \in \{0,1\}$

$$P(X=1) = p \qquad P(X=0) = 1-p$$

$$\mathbb{E}(X) = p$$

$$\mathbb{E}(X^2) = \sum_k k^2 P(X=k) = 0^2 \cdot (1-p) + 1^2 \cdot (p)$$

$$= p$$

$$\mathrm{Var}(X) = \mathbb{E}(X^2) - (\mathbb{E}[X])^2 = p - p^2$$

$$\mathrm{Var}(X) = p(1-p) = P(X=1)P(X=0)$$

$$X = I_A$$

$$\mathrm{Var}[I_A] = \underbrace{P(A)}_{P(I_A = 1)} \cdot \underbrace{P(A^c)}_{P(I_A = 0)}$$

③ $X \sim \text{Bin}(n,p)$

$$\mathbb{E}(X) = np$$

$$\mathbb{E}(X^2) = n(n-1)p^2 + np$$

$$\text{Var}(X) = \underbrace{n(n-1)p^2 + np}_{\mathbb{E}(X^2)} - \underbrace{n^2p^2}_{\mathbb{E}(X)^2}$$

$$= \cancel{n^2p^2} - np^2 + np - \cancel{n^2p^2}$$

$$\text{Var}(X) = np[1-p] = n \, \text{Var}[\,\text{Be}(p)\,]$$

(4) $X \sim \text{Geom}(p)$

$$\mathbb{E}(X) = \frac{1}{p}$$

$q := 1 - p$

$$\mathbb{E}(X^2) = \frac{1+q}{p^2} = \frac{2-p}{p^2}$$

$$\text{Var}(X) = \mathbb{E}(X^2) - \mathbb{E}(X)^2 = \frac{2-p}{p^2} - \frac{1}{p^2}$$

$$= \frac{1-p}{p^2} = q/p^2$$

VARIANCE MEAUSURES SPREAD

Fact 3.54. For a random variable X, $\mathrm{Var}(X) = 0$ if and only if $P(X = a) = 1$ for some real value a.

DEGENERATE

$P(X=a)=1$

$\mathbb{E}(X) = a \cdot P(X=a) = a$

$\mathbb{E}(X^2) = a^2 \cdot P(X=a) = a^2$

DISCRETE

$\mathrm{Var}(X) = a^2 - (a)^2 = 0$

$$\mathrm{Var}(X) = 0$$

$$\downarrow$$

$$\underbrace{\sum_k \underbrace{(k-\mu)^2}_{\geq 0} \; \underbrace{P(X=k)}_{\geq 0}}_{=0} = 0$$

$(k-\mu)^2 P(X=k)$ FOR EVERY k

$k = \mu$ $\qquad$ $P(X=k) = 0$ $\Rightarrow$ $P(X=k) = 0$ UNLESS $k=\mu$

$P(X=\mu) = 1$

AFFINE

WILL BE GENERALIZED BY LINEARITY OF EXP.

Fact 3.52. Let X be a random variable and a and b real numbers. Then

$$E(aX + b) = aE(X) + b \tag{3.36}$$

and

$$\operatorname{Var}(aX + b) = a^2 \operatorname{Var}(X) \tag{3.37}$$

provided the mean and variance are well defined.

Pf (FOR DISC.)

$$t \mapsto at + b \quad (\text{AFFINE})$$

$$\mathbb{E}[aX+b] = \sum_k (ak+b)\cdot P(X=k) = a\Big(\underbrace{\sum_k k\,P(X=k)}_{\mathbb{E}(X)}\Big) + b\underbrace{\sum_k P(X=k)}_{1}$$

$$= a\,\mathbb{E}(X) + b$$

$$\mathrm{Var}(aX+b) = \mathbb{E}\left[(aX+b)^2\right] - \Big(\underbrace{\mathbb{E}[aX+b]}_{a\,\mathbb{E}(X)\;+\;b}\Big)^2$$

$$\mathbb{E}\left[(aX+b)^2\right] = \sum_k (ak+b)^2 \; P(X=k)$$

$$= \sum_k \left[a^2k^2 + 2abk + b^2\right] P(X=k)$$

$$= a^2 \sum_k \underbrace{k^2 P(X=k)}_{\mathbb{E}(X^2)} + 2ab \underbrace{\sum_k k\, P(X=k)}_{\mathbb{E}(X)} + b^2 \overbrace{\sum_k P(X=k)}^{1}$$

$$\mathbb{E}\left((aX+b)^2\right) = \boxed{a^2\,\mathbb{E}(X^2) + 2ab\,\mathbb{E}(X) + b^2}$$

$$\left[\mathbb{E}(aX+b)\right]^2 = \left(a\,\mathbb{E}(X)+b\right)^2$$

$$= \boxed{a^2\,\mathbb{E}(X)^2 + 2ab\,\mathbb{E}(X) + b^2}$$

$$\mathrm{Var}(aX+b) = \boxed{\quad} - \boxed{\quad}$$

$$= a^2\,\mathbb{E}(X^2) - a^2\,\mathbb{E}(X)^2$$

$$= a^2\left[\mathbb{E}(X^2) - \mathbb{E}(X)^2\right] = a^2\,\mathrm{Var}(X)$$

SEE YOU ON

THURSDAY