MATH 201 (SUMMER 2023, SESH A2)

LECTURE 8 : 05/25/23

ANURAG SAHAY
OFF HRS: BY APPT (VIA ZOOM)

email: anuragsahay@rochester.edu

Zoom ID: 979-4693-0650

LECTURES:
9:00 AM – 11:15 AM (ET)
M, T, W, R

COURSE WEB PAGE: https://people.math.rochester.edu/grads/asahay/summer2023/math201/index.html

ALL PHOTOS TAKEN FROM TEXTBOOK

ANNOUNCEMENTS

LECTURE 7

1. NEW H.W. IS UPLOADED., NO CLASS ON MONDAY (MEMORIAL DAY)
2. OFFICE HOURS : TODAY — AFTER CLASS. (11:15 – 12:15)
3. UPCOMING DEADLINES :
   a. WW04 – FRI, MAY 26th
   b. HW03 – SAT, MAY 27th
   c. WW05 – TUES, MAY 30th
   d. HW04 – TUES, MAY 30th } → TO BE UPLOADED
4. ERROR IN HW 2 — SEE ANNOUNCEMENT ON BLACKBOARD
5. DETAILS ABOUT EXAMS → SEE WEBPAGE. → IN CLASS MIDTERM – 2h, FINAL – 3h
6. PLEASE KEEP VIDEOS ON, IF POSSIBLE!

§ 3.5 NORMAL DISTRIBUTION

SO FAR :

WE HAVE ONLY SEEN <u>ONE</u> EXAMPLE OF A CONTINUOUS R.V. (i.e. Unif $[a, b]$)

MOST IMP. CONCEPT $\longrightarrow$ $\underbrace{\text{NORMAL}}_{\text{CONT. R.V.}}$ DISTRIBUTION.

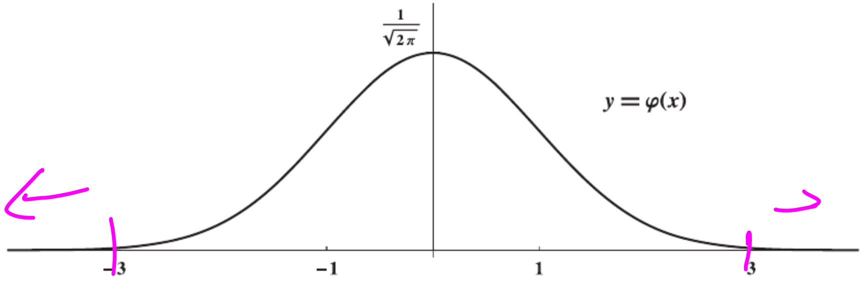
C.L.T. $\rightarrow$ MANY DISTRIB. ARE WELL-APPROX. BY NORMALS.

Definition 3.55. A random variable Z has the **standard normal distribution** (also called **standard Gaussian distribution**) if Z has density function

$$\varphi(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \tag{3.39}$$

p.d.f.

on the real line. Abbreviate this by $Z \sim \mathcal{N}(0, 1)$.



BELL CURVE

Figure 3.9. The probability density function φ of the standard normal distribution.

NOTES :

① A p.d.f SATISFIES

$$\int_{-\infty}^{\infty} f(x)\, dx = P(X \in \mathbb{R}) = 1$$

$z \sim \phi$

$$\boxed{\int_{-\infty}^{\infty} e^{-x^2/2}\, dx = \sqrt{2\pi}}$$

TO BE DONE NEXT WEEK

② C.D.F? → APPENDIX E OF THE BOOK

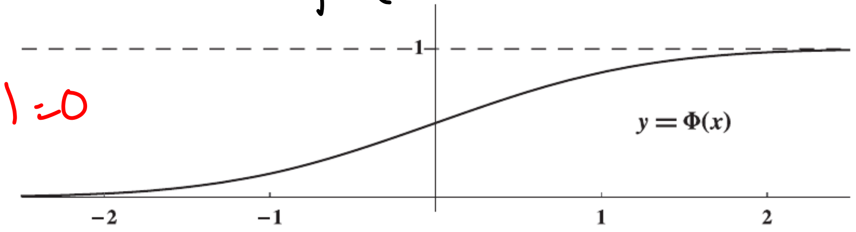
$$\Phi(t) = P(Z \le t) = \int_{-\infty}^{t} \frac{1}{\sqrt{2\pi}}\, e^{-x^2/2}\, dx$$

erf(t)

$$= \frac{1}{\sqrt{2\pi}} \boxed{\int_{-\infty}^{t} e^{-x^2/2}\, dx}$$

ANTI-DERI. OF $e^{-x^2/2}$

$\lim_{x \to -\infty} \Phi(x) = 0$



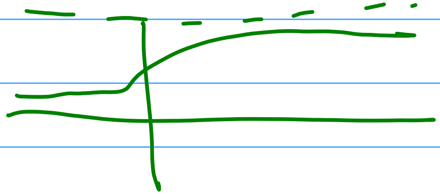
$\lim_{x \to \infty} \Phi(x) = 1$

Figure 3.10. The cumulative distribution function Φ of the standard normal distribution. It satisfies $0 < \Phi(x) < 1$ for all x.

(3) ϕ & Φ $\Big\}\rightarrow$ p.d.f & c.d.f. of $N(0,1)$

$$\phi(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

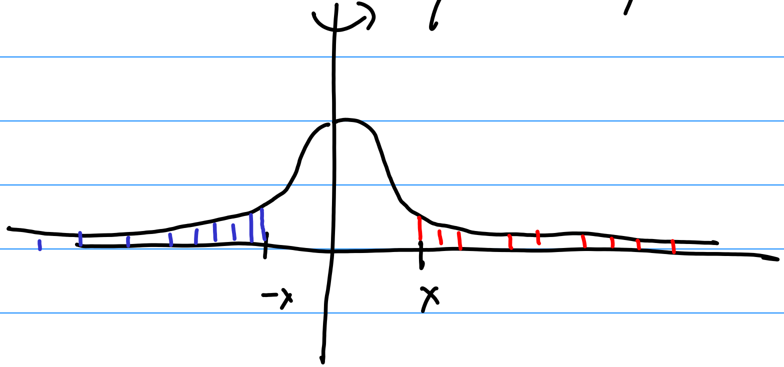
$\Phi \rightarrow$ APPENDIX E



(4) $\Phi(x) + \Phi(-x) = 1$ $\phi(x) = \phi(-x)$

$$\Phi(-x) = \int_{-\infty}^{-x} \frac{e^{-x^2/2}}{\sqrt{2\pi}}\,dx$$

$$= \int_{x}^{\infty} \frac{e^{-x^2/2}}{\sqrt{2\pi}}\,dx$$



$$\Phi(x) = \int_{-\infty}^{x} \phi(x)\,dx$$

$$\Phi(x) + \Phi(-x) = \int_{-\infty}^{x} \phi(x)\,dx + \int_{x}^{\infty} \phi(x)\,dx$$

$$= \int_{-\infty}^{\infty} \underbrace{\phi(x)\,dx}_{\text{p.d.f.}} = 1$$

$$\boxed{\therefore\ \Phi(-x) = 1 - \Phi(x)}$$

(5) APPENDIX E

$\Phi(t)$ FOR $0 \le t \le 3.49$ [$t \sim 2$ DECIMAL]

2nd DECIMAL OF t

1st DECIMAL OF t

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817

$\Phi(\boxed{1.1}7) \approx 0.8790$

$\Phi(1.2) \approx 0.8849$

$\Phi(-0.03) = 1 - \Phi(0.03)$

$\approx 1 - 0.5120$

$= 0.4880$

Example 3.57. Let $Z \sim \mathcal{N}(0, 1)$. Find the numerical value of $P(-1 \leq Z \leq 1.5)$.

$$P(-1 \leq Z \leq 1.5) = P(-1 < Z \leq 1.5)$$

$P(Z = -1) = 0$

$$= P(Z \leq 1.5) - P(Z \leq 1)$$

$$= \Phi(1.5) - \Phi(1)$$

$$\simeq 0.9332 - 0.8413$$

$$= 0.0919$$

Example 3.58. Find $z > 0$ so that a standard normal random variable Z has approximately $2/3$ probability of being in the interval $(-z, z)$.

$$P(Z \in (-z, z)) \approx 2/3$$

$$2/3 = P(-z < Z \underset{\le z}{<} z) = \Phi(z) - \underbrace{\Phi(-z)}_{1-\Phi(z)}$$

$$= \Phi(z) - (1 - \Phi(z))$$

$$= 2\Phi(z) - 1$$

$$2\Phi(z) - 1 \approx 2/3$$

$$\Phi(z) \approx \frac{1}{2}\left[\frac{2}{3}+1\right] \qquad \left(2\Phi(z)-1 \approx 2/3\right)$$

$$= 5/6 \approx 0.833$$

$$\Phi(0.97) \approx 0.8340 \quad , \quad \Phi(0.96) \approx 0.8315$$

$$0.96 < z < 0.97$$

$$\boxed{z \approx 0.97} \qquad (-0.97, 0.97)$$

MEAN & VARIANCE

Fact 3.59. Let $Z \sim \mathcal{N}(0, 1)$. Then $E(Z) = 0$ and $\mathrm{Var}(Z) = E(Z^2) = 1$.

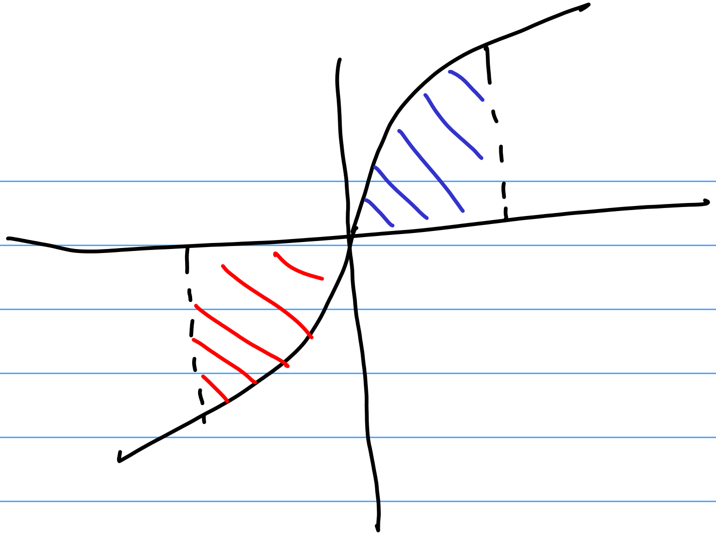
$\mathcal{N}(\underset{\text{MEAN}}{0}, \overset{\text{VARIANCE}}{1})$

Pf.

$$\mathbb{E}(Z) = \int_{-\infty}^{\infty} x\,\phi(x)\,dx = \int_{-\infty}^{\infty} x\,\frac{e^{-x^2/2}}{\sqrt{2\pi}}\,dx$$

$$= \lim_{a\to\infty} \int_{-a}^{a} \boxed{\frac{x\,e^{-x^2/2}}{\sqrt{2\pi}}}\,dx = \lim_{a\to\infty} 0 = 0$$

ODD FUNCTION



$$\mathrm{Var}(Z) = \mathbb{E}(Z^2) - \cancel{E(Z)}^2$$

$$= \int_{-\infty}^{\infty} \frac{x^2 \quad e^{-x^2/2}}{\sqrt{2\pi}} dx = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \overbrace{x}^{u} \cdot \overbrace{\left(x e^{-x^2/2}\right)}^{dv} dx$$

$$\int \overbrace{x}^{u} \cdot \overbrace{\left(x e^{-x^2/2}\right) dx}^{dv} = 1 \cdot \left(-e^{-x^2/2}\right) - \int \left(-e^{-x^2/2}\right) dx$$

$$du = dx \qquad v = -e^{-x^2/2}$$

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} x^2 e^{-x^2/2} dx = \underbrace{-e^{-x^2/2} \Big]_{-\infty}^{\infty}}_{=0} + \underbrace{\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-x^2/2} dx}_{1.}$$

$$\lim_{x \to \pm\infty} e^{-x^2/2} = 0$$

$$\mathbb{E}(Z^2) = 1$$

$$\mathrm{Var}(Z) = 1 - 0 = 1$$

$\blacksquare$

AFFINE TRANSFORM

LET $X = \underbrace{\sigma Z + \mu}_{\text{AFFINE TRANSFORM}}$ $\qquad Z \sim \mathcal{N}(0,1)$

$\sigma > 0, \ \mu \in \mathbb{R}$

$\mathbb{E}(X), \ \mathrm{Var}(X), \ f_X, \ F_X \ ?$

$$\mathbb{E}(X) = \mathbb{E}(\sigma Z + \mu) = \sigma \cancel{\mathbb{E}(Z)} + \mu = \mu$$

$$\mathrm{Var}(X) = \mathrm{Var}(\sigma Z + \mu) = \sigma^2 \underbrace{\boxed{\mathrm{Var}(Z)}}_{1} = \sigma^2$$

$Z \sim N(0,1)$

$$F_X(t) = P(X \leq t)$$

$$= P(\sigma Z + \mu \leq t)$$

$$= P(\sigma Z \leq t - \mu)$$

$$= P\left(Z \leq \frac{t-\mu}{\sigma}\right)$$

$$\boxed{F_X(t) = \Phi\left(\frac{t-\mu}{\sigma}\right)}$$

$Z \to$ CONT., $X \to$ CONT.

$$\Rightarrow f_X(t) = F_X'(t)$$

$$= \frac{d}{dt} \Phi\left(\frac{t-\mu}{\sigma}\right)$$

$$= \frac{d}{dt}\left[\frac{t-\mu}{\sigma}\right] \cdot \Phi'\left(\frac{t-\mu}{\sigma}\right)$$

[CHAIN RULE]

$$= \frac{1}{\sigma}\,\phi\left(\frac{t-\mu}{\sigma}\right) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-\frac{(t-\mu)^2}{2\sigma^2}}$$

FACT:

$$\phi = \Phi'$$

Definition 3.60. Let μ be real and $\sigma > 0$. A random variable X has the *normal distribution with mean μ and variance σ^2* if X has density function

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \tag{3.44}$$

on the real line. Abbreviate this by $X \sim \mathcal{N}(\mu, \sigma^2)$.

$\mu = 0$

$\sigma^2 = 1$

$\mathbb{E}(X) = \mu, \quad \mathrm{Var}(X) = \sigma^2$

RULE OF THUMB $\rightarrow$ STANDARDIZE.

FACT: $X \sim N(\mu, \sigma^2)$, THEN $\dfrac{X-\mu}{\sigma} = Z \sim N(0,1)$

Pf: REVERSE $X = \sigma Z + \mu$

Example 3.62. Suppose $X \sim \mathcal{N}(-3, 4)$. Find the probability $P(X \le -1.7)$.

$\mu = -3$, $\sigma^2 = 4$ $(\sigma = 2)$.

$P(X \le -1.7)$

$$Z = \frac{X-\mu}{\sigma} = \frac{X-(-3)}{2} = \frac{X+3}{2}$$

$X \le -1.7 \longleftrightarrow Z \le \dots$

$$X \leq -1.7 \iff X + 3 \leq 3 - 1.7$$

$$\iff \frac{X+3}{2} \leq \frac{3-1.7}{2} = \frac{1.3}{2} = 0.$$

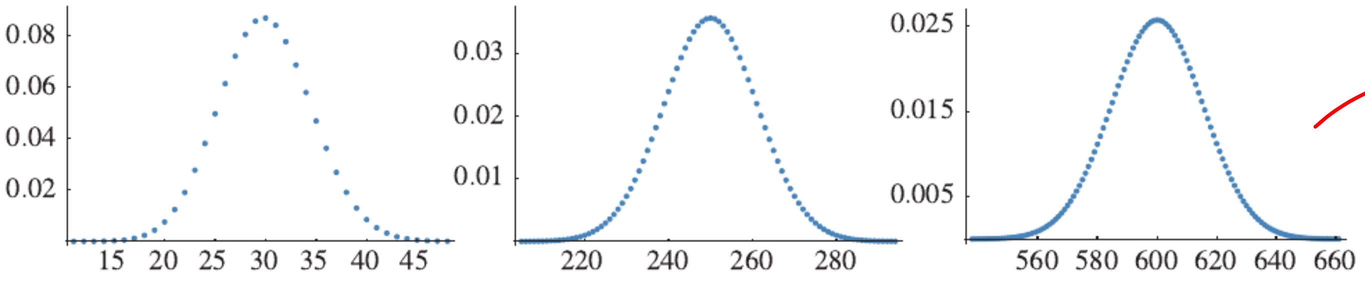
$$P(X \leq -1.7) = P\Bigg(\underbrace{\boxed{\frac{X+3}{2}}}_{\sim N(0,1)} \leq 0.65\Bigg) = \Phi(0.65)$$

$$= 0.7422$$

BREAK TILL
10:15 AM ET

§ 4.1 NORMAL APPROXIMATION

Q WHAT MAKES $N(\mu, \sigma^2)$ INTERESTING?



BELL CURVE.

$n=100$
$p=0.3$

Figure 4.1. The probability mass function of the Bin(n, p) distribution with parameter values (100, 0.3), (500, 0.5), and (1000, 0.6). In each case the graph shows the function on the interval $(np - 4\sqrt{np(1-p)}, np + 4\sqrt{np(1-p)})$ and the scale on the y-axis changes also.

SEEMS LIKE $\mathrm{Bin}(n,p) \approx N(\mu, \sigma^2)$

AS $n \longrightarrow \infty$

μ = MEAN OF $\mathrm{Bin}(n,p)$, σ^2 = VARIANCE OF $\mathrm{Bin}(n,p)$

$= np$

$= npq$ $(q : 1-p)$

$= np(1-p)$

$$\mathrm{Bin}(n,p) \approx N\big(np,\ np(1-p)\big)$$

$\mu = n \cdot p$
$= 1000 \times 0.6$
$= 600$

$\sigma^2 = n \cdot p(1-p)$
$= (1000)(0.6)(0.4)$
$= 240$

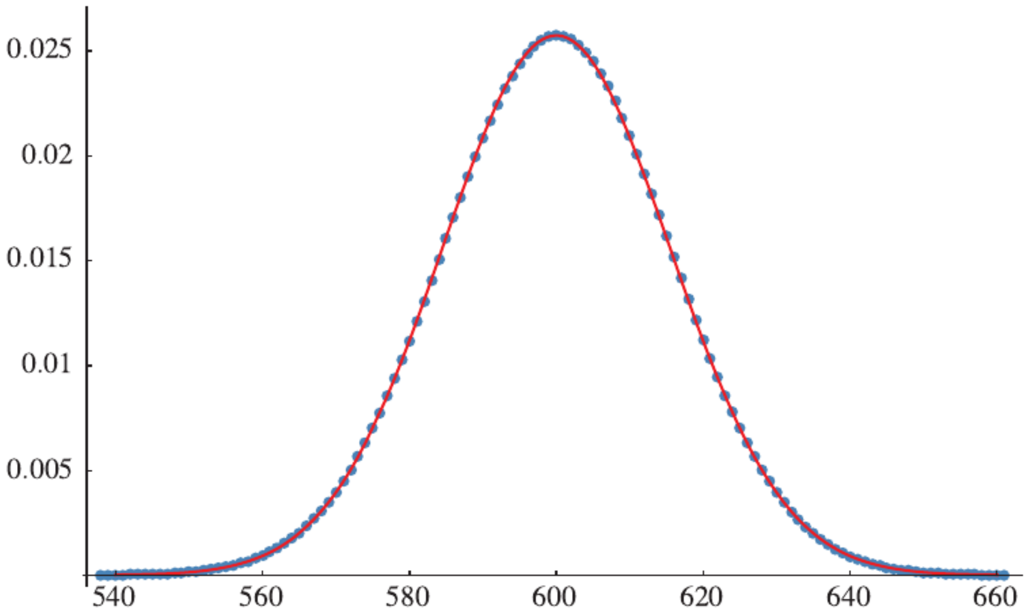


Figure 4.3. The bullets represent the probability mass function of $\boxed{\mathrm{Bin}(1000, 0.6)}$ and the continuous curve is the density function of $\boxed{\mathcal{N}(600, 240)}$ in the interval [540, 660].

$\mathcal{N}(np, np(1-p))$

. . . .

$S_n \sim \mathrm{Bin}(n, p)$, $X \sim \mathcal{N}(np, npq) \Rightarrow S_n \to X$ AS $n \to \infty$

PROBLEM : S_n & X BOTH HAVE GROWING μ, σ

FIX : RENORMALIZE. / STANDARDIZE.

FOR Y w/ $\mathbb{E}(Y) = \mu$, $\text{Var}(Y) = \sigma^2$

$$\hat{Y} = \frac{Y - \mu}{\sigma}$$

AFFINE FUNCTION OF Y

$$\mathbb{E}(\hat{Y}) = \frac{\mathbb{E}(Y) - \mu}{\sigma} = \frac{\mu - \mu}{\sigma} = 0$$

$$\text{Var}(\hat{Y}) = \frac{1}{\sigma^2} \text{Var}(Y) = \frac{1}{\sigma^2} \cdot \sigma^2 = 1$$

$$Y \longrightarrow \hat{Y} = \frac{Y - \mu}{\sigma}$$

$\mathbb{E}(Y) = \mu$

$\mathrm{Var}(Y) = \sigma^2$

$\mathbb{E}(\hat{Y}) = 0$

$\mathrm{Var}(\hat{Y}) = 1$

$$S_n \sim \mathrm{Bin}(n, p) \Rightarrow \hat{S}_n = \frac{S_n - \mathbb{E}(S_n)}{\sqrt{\mathrm{Var}(S_n)}} = \frac{S_n - np}{\sqrt{np(1-p)}}$$

CLT FOR $\text{Bin}(n,p) \approx \hat{S}_n \longrightarrow N(0,1)$

Theorem 4.1. (Central limit theorem for binomial random variables) *Let $0 < p < 1$ be fixed and suppose that $S_n \sim \text{Bin}(n,p)$. Then for any fixed $-\infty \le a \le b \le \infty$ we have the limit*

$$\lim_{n\to\infty} P\left(a \le \frac{S_n - np}{\sqrt{np(1-p)}} \le b\right) = \int_a^b \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}\,dx. \tag{4.1}$$

$$P\left(a \le \hat{S}_n \le b\right) \xrightarrow[(n \to \infty)]{} P\left(a \le Z \le b\right)$$

Pf $\rightarrow$ SEE BOOK.

Normal approximation of the binomial distribution. Suppose that $S_n \sim \text{Bin}(n, p)$ with n large and p not too close to 0 or 1. Then

$$P\left(a \leq \frac{S_n - np}{\sqrt{np(1-p)}} \leq b\right) \text{ is close to } \Phi(b) - \Phi(a). \tag{4.3}$$

As a rule of thumb, the approximation is good if $\boxed{np(1-p) > 10.}$

$P(a \leq \hat{S}_n \leq b)$

$\|$

$P(a \leq Z \leq b)$

$= \Phi(b) - \Phi(a)$

Example 4.2. A fair coin is flipped 10,000 times. Estimate the probability that the number of heads is between 4850 and 5100.

Example 4.2. A fair coin is flipped 10,000 times. Estimate the probability that the number of heads is between 4850 and 5100.

$S \to$ # OF HEADS

$P(4850 \le S \le 5100)$

$S \sim \text{Bin}(\underbrace{10000}_{n}, \underbrace{1/2}_{p})$

$\mu = \mathbb{E}(S) = np$

$= (10000) \cdot 1/2 = 5000$

$\sigma^2 = \text{Var}(S) = n \cdot p(1-p) = (10000)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = 2500$

$\Rightarrow \sigma = \sqrt{2500} = 50$

$S \approx N(5000, 2500)$

$$P(4850 \le S \le 5100)$$

$$\underbrace{\frac{4850-5000}{50}}_{=\,-3} \le \frac{S-5000}{50} \le \underbrace{\frac{5100-5000}{50}}_{\downarrow\, 2}$$

C.L.T.

$$P\left(-3 \le \underbrace{\frac{S-5000}{50}}_{\hat{S}} \le 2\right) \approx \Phi(2) - \Phi(-3)$$

$$\approx 0.9772 - (1 - 0.9987)$$

$$\approx 0.9759$$

$$P(4850 \le S \le 5100) = \sum_{4850 \le k \le 5100} \underbrace{\binom{10000}{k} \left(\frac{1}{2}\right)^{k} \left(\frac{1}{2}\right)^{10000-k}}_{\text{p.m.f OF } Bin(10000,\, 1/2)}$$

$$= \sum_{k=4850}^{5100} \binom{10000}{k} \cdot 2^{-10000}$$

$$\left.\begin{array}{l} \text{ACTUAL} = 0.9765 \\ \text{APPROX} = 0.9759 \end{array}\right\} \to 0.06\,\%$$

SKIP

Example 4.3. Suppose a game piece moves along a board according to the following rule: if a roll of a die gives 1 or 2 then take two steps ahead, otherwise take three steps. Approximate the probability that after 120 rolls the piece has moved more than 315 steps.

CONTINUITY CORRECTION

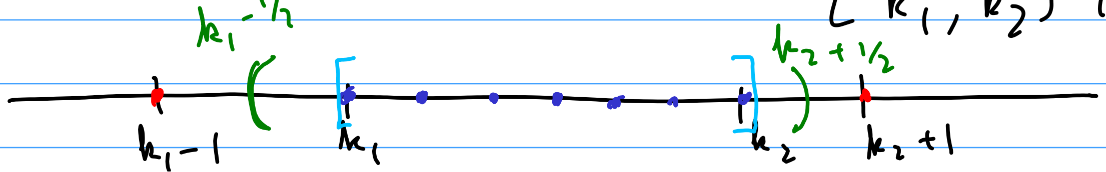
$$S_n \sim \mathrm{Bin}(n,p) \Rightarrow S_n \in \{0, \dots, n\}$$

INTEGER - VALUED.

$$\therefore \quad P(k_1 \leq S_n \leq k_2) = P(k_1 - 1/2 \leq S_n \leq k_2 + 1/2)$$

$(k_1, k_2 \in \mathbb{Z})$

$$[k_1, k_2] \cap \mathbb{Z} = [k_1 - 1/2, k_2 + 1/2] \cap \mathbb{Z}$$



$$P(k_1 \le S_n \le k_2) = P(k_1 - 1/2 \le S_n \le k_2 + 1/2)$$

$$\approx P(k_1 \le X \le k_2) \xrightarrow{?} P(k_1 - 1/2 \le X \le k_2 + 1/2)$$

$$X \sim N(np,\ np(1-p))$$

BETTER APPROX.

CONT. CORRECTION.

Example 4.5. Roll a fair die 720 times. Estimate the probability that we have exactly 113 sixes.

X → # OF SIXES.

$$\mu = 720 \cdot \frac{1}{6} = 120$$

$$X \sim Bin\left(720, \frac{1}{6}\right)$$

$$\sigma^2 = 720 \cdot \frac{1}{6} \cdot \frac{5}{6} = 100$$

$$P(X = 113) = P(112.5 \le X \le 113.5)$$

$$\boxed{\frac{X - \overset{\mu}{120}}{\underset{\sigma}{10}}} = P\left(\underbrace{\frac{112.5 - 120}{10}}_{-0.75} \le \underbrace{\frac{X - 120}{10}}_{\tilde{X}} \le \underbrace{\frac{113.5 - 120}{10}}_{-0.65}\right)$$

$$P(X=113) = P\left(-0.75 \le \hat{X} \le -0.65\right)$$

$$\approx P\left(-0.75 \le Z \le -0.65\right)$$

$$= \Phi(-0.65) - \Phi(-0.75)$$

$$= (1-0.7422) - (1-0.7734)$$

$$= 0.0312$$

$$p_X(113) = \binom{720}{113}\frac{5^{607}}{6^{720}} \qquad \left(\text{p.m.f. of } \mathrm{Bin}(n,p) \quad \binom{n}{k}\cdot p^k \cdot (1-p)^{n-k}\right)$$

$$\approx 0.0318$$

REDO

(ACCOUNT FOR C.C.)

Example 4.2. A fair coin is flipped 10,000 times. Estimate the probability that the number of heads is between 4850 and 5100.

$$\mu = 10000 \cdot \tfrac{1}{2} = 5000$$

$$\sigma = \sqrt{10000 \cdot \tfrac{1}{2} \cdot \tfrac{1}{2}} = 50$$

$$P(4850 \le S \le 5100) = P(4849.5 \le S \le 5100.5)$$

$$= P\Bigg(\underbrace{\frac{4849.5-5000}{50}}_{-3.01} \le \underbrace{\boxed{\frac{S-5000}{50}}}_{\tilde{S}} \le \underbrace{\frac{5100.5-5000}{50}}_{2.01}\Bigg)$$

$$\approx \Phi(2.01) - \Phi(-3.01)$$

$$\approx (0.9778) - \underbrace{(1 - 0.9987)}_{\Phi(-3.01) = 1 - \Phi(3.01)}$$

$\Phi(2.01)$

$$= 0.9765$$

ACTUAL $= 0.9765$

APPROX (w/o CORR.) $= 0.9759$

APROX. (w/ CORR.) $= 0.9765$

REMINDER : NO CLASS
ON MONDAY