

Chapter 4

Life Insurance and Endowments

Type of Coverage	Payable at Moment of Death	Payable at End of Year	Payable at End of Each mth of a Year
whole life	$\bar{A}_x = \int_0^{\infty} v^t \cdot {}_tP_x \cdot \mu_{x+t} dt$	$A_x = \sum_{k=0}^{\infty} v^{k+1} \cdot {}_kP_x \cdot q_{x+k}$	$A_x^{(m)} = \sum_{k=0}^{\infty} v^{k+1/m} \cdot {}_{k/m}P_x \cdot {}_{1/m}q_{x+k/m}$
n-year term insurance	$\bar{A}_{x:n}^1 = \int_0^n v^t \cdot {}_tP_x \cdot \mu_{x+t} dt$	$A_{x:n}^1 = \sum_{k=0}^{n-1} v^{k+1} \cdot {}_kP_x \cdot q_{x+k}$	$A_{x:n}^{(m)} = \sum_{k=0}^{nm-1} v^{k+1/m} \cdot {}_{k/m}P_x \cdot {}_{1/m}q_{x+k/m}$
n-year pure endowment	$A_{x:n}^{\bar{1}} = {}_nE_x = v^n \cdot {}_nP_x$		
n-year endowment insurance	$\bar{A}_{x:n} = \bar{A}_{x:n}^1 + A_{x:n}^{\bar{1}} = \bar{A}_{x:n}^1 + {}_nE_x$	$A_{x:n} = A_{x:n}^1 + A_{x:n}^{\bar{1}} = A_{x:n}^1 + {}_nE_x$	$A_{x:n}^{(m)} = A_{x:n}^{(m)} + {}_nE_x$
u-year deferred n-year term	${}_u \bar{A}_{x:n}^1 = {}_uE_x \cdot \bar{A}_{x+u:n}^1$	${}_u A_{x:n}^1 = {}_uE_x \cdot A_{x+u:n}^1$	${}_u A_{x:n}^{(m)} = {}_uE_x \cdot A_{x+u:n}^{(m)}$
n-year deferred whole life	${}_n \bar{A}_x = {}_nE_x \cdot \bar{A}_{x+n} = v^n \cdot {}_nP_x \cdot \bar{A}_{x+n}$	${}_n A_x = {}_nE_x \cdot A_{x+n} = v^n \cdot {}_nP_x \cdot A_{x+n}$	${}_n A_x^{(m)} = {}_nE_x \cdot A_{x+n}^{(m)}$

Variances

Type of Coverage	Payable at Moment of Death	Payable at End of Year	Payable at End of Each mth of a Year
whole life	${}^2\bar{A}_x - (\bar{A}_x)^2$	${}^2A_x - (A_x)^2$	${}^2A_x^{(m)} - (A_x^{(m)})^2$
n-year term insurance	${}^2\bar{A}_{x:n}^1 - (\bar{A}_{x:n}^1)^2$	${}^2A_{x:n}^1 - (A_{x:n}^1)^2$	${}^2A_{x:n}^{(m)} - (A_{x:n}^{(m)})^2$
n-year pure endowment	${}^2A_{x:n}^{\bar{1}} - (A_{x:n}^{\bar{1}})^2 = v^{2n} \cdot {}_nP_x \cdot q_x = e^{-2\delta n} \cdot {}_nP_x \cdot q_x$		
n-year endowment insurance	${}^2\bar{A}_{x:n} - (\bar{A}_{x:n})^2$	${}^2A_{x:n} - (A_{x:n})^2$	${}^2A_{x:n}^{(m)} - (A_{x:n}^{(m)})^2$
n-year deferred whole life	${}^2{}_n \bar{A}_x - ({}_n \bar{A}_x)^2$	${}^2{}_n A_x - ({}_n A_x)^2$	${}^2{}_n A_x^{(m)} - ({}_n A_x^{(m)})^2$

Relationships

Payable at Moment of Death	Payable at End of Year
$\bar{A}_x = \bar{A}_{x:n}^1 + {}_n \bar{A}_x = \bar{A}_{x:n}^1 + {}_nE_x \cdot \bar{A}_{x+n} = \bar{A}_{x:n}^1 + v^n \cdot {}_nP_x \cdot \bar{A}_{x+n}$	$A_x = A_{x:n}^1 + {}_n A_x = A_{x:n}^1 + {}_nE_x \cdot A_{x+n} = A_{x:n}^1 + v^n \cdot {}_nP_x \cdot A_{x+n}$

Payable at End of Year Only		
$A_x = v \cdot q_x + v \cdot p_x \cdot A_{x+1}$	${}^2A_x = v^2 \cdot q_x + v^2 \cdot p_x \cdot {}^2A_{x+1}$	$A_x^{(m)} = v^{1/m} \cdot {}_{1/m}q_x + v^{1/m} \cdot {}_{1/m}p_x \cdot A_{x+1/m}^{(m)}$

Chapter 4

Assuming UDD

$$\bar{A}_x = i/\delta A_x ; \bar{A}_{x:\overline{n}|}^1 = i/\delta A_{x:\overline{n}|}^1 ; \bar{A}_{x:\overline{n}|} = i/\delta A_{x:\overline{n}|}^1 + {}_nE_x ; A_x^{(m)} = i/i^{(m)} A_x$$

Varying Death Benefit

General Formula – Payable at Moment of Death	$\int_0^{\infty} b_t \cdot v^t \cdot {}_tP_x \cdot \mu_{x+t} dt$
General Formula – Payable at End of Year of Death	$\sum_{k=0}^{\infty} b_{k+1} \cdot v^{k+1} \cdot {}_kP_x \cdot q_{x+k}$
$(\bar{IA})_x$	$\int_0^{\infty} t \cdot v^t \cdot {}_tP_x \cdot \mu_{x+t} dt$
$(\bar{IA})_{x:\overline{n} }^1$	$\int_0^n t \cdot v^t \cdot {}_tP_x \cdot \mu_{x+t} dt$
$(IA)_x$	$\sum_{k=0}^{\infty} (k+1) \cdot v^{k+1} \cdot {}_kP_x \cdot q_{x+k}$
$(IA)_{x:\overline{n} }^1$	$\sum_{k=0}^{n-1} (k+1) \cdot v^{k+1} \cdot {}_kP_x \cdot q_{x+k}$