SOCIETY OF ACTUARIES

EXAM FM FINANCIAL MATHEMATICS

EXAM FM SAMPLE QUESTIONS

This set of sample questions includes those published on the interest theory topic for use with previous versions of this examination. Questions from previous versions of this document that are not relevant for the syllabus effective with the October 2022 administration have been deleted. The questions have been renumbered.

Some of the questions in this study note are taken from past SOA examinations.

These questions are representative of the types of questions that might be asked of candidates sitting for the Financial Mathematics (FM) Exam. These questions are intended to represent the depth of understanding required of candidates. The distribution of questions by topic is not intended to represent the distribution of questions on future exams.

Update history:
October 2022: Questions 208-275 were added

1.
Bruce deposits 100 into a bank account. His account is credited interest at an annual nominal rate of interest of 4% convertible semiannually.

At the same time, Peter deposits 100 into a separate account. Peter's account is credited interest at an annual force of interest of δ.

After 7.25 years, the value of each account is the same.

Calculate δ.

(A) 0.0388
(B) 0.0392
(C) 0.0396
(D) 0.0404
(E) 0.0414

2.
Kathryn deposits 100 into an account at the beginning of each 4-year period for 40 years. The account credits interest at an annual effective interest rate of *i*.

The accumulated amount in the account at the end of 40 years is *X*, which is 5 times the accumulated amount in the account at the end of 20 years.

Calculate *X*.

(A) 4695
(B) 5070
(C) 5445
(D) 5820
(E) 6195

3.
Eric deposits 100 into a savings account at time 0, which pays interest at an annual nominal rate of *i*, compounded semiannually.

Mike deposits 200 into a different savings account at time 0, which pays simple interest at an annual rate of *i*.

Eric and Mike earn the same amount of interest during the last 6 months of the 8^{th} year.

Calculate *i*.

(A) 9.06%
(B) 9.26%
(C) 9.46%
(D) 9.66%
(E) 9.86%

4.
A perpetuity costs 77.1 and makes end-of-year payments. The perpetuity pays 1 at the end of year 2, 2 at the end of year 3, …., *n* at the end of year (n+1). After year (n+1), the payments remain constant at *n*. The annual effective interest rate is 10.5%.

Calculate *n*.

(A) 17
(B) 18
(C) 19
(D) 20
(E) 21

5.
1000 is deposited into Fund X, which earns an annual effective rate of 6%. At the end of each year, the interest earned plus an additional 100 is withdrawn from the fund. At the end of the tenth year, the fund is depleted.

The annual withdrawals of interest and principal are deposited into Fund Y, which earns an annual effective rate of 9%.

Calculate the accumulated value of Fund Y at the end of year 10.

(A) 1519
(B) 1819
(C) 2085
(D) 2273
(E) 2431

6.
A 20-year loan of 1000 is repaid with payments at the end of each year.

Each of the first ten payments equals 150% of the amount of interest due. Each of the last ten payments is *X*.

The lender charges interest at an annual effective rate of 10%.

Calculate *X*.

(A) 32
(B) 57
(C) 70
(D) 97
(E) 117

7.
A 10,000 par value 10-year bond with 8% annual coupons is bought at a premium to yield an annual effective rate of 6%.

Calculate the interest portion of the 7th coupon.

(A) 632
(B) 642
(C) 651
(D) 660
(E) 667

8.

A perpetuity-immediate pays 100 per year. Immediately after the fifth payment, the perpetuity is exchanged for a 25-year annuity-immediate that will pay X at the end of the first year. Each subsequent annual payment will be 8% greater than the preceding payment.

The annual effective rate of interest is 8%.

Calculate X.

(A) 54
(B) 64
(C) 74
(D) 84
(E) 94

9.
Jeff deposits 10 into a fund today and 20 fifteen years later. Interest for the first 10 years is credited at a nominal discount rate of d compounded quarterly, and thereafter at a nominal interest rate of 6% compounded semiannually. The accumulated balance in the fund at the end of 30 years is 100.

Calculate d.

(A) 4.33%
(B) 4.43%
(C) 4.53%
(D) 4.63%
(E) 4.73%

10.
Ernie makes deposits of 100 at time 0, and X at time 3. The fund grows at a force of interest

$\delta_t = \dfrac{t^2}{100}$, $t > 0$.

The amount of interest earned from time 3 to time 6 is also X.

Calculate X.

(A) 385
(B) 485
(C) 585
(D) 685
(E) 785

11.
Mike buys a perpetuity-immediate with varying annual payments. During the first 5 years, the payment is constant and equal to 10. Beginning in year 6, the payments start to increase. For year 6 and all future years, the payment in that year is K% larger than the payment in the year immediately preceding that year, where $K < 9.2$.

At an annual effective interest rate of 9.2%, the perpetuity has a present value of 167.50.

Calculate K.

(A) 4.0
(B) 4.2
(C) 4.4
(D) 4.6
(E) 4.8

12.
A 10-year loan of 2000 is to be repaid with payments at the end of each year. It can be repaid under the following two options:

(i) Equal annual payments at an annual effective interest rate of 8.07%.
(ii) Installments of 200 each year plus interest on the unpaid balance at an annual effective interest rate of i.

The sum of the payments under option (i) equals the sum of the payments under option (ii).

Calculate i.

(A) 8.75%
(B) 9.00%
(C) 9.25%
(D) 9.50%
(E) 9.75%

13.
A loan is amortized over five years with monthly payments at an annual nominal interest rate of 9% compounded monthly. The first payment is 1000 and is to be paid one month from the date of the loan. Each succeeding monthly payment will be 2% lower than the prior payment.

Calculate the outstanding loan balance immediately after the 40^{th} payment is made.

(A) 6750
(B) 6890
(C) 6940
(D) 7030
(E) 7340

14.
To accumulate 8000 at the end of $3n$ years, deposits of 98 are made at the end of each of the first n years and 196 at the end of each of the next $2n$ years.

The annual effective rate of interest is i. You are given $(1+i)^n = 2.0$.

Calculate i.

(A) 11.25%
(B) 11.75%
(C) 12.25%
(D) 12.75%
(E) 13.25%

15.
Olga buys a 5-year increasing annuity for *X*.

Olga will receive 2 at the end of the first month, 4 at the end of the second month, and for each month thereafter the payment increases by 2.

The annual nominal interest rate is 9% convertible quarterly.

Calculate *X*.

(A) 2680
(B) 2730
(C) 2780
(D) 2830
(E) 2880

16.
David can receive one of the following two payment streams:

(i) 100 at time 0, 200 at time n years, and 300 at time $2n$ years
(ii) 600 at time 10 years

At an annual effective interest rate of i, the present values of the two streams are equal.

Given $v^n = 0.76$, calculate i.

(A) 3.5%
(B) 4.0%
(C) 4.5%
(D) 5.0%
(E) 5.5%

17.
Payments are made to an account at a continuous rate of (8*k* + *tk*), where $0 \le t \le 10$.

Interest is credited at a force of interest $\delta_t = \dfrac{1}{8+t}$.

After time 10, the account is worth 20,000.

Calculate *k*.

(A) 111
(B) 116
(C) 121
(D) 126
(E) 131

18.
You have decided to invest in Bond X, an *n*-year bond with semi-annual coupons and the following characteristics:

(i) Par value is 1000.
(ii) The ratio of the semi-annual coupon rate, *r*, to the desired semi-annual yield rate, *i*, is 1.03125.
(iii) The present value of the redemption value is 381.50.

Given $(1+i)^{-n} = 0.5889$, calculate the price of bond X.

(A) 1019
(B) 1029
(C) 1050
(D) 1055
(E) 1072

19.
Project P requires an investment of 4000 today. The investment pays 2000 one year from today and 4000 two years from today.

Project Q requires an investment of *X* two years from today. The investment pays 2000 today and 4000 one year from today.

The net present values of the two projects are equal at an annual effective interest rate of 10%.

Calculate *X*.

(A) 5400
(B) 5420
(C) 5440
(D) 5460
(E) 5480

20.
A perpetuity-immediate pays *X* per year. Brian receives the first *n* payments, Colleen receives the next *n* payments, and a charity receives the remaining payments. Brian's share of the present value of the original perpetuity is 40%, and the charity's share is *K*.

Calculate *K*.

(A) 24%
(B) 28%
(C) 32%
(D) 36%
(E) 40%

21.
Seth, Janice, and Lori each borrow 5000 for five years at an annual nominal interest rate of 12%, compounded semi-annually.

Seth has interest accumulated over the five years and pays all the interest and principal in a lump sum at the end of five years.

Janice pays interest at the end of every six-month period as it accrues and the principal at the end of five years.

Lori repays her loan with 10 level payments at the end of every six-month period.

Calculate the total amount of interest paid on all three loans.

(A) 8718
(B) 8728
(C) 8738
(D) 8748
(E) 8758

22.
Bruce and Robbie each open up new bank accounts at time 0. Bruce deposits 100 into his bank account, and Robbie deposits 50 into his. Each account earns the same annual effective interest rate.

The amount of interest earned in Bruce's account during the 11th year is equal to X. The amount of interest earned in Robbie's account during the 17th year is also equal to X.

Calculate X.

(A) 28.00
(B) 31.30
(C) 34.60
(D) 36.70
(E) 38.90

23.
Ron is repaying a loan with payments of 1 at the end of each year for n years. The annual effective interest rate on the loan is i. The amount of interest paid in year t plus the amount of principal repaid in year t + 1 equals X.

Determine which of the following is equal to X.

(A) $1+\dfrac{v^{n-t}}{i}$

(B) $1+\dfrac{v^{n-t}}{d}$

(C) $1+v^{n-t}i$

(D) $1+v^{n-t}d$

(E) $1+v^{n-t}$

24.
At an annual effective interest rate of i, i > 0%, the present value of a perpetuity paying 10 at the end of each 3-year period, with the first payment at the end of year 3, is 32.

At the same annual effective rate of i, the present value of a perpetuity paying 1 at the end of each 4-month period, with first payment at the end of 4 months, is X.

Calculate X.

(A) 31.6
(B) 32.6
(C) 33.6
(D) 34.6
(E) 35.6

25.
As of 12/31/2013, an insurance company has a known obligation to pay 1,000,000 on 12/31/2017. To fund this liability, the company immediately purchases 4-year 5% annual coupon bonds totaling 822,703 of par value. The company anticipates reinvestment interest rates to remain constant at 5% through 12/31/2017. The maturity value of the bond equals the par value.

Consider two reinvestment interest rate movement scenarios effective 1/1/2014. Scenario A has interest rates drop by 0.5%. Scenario B has interest rates increase by 0.5%.

Determine which of the following best describes the insurance company's profit or (loss) as of 12/31/2017 after the liability is paid.

(A) Scenario A – 6,610, Scenario B – 11,150
(B) Scenario A – (14,760), Scenario B – 14,420
(C) Scenario A – (18,910), Scenario B – 19,190
(D) Scenario A – (1,310), Scenario B – 1,320
(E) Scenario A – 0, Scenario B – 0

26.
An insurance company has an obligation to pay the medical costs for a claimant. Annual claim costs today are 5000, and medical inflation is expected to be 7% per year. The claimant will receive 20 payments.

Claim payments are made at yearly intervals, with the first claim payment to be made one year from today.

Calculate the present value of the obligation using an annual effective interest rate of 5%.

(A) 87,900
(B) 102,500
(C) 114,600
(D) 122,600
(E) Cannot be determined

27.
An investor pays 100,000 today for a 4-year investment that returns cash flows of 60,000 at the end of each of years 3 and 4. The cash flows can be reinvested at 4.0% per annum effective.

Using an annual effective interest rate of 5.0%, calculate the net present value of this investment today.

(A) –1,398
(B) –699
(C) 699
(D) 1,398
(E) 2,629

28.
You are given the following information with respect to a bond:

(i) par value: 1000
(ii) term to maturity: 3 years
(iii) annual coupon rate: 6% payable annually

You are also given that the one, two, and three year annual spot interest rates are 7%, 8%, and 9% respectively.

Calculate the value of the bond.

(A) 906
(B) 926
(C) 930
(D) 950
(E) 1000

29.
You are given the following information with respect to a bond:

(i) par value: 1000
(ii) term to maturity: 3 years
(iii) annual coupon rate: 6% payable annually

You are also given that the one, two, and three year annual spot interest rates are 7%, 8%, and 9% respectively.

The bond is sold at a price equal to its value.

Calculate the annual effective yield rate for the bond i.

(A) 8.1%
(B) 8.3%
(C) 8.5%
(D) 8.7%
(E) 8.9%

30.
The current price of an annual coupon bond is 100. The yield to maturity is an annual effective rate of 8%. The derivative of the price of the bond with respect to the yield to maturity is -700.

Using the bond's yield rate, calculate the Macaulay duration of the bond in years.

(A) 7.00
(B) 7.49
(C) 7.56
(D) 7.69
(E) 8.00

31.
A common stock pays a constant dividend at the end of each year into perpetuity.

Using an annual effective interest rate of 10%, calculate the Macaulay duration of the stock.

(A) 7 years
(B) 9 years
(C) 11 years
(D) 19 years
(E) 27 years

32.
A common stock pays dividends at the end of each year into perpetuity. Assume that the dividend increases by 2% each year.

Using an annual effective interest rate of 5%, calculate the Macaulay duration of the stock in years.

(A) 27
(B) 35
(C) 44
(D) 52
(E) 58

33.
Seth borrows X for four years at an annual effective interest rate of 8%, to be repaid with equal payments at the end of each year. The outstanding loan balance at the end of the third year is 559.12.

Calculate the principal repaid in the first payment.

(A) 444
(B) 454
(C) 464
(D) 474
(E) 484

34.
Bill buys a 10-year 1000 par value bond with semi-annual coupons paid at an annual rate of 6%. The price assumes an annual nominal yield of 6%, compounded semi-annually.

As Bill receives each coupon payment, he immediately puts the money into an account earning interest at an annual effective rate of i.

At the end of 10 years, immediately after Bill receives the final coupon payment and the redemption value of the bond, Bill has earned an annual effective yield of 7% on his investment in the bond.

Calculate i.

(A) 9.50%
(B) 9.75%
(C) 10.00%
(D) 10.25%
(E) 10.50%

35.
A man turns 40 today and wishes to provide supplemental retirement income of 3000 at the beginning of each month starting on his 65th birthday. Starting today, he makes monthly contributions of *X* to a fund for 25 years. The fund earns an annual nominal interest rate of 8% compounded monthly.

On his 65th birthday, each 1000 of the fund will provide 9.65 of income at the beginning of each month starting immediately and continuing as long as he survives.

Calculate *X*.

(A) 324.70
(B) 326.90
(C) 328.10
(D) 355.50
(E) 450.70

36.
Happy and financially astute parents decide at the birth of their daughter that they will need to provide 50,000 at each of their daughter's 18th, 19th, 20th and 21st birthdays to fund her college education. They plan to contribute *X* at each of their daughter's 1st through 17th birthdays to fund the four 50,000 withdrawals. They anticipate earning a constant 5% annual effective interest rate on their contributions.
Let $v = 1/1.05$.

Determine which of the following equations of value can be used to calculate *X*.

(A) $X\sum_{k=1}^{17} v^k = 50,000[v + v^2 + v^3 + v^4]$

(B) $X\sum_{k=1}^{16} 1.05^k = 50,000[1 + v + v^2 + v^3]$

(C) $X\sum_{k=0}^{17} 1.05^k = 50,000[1 + v + v^2 + v^3]$

(D) $X\sum_{k=1}^{17} 1.05^k = 50,000[1 + v + v^2 + v^3]$

(E) $X\sum_{k=0}^{17} v^k = 50,000[v^{18} + v^{19} + v^{20} + v^{21} + v^{22}]$

37.
Joe must pay liabilities of 1,000 due 6 months from now and another 1,000 due one year from now. There are two available investments:

Bond I: A 6-month bond with face amount of 1,000, an 8% nominal annual coupon rate convertible semiannually, and a 6% nominal annual yield rate convertible semiannually;
Bond II: A one-year bond with face amount of 1,000, a 5% nominal annual coupon rate convertible semiannually, and a 7% nominal annual yield rate convertible semiannually.

Calculate the amount of each bond that Joe should purchase to exactly match the liabilities.

(A) Bond I – 1, Bond II – 0.97561
(B) Bond I – 0.93809, Bond II – 1
(C) Bond I – 0.97561, Bond II – 0.94293
(D) Bond I – 0.93809, Bond II – 0.97561
(E) Bond I – 0.98345, Bond II – 0.97561

38.
Joe must pay liabilities of 2000 due one year from now and another 1000 due two years from now. He exactly matches his liabilities with the following two investments:

Mortgage I: A one year mortgage in which X is lent. It is repaid with a single payment at time one. The annual effective interest rate is 6%.

Mortgage II: A two-year mortgage in which Y is lent. It is repaid with two equal annual payments. The annual effective interest rate is 7%.

Calculate $X + Y$.

(A) 2600
(B) 2682
(C) 2751
(D) 2825
(E) 3000

39.
Joe must pay liabilities of 1,000 due one year from now and another 2,000 due three years from now. There are two available investments:

Bond I: A one-year zero-coupon bond that matures for 1000. The yield rate is 6% per year
Bond II: A two-year zero-coupon bond with face amount of 1,000. The yield rate is 7% per year.

At the present time the one-year forward rate for an investment made two years from now is 6.5%

Joe plans to buy amounts of each bond. He plans to reinvest the proceeds from Bond II in a one-year zero-coupon bond. Assuming the reinvestment earns the forward rate, calculate the total purchase price of Bond I and Bond II where the amounts are selected to exactly match the liabilities.

(A) 2584
(B) 2697
(C) 2801
(D) 2907
(E) 3000

40.
Matt purchased a 20-year par value bond with an annual nominal coupon rate of 8% payable semiannually at a price of 1722.25. The bond can be called at par value X on any coupon date starting at the end of year 15 after the coupon is paid. The lowest yield rate that Matt can possibly receive is a nominal annual interest rate of 6% convertible semiannually.

Calculate X.

(A) 1400
(B) 1420
(C) 1440
(D) 1460
(E) 1480

41.
Toby purchased a 20-year par value bond with semiannual coupons of 40 and a redemption value of 1100. The bond can be called at 1200 on any coupon date prior to maturity, starting at the end of year 15.

Calculate the maximum price of the bond to guarantee that Toby will earn an annual nominal interest rate of at least 6% convertible semiannually.

(A) 1251
(B) 1262
(C) 1278
(D) 1286
(E) 1295

42.
Sue purchased a 10-year par value bond with an annual nominal coupon rate of 4% payable semiannually at a price of 1021.50. The bond can be called at par value X on any coupon date starting at the end of year 5. The lowest yield rate that Sue can possibly receive is an annual nominal rate of 6% convertible semiannually.

Calculate X.

(A) 1120
(B) 1140
(C) 1160
(D) 1180
(E) 1200

43.
Mary purchased a 10-year par value bond with an annual nominal coupon rate of 4% payable semiannually at a price of 1021.50. The bond can be called at 100 over the par value of 1100 on any coupon date starting at the end of year 5 and ending six months prior to maturity.

Calculate the minimum yield that Mary could receive, expressed as an annual nominal rate of interest convertible semiannually.

(A) 4.7%
(B) 4.9%
(C) 5.1%
(D) 5.3%
(E) 5.5%

44.
A liability consists of a series of 15 annual payments of 35,000 with the first payment to be made one year from now.

The assets available to immunize this liability are five-year and ten-year zero-coupon bonds.

The annual effective interest rate used to value the assets and the liability is 6.2%. The liability has the same present value and duration as the asset portfolio.

Calculate the amount invested in the five-year zero-coupon bonds.

(A) 127,000
(B) 167,800
(C) 208,600
(D) 247,900
(E) 292,800

45.
You are given the following information about a loan of L that is to be repaid with a series of 16 annual payments:

(i) The first payment of 2000 is due one year from now.
(ii) The next seven payments are each 3% larger than the preceding payment.
(iii) From the 9^{th} to the 16^{th} payment, each payment will be 3% less than the preceding payment.
(iv) The loan has an annual effective interest rate of 7%.

Calculate L.

(A) 20,689
(B) 20,716
(C) 20,775
(D) 21,147
(E) 22,137

46.
The annual force of interest credited to a savings account is defined by

$$\delta_t = \frac{\frac{t^2}{100}}{3 + \frac{t^3}{150}}$$

with t in years. Austin deposits 500 into this account at time 0.

Calculate the time in years it will take for the fund to be worth 2000.

(A) 6.7
(B) 8.8
(C) 14.2
(D) 16.5
(E) 18.9

47.
A 40-year bond is purchased at a discount. The bond pays annual coupons. The amount for accumulation of discount in the 15th coupon is 194.82. The amount for accumulation of discount in the 20th coupon is 306.69.

Calculate the amount of discount in the purchase price of this bond.

(A) 13,635
(B) 13,834
(C) 16,098
(D) 19,301
(E) 21,135

48.
Tanner takes out a loan today and repays the loan with eight level annual payments, with the first payment one year from today. The payments are calculated based on an annual effective interest rate of 4.75%. The principal portion of the fifth payment is 699.68.

Calculate the total amount of interest paid on this loan.

(A) 1239
(B) 1647
(C) 1820
(D) 2319
(E) 2924

49.
Turner buys a new car and finances it with a loan of 22,000. He will make n monthly payments of 450.30 starting in one month. He will make one larger payment in n+1 months to pay off the loan. Payments are calculated using an annual nominal interest rate of 8.4%, convertible monthly. Immediately after the 18th payment he refinances the loan to pay off the remaining balance with 24 monthly payments starting one month later. This refinanced loan uses an annual nominal interest rate of 4.8%, convertible monthly.

Calculate the amount of the new monthly payment.

(A) 668
(B) 693
(C) 702
(D) 715
(E) 742

50.
Kylie bought a 7-year, 5000 par value bond with an annual coupon rate of 7.6% paid semiannually. She bought the bond with no premium or discount.

Calculate the Macaulay duration of this bond with respect to the yield rate on the bond.

(A) 5.16
(B) 5.35
(C) 5.56
(D) 5.77
(E) 5.99

51.
Krishna buys an n-year 1000 bond at par. The Macaulay duration is 7.959 years using an annual effective interest rate of 7.2%.

Calculate the estimated price of the bond, using the first-order modified approximation, if the interest rate rises to 8.0%.

(A) 940.60
(B) 942.88
(C) 944.56
(D) 947.03
(E) 948.47

52.
The prices of zero-coupon bonds are:

Maturity	Price
1	0.95420
2	0.90703
3	0.85892

Calculate the one-year forward rate, deferred two years.

(A) 0.048
(B) 0.050
(C) 0.052
(D) 0.054
(E) 0.056

53.
Sam buys an eight-year, 5000 par bond with an annual coupon rate of 5%, paid annually. The bond sells for 5000. Let d_1 be the Macaulay duration just before the first coupon is paid. Let d_2 be the Macaulay duration just after the first coupon is paid.

Calculate $\frac{d_1}{d_2}$.

(A) 0.91
(B) 0.93
(C) 0.95
(D) 0.97
(E) 1.00

54.
An insurance company must pay liabilities of 99 at the end of one year, 102 at the end of two years and 100 at the end of three years. The only investments available to the company are the following three bonds. Bond A and Bond C are annual coupon bonds. Bond B is a zero-coupon bond.

Bond	Maturity (in years)	Yield-to-Maturity (Annualized)	Coupon Rate
A	1	6%	7%
B	2	7%	0%
C	3	9%	5%

All three bonds have a par value of 100 and will be redeemed at par.

Calculate the number of units of Bond A that must be purchased to match the liabilities exactly.

(A) 0.8807
(B) 0.8901
(C) 0.8975
(D) 0.9524
(E) 0.9724

55.
Determine which of the following statements is false with respect to Redington immunization.

(A) Modified duration may change at different rates for each of the assets and liabilities as time goes by.
(B) Redington immunization requires infrequent rebalancing to keep modified duration of assets equal to modified duration of liabilities.
(C) This technique is designed to work only for small changes in the interest rate.
(D) The yield curve is assumed to be flat.
(E) The yield curve shifts in parallel when the interest rate changes.

56.
Aakash has a liability of 6000 due in four years. This liability will be met with payments of A in two years and B in six years. Aakash is employing a full immunization strategy using an annual effective interest rate of 5%.

Calculate $|A-B|$.

(A) 0
(B) 146
(C) 293
(D) 586
(E) 881

57.
Jia Wen has a liability of 12,000 due in eight years. This liability will be met with payments of 5000 in five years and B in $8+b$ years. Jia Wen is employing a full immunization strategy using an annual effective interest rate of 3%.

Calculate $\frac{B}{b}$.

(A) 2807
(B) 2873
(C) 2902
(D) 2976
(E) 3019

58.
Trevor has assets at time 2 of A and at time 9 of B. He has a liability of 95,000 at time 5. Trevor has achieved Redington immunization in his portfolio using an annual effective interest rate of 4%.

Calculate $\frac{A}{B}$.

(A) 0.7307
(B) 0.9670
(C) 1.0000
(D) 1.0132
(E) 1.3686

59.
You are given the following information about two bonds, Bond A and Bond B:

i) Each bond is a 10-year bond with semiannual coupons redeemable at its par value of 10,000, and is bought to yield an annual nominal interest rate of i, convertible semiannually.
ii) Bond A has an annual coupon rate of (i + 0.04), paid semiannually.
iii) Bond B has an annual coupon rate of (i – 0.04), paid semiannually.
iv) The price of Bond A is 5,341.12 greater than the price of Bond B.

Calculate i.

(A) 0.042
(B) 0.043
(C) 0.081
(D) 0.084
(E) 0.086

60.
A borrower takes out a 15-year loan for 400,000, with level end-of-month payments, at an annual nominal interest rate of 9% convertible monthly.

Immediately after the 36th payment, the borrower decides to refinance the loan at an annual nominal interest rate of j, convertible monthly. The remaining term of the loan is kept at twelve years, and level payments continue to be made at the end of the month. However, each payment is now 409.88 lower than each payment from the original loan.

Calculate j.

(A) 4.72%
(B) 5.75%
(C) 6.35 %
(D) 6.90%
(E) 9.14%

61.
Consider two 30-year bonds with the same purchase price. Each has an annual coupon rate of 5% paid semiannually and a par value of 1000.

The first bond has an annual nominal yield rate of 5% compounded semiannually, and a redemption value of 1200.

The second bond has an annual nominal yield rate of j compounded semiannually, and a redemption value of 800.

Calculate j.

(A) 2.20%
(B) 2.34%
(C) 3.53%
(D) 4.40%
(E) 4.69%

62.
Lucas opens a bank account with 1000 and lets it accumulate at an annual nominal interest rate of 6% convertible semiannually. Danielle also opens a bank account with 1000 at the same time as Lucas, but it grows at an annual nominal interest rate of 3% convertible monthly.

For each account, interest is credited only at the end of each interest conversion period.

Calculate the number of months required for the amount in Lucas's account to be at least double the amount in Danielle's account.

(A) 276
(B) 282
(C) 285
(D) 286
(E) 288

63.
Bill and Joe each put 10 into separate accounts at time $t = 0$, where t is measured in years. Bill's account earns interest at a constant annual effective interest rate of $K/25$, $K > 0$.

Joe's account earns interest at a force of interest, $\delta_t = \dfrac{1}{K + 0.25t}$.

At the end of four years, the amount in each account is X.

Calculate X.

(A) 20.7
(B) 21.7
(C) 22.7
(D) 23.7
(E) 24.7

64.
A borrower takes out a 50-year loan, to be repaid with payments at the end of each year. The loan payment is 2500 for each of the first 26 years. Thereafter, the payments decrease by 100 per year. Interest on the loan is charged at an annual effective rate of i ($0\% < i < 10\%$).
The principal repaid in year 26 is X.

Determine the amount of interest paid in the first year.

(A) Xv^{25}
(B) $2500v^{25} - Xv^{25}$
(C) $2500 - X$
(D) $2500 - Xv^{25}$
(E) $25Xi$

65.
John made a deposit of 1000 into a fund at the beginning of each year for 20 years.

At the end of 20 years, he began making semiannual withdrawals of 3000 at the beginning of each six months, with a smaller final withdrawal to exhaust the fund. The fund earned an annual effective interest rate of 8.16%.

Calculate the amount of the final withdrawal.

(A) 561
(B) 1226
(C) 1430
(D) 1488
(E) 2240

66.
The present value of a perpetuity paying 1 every two years with first payment due immediately is 7.21 at an annual effective rate of i.

Another perpetuity paying R every three years with the first payment due at the beginning of year two has the same present value at an annual effective rate of $i + 0.01$.

Calculate R.

(A) 1.23
(B) 1.56
(C) 1.60
(D) 1.74
(E) 1.94

67.
A loan of 10,000 is repaid with a payment made at the end of each year for 20 years. The payments are 100, 200, 300, 400, and 500 in years 1 through 5, respectively. In the subsequent 15 years, equal annual payments of X are made.
The annual effective interest rate is 5%.

Calculate X.

(A) 842
(B) 977
(C) 1017
(D) 1029
(E) 1075

68.
An investor wishes to accumulate 5000 in a fund at the end of 15 years. To accomplish this, she plans to make equal deposits of X at the end of each year for the first ten years. The fund earns an annual effective rate of 6% during the first ten years and 5% for the next five years.

Calculate X.

(A) 224
(B) 284
(C) 297
(D) 312
(E) 379

69.
A borrower takes out a 15-year loan for 65,000, with level end-of-month payments. The annual nominal interest rate of the loan is 8%, convertible monthly.

Immediately after the 12th payment is made, the remaining loan balance is reamortized. The maturity date of the loan remains unchanged, but the annual nominal interest rate of the loan is changed to 6%, convertible monthly.

Calculate the new end-of-month payment.

(A) 528
(B) 534
(C) 540
(D) 546
(E) 552

70.
College tuition is 6000 for the current school year, payable in full at the beginning of the school year. College tuition will grow at an annual rate of 5%. A parent sets up a college savings fund earning interest at an annual effective rate of 7%. The parent deposits 750 at the beginning of each school year for 18 years, with the first deposit made at the beginning of the current school year. Immediately following the 18th deposit, the parent pays tuition for the 18th school year from the fund.

The amount of money needed, in addition to the balance in the fund, to pay tuition at the beginning of the 19^{th} school year is *X*.

Calculate *X*.

(A) 1439
(B) 1545
(C) 1664
(D) 1785
(E) 1870

71.
A 1000 par value 20-year bond sells for *P* and yields a nominal interest rate of 10% convertible semiannually. The bond has 9% coupons payable semiannually and a redemption value of 1200.

Calculate *P*.

(A) 914
(B) 943
(C) 1013
(D) 1034
(E) 1097

72.
An investor purchases a 10-year callable bond with face amount of 1000 for price *P*. The bond has an annual nominal coupon rate of 10% paid semi-annually.

The bond may be called at par by the issuer on every other coupon payment date, beginning with the second coupon payment date.

The investor earns at least an annual nominal yield of 12% compounded semi-annually regardless of when the bond is redeemed.

Calculate the largest possible value of *P*.

(A) 885
(B) 892
(C) 926
(D) 965
(E) 982

73.
You are given the following term structure of interest rates:

Length of investment in years	Spot rate
1	7.50%
2	8.00%
3	8.50%
4	9.00%
5	9.50%
6	10.00%

Calculate the one-year forward rate, deferred four years, implied by this term structure.

(A) 9.5%
(B) 10.0%
(C) 11.5%
(D) 12.0%
(E) 12.5%

74.
Seth has two retirement benefit options.

His first option is to receive a lump sum of 374,500 at retirement.

His second option is to receive monthly payments for 25 years starting one month after retirement. For the first year, the amount of each monthly payment is 2000. For each subsequent year, the monthly payments are 2% more than the monthly payments from the previous year.

Using an annual nominal interest rate of 6%, compounded monthly, the present value of the second option is P.

Determine which of the following is true.

(A) P is 323,440 more than the lump sum option amount.
(B) P is 107,170 more than the lump sum option amount.
(C) The lump sum option amount is equal to P.
(D) The lump sum option amount is 60 more than P.
(E) The lump sum option amount is 64,090 more than P.

75.
A couple decides to save money for their child's first year college tuition.

The parents will deposit 1700 n months from today and another 3400 $2n$ months from today.

All deposits earn interest at a nominal annual rate of 7.2%, compounded monthly.

Calculate the maximum integral value of n such that the parents will have accumulated at least 6500 five years from today.

(A) 11
(B) 12
(C) 18
(D) 24
(E) 25

76.
Let S be the accumulated value of 1000 invested for two years at a nominal annual rate of discount d convertible semiannually, which is equivalent to an annual effective interest rate of i.

Let T be the accumulated value of 1000 invested for one year at a nominal annual rate of discount d convertible quarterly.

$S / T = (39 / 38)^4$.

Calculate i.

(A) 10.0%
(B) 10.3%
(C) 10.8%
(D) 10.9%
(E) 11.1%

77.
An investor's retirement account pays an annual nominal interest rate of 4.2%, convertible monthly.

On January 1 of year y, the investor's account balance was X. The investor then deposited 100 at the end of every quarter. On May 1 of year $(y + 10)$, the account balance was $1.9X$.

Determine which of the following is an equation of value that can be used to solve for X.

(A) $\dfrac{1.9X}{(1.0105)^{\frac{124}{3}}} + \sum_{k=1}^{42} \dfrac{100}{(1.0105)^{k-1}} = X$

(B) $X + \sum_{k=1}^{42} \dfrac{100}{(1.0035)^{3(k-1)}} = \dfrac{1.9X}{(1.0035)^{124}}$

(C) $X + \sum_{k=1}^{41} \dfrac{100}{(1.0035)^{3k}} = \dfrac{1.9X}{(1.0035)^{124}}$

(D) $X + \sum_{k=1}^{41} \dfrac{100}{(1.0105)^{k-1}} = \dfrac{1.9X}{(1.0105)^{\frac{124}{3}}}$

(E) $X + \sum_{k=1}^{42} \dfrac{100}{(1.0105)^{k-1}} = \dfrac{1.9X}{(1.0105)^{\frac{124}{3}}}$

78.
Five deposits of 100 are made into a fund at two-year intervals with the first deposit at the beginning of the first year.

The fund earns interest at an annual effective rate of 4% during the first six years and at an annual effective rate of 5% thereafter.

Calculate the annual effective yield rate earned over the investment period ending at the end of the tenth year.

(A) 4.18%
(B) 4.40%
(C) 4.50%
(D) 4.58%
(E) 4.78%

79.
John finances his daughter's college education by making deposits into a fund earning interest at an annual effective rate of 8%. For 18 years he deposits *X* at the beginning of each month.

In the 16th through the 19th years, he makes a withdrawal of 25,000 at the beginning of each year. The final withdrawal reduces the fund balance to zero.

Calculate *X*.

(A) 207
(B) 223
(C) 240
(D) 245
(E) 260

80.
Jack inherited a perpetuity-due, with annual payments of 15,000. He immediately exchanged the perpetuity for a 25-year annuity-due having the same present value. The annuity-due has annual payments of *X*.

All the present values are based on an annual effective interest rate of 10% for the first 10 years and 8% thereafter.

Calculate *X*.

(A) 16,942
(B) 17,384
(C) 17,434
(D) 17,520
(E) 18,989

81.
An investor owns a bond that is redeemable for 300 in seven years. The investor has just received a coupon of 22.50 and each subsequent semiannual coupon will be *X* more than the preceding coupon. The present value of this bond immediately after the payment of the coupon is 1050.50 assuming an annual nominal yield rate of 6% convertible semiannually.

Calculate *X*.

(A) 7.54
(B) 10.04
(C) 22.37
(D) 34.49
(E) 43.98

82.
A 30-year annuity is arranged to pay off a loan taken out today at a 5% annual effective interest rate. The first payment of the annuity is due in ten years in the amount of 1,000. The subsequent payments increase by 500 each year.

Calculate the amount of the loan.

(A) 58,283
(B) 61,197
(C) 64,021
(D) 64,257
(E) 69,211

83.
A woman worked for 30 years before retiring. At the end of the first year of employment she deposited 5000 into an account for her retirement. At the end of each subsequent year of employment, she deposited 3% more than the prior year. The woman made a total of 30 deposits.

She will withdraw 50,000 at the beginning of the first year of retirement and will make annual withdrawals at the beginning of each subsequent year for a total of 30 withdrawals. Each of these subsequent withdrawals will be 3% more than the prior year. The final withdrawal depletes the account.

The account earns a constant annual effective interest rate.

Calculate the account balance after the final deposit and before the first withdrawal.

(A) 760,694
(B) 783,948
(C) 797,837
(D) 805,541
(E) 821,379

84.
An insurance company purchases a perpetuity-due providing a geometric series of quarterly payments for a price of 100,000 based on an annual effective interest rate of i. The first and second quarterly payments are 2000 and 2010, respectively.

Calculate i.

(A) 10.0%
(B) 10.2%
(C) 10.4%
(D) 10.6%
(E) 10.8%

85.
A perpetuity provides for continuous payments. The annual rate of payment at time t is

$$\begin{cases} 1, & \text{for } 0 \le t < 10, \\ (1.03)^{t-10}, & \text{for } t \ge 10. \end{cases}$$

Using an annual effective interest rate of 6%, the present value at time t = 0 of this perpetuity is x.

Calculate x.

(A) 27.03
(B) 30.29
(C) 34.83
(D) 38.64
(E) 42.41

86.
A bank agrees to lend 10,000 now and X three years from now in exchange for a single repayment of 75,000 at the end of 10 years. The bank charges interest at an annual effective rate of 6% for the first 5 years and at a force of interest $\delta_t = \dfrac{1}{t+1}$ for $t \ge 5$.

Calculate X.

(A) 23,500
(B) 24,000
(C) 24,500
(D) 25,000
(E) 25,500

87.
A company takes out a loan of 15,000,000 at an annual effective discount rate of 5.5%. You are given:

i) The loan is to be repaid with n annual payments of 1,200,000 plus a drop payment one year after the nth payment.
ii) The first payment is due three years after the loan is taken out.

Calculate the amount of the drop payment.

(A) 79,100
(B) 176,000
(C) 321,300
(D) 959,500
(E) 1,180,300

88.
Tim takes out an n-year loan with equal annual payments at the end of each year.

The interest portion of the payment at time $(n-1)$ is equal to 0.5250 of the interest portion of the payment at time $(n-3)$ and is also equal to 0.1427 of the interest portion of the first payment.

Calculate n.

(A) 18
(B) 20
(C) 22
(D) 24
(E) 26

89.
On January 1, 2003 Mike took out a 30-year mortgage loan in the amount of 200,000 at an annual nominal interest rate of 6% compounded monthly. The loan was to be repaid by level end-of-month payments with the first payment on January 31, 2003.

Mike repaid an extra 10,000 in addition to the regular monthly payment on each December 31 in the years 2003 through 2007.

Determine the date on which Mike will make his last payment (which is a drop payment).

(A) July 31, 2013
(B) November 30, 2020
(C) December 31, 2020
(D) December 31, 2021
(E) January 31, 2022

90.
A 5-year loan of 500,000 with an annual effective discount rate of 8% is to be repaid by level end-of-year payments.

If the first four payments had been rounded up to the next multiple of 1,000, the final payment would be X.

Calculate X.

(A) 103,500
(B) 111,700
(C) 115,200
(D) 125,200
(E) 127,500

91.
A company plans to invest X at the beginning of each month in a zero-coupon bond in order to accumulate 100,000 at the end of six months. The price of each bond as a percentage of redemption value is given in the following chart:

Maturity (months)	1	2	3	4	5	6
Price	99%	98%	97%	96%	95%	94%

Calculate X given that the bond prices will not change during the six-month period.

(A) 15,667
(B) 16,078
(C) 16,245
(D) 16,667
(E) 17,271

92.
A loan of X is repaid with level annual payments at the end of each year for 10 years. You are given:

i) The interest paid in the first year is 3600; and
ii) The principal repaid in the 6^{th} year is 4871.

Calculate X.

(A) 44,000
(B) 45,250
(C) 46,500
(D) 48,000
(E) 50,000

93.
An investor purchased a 25-year bond with semiannual coupons, redeemable at par, for a price of 10,000. The annual effective yield rate is 7.05%, and the annual coupon rate is 7%.

Calculate the redemption value of the bond.

(A) 9,918
(B) 9,942
(C) 9,981
(D) 10,059
(E) 10,083

94.
Jeff has 8000 and would like to purchase a 10,000 bond. In doing so, Jeff takes out a 10 year loan of 2000 from a bank and will make interest-only payments at the end of each month at a nominal rate of 8.0% convertible monthly. He immediately pays 10,000 for a 10-year bond with a par value of 10,000 and 9.0% coupons paid monthly.

Calculate the annual effective yield rate that Jeff will realize on his 8000 over the 10-year period.

(A) 9.30%
(B) 9.65%
(C) 10.00%
(D) 10.35%
(E) 10.70%

95.
A bank issues three annual coupon bonds redeemable at par, all with the same term, price, and annual effective yield rate.

The first bond has face value 1000 and annual coupon rate 5.28%.

The second bond has face value 1100 and annual coupon rate 4.40%.

The third bond has face value 1320 and annual coupon rate r.

Calculate r.

(A) 2.46%
(B) 2.93%
(C) 3.52%
(D) 3.67%
(E) 4.00%

96.
An investor owns a bond that is redeemable for 250 in 6 years from now. The investor has just received a coupon of c and each subsequent semiannual coupon will be 2% larger than the preceding coupon. The present value of this bond immediately after the payment of the coupon is 582.53 assuming an annual effective yield rate of 4%.

Calculate c.

(A) 32.04
(B) 32.68
(C) 40.22
(D) 48.48
(E) 49.45

97.
An n-year bond with annual coupons has the following characteristics:

i) The redemption value at maturity is 1890;
ii) The annual effective yield rate is 6%;
iii) The book value immediately after the third coupon is 1254.87; and
iv) The book value immediately after the fourth coupon is 1277.38.

Calculate n.

(A) 16
(B) 17
(C) 18
(D) 19
(E) 20

98.
An n-year bond with semiannual coupons has the following characteristics:

i) The par value and redemption value are 2500;
ii) The annual coupon rate is 7% payable semi-annually;
iii) The annual nominal yield to maturity is 8% convertible semiannually; and
iv) The book value immediately after the fourth coupon is 8.44 greater than the book value immediately after the third coupon.

Calculate n.

(A) 6.5
(B) 7.0
(C) 9.5
(D) 12.0
(E) 14.0

99.
The one-year forward rates, deferred *t* years, are estimated to be:

Year (*t*)	0	1	2	3	4
Forward Rate	4%	6%	8%	10%	12%

Calculate the spot rate for a zero-coupon bond maturing three years from now.

(A) 4%
(B) 5%
(C) 6%
(D) 7%
(E) 8%

100.
Annuity A pays 1 at the beginning of each year for three years. Annuity B pays 1 at the beginning of each year for four years.

The Macaulay duration of Annuity A at the time of purchase is 0.93. Both annuities offer the same yield rate.

Calculate the Macaulay duration of Annuity B at the time of purchase.

(A) 1.240
(B) 1.369
(C) 1.500
(D) 1.930
(E) 1.965

101.
Cash flows are 40,000 at time 2 (in years), 25,000 at time 3, and 100,000 at time 4. The annual effective yield rate is 7.0%.

Calculate the Macaulay duration.

(A) 2.2
(B) 2.3
(C) 3.1
(D) 3.3
(E) 3.4

102.
Rhonda purchases a perpetuity providing a payment of 1 at the beginning of each year. The perpetuity's Macaulay duration is 30 years.

Calculate the modified duration of this perpetuity.

(A) 28.97
(B) 29.00
(C) 29.03
(D) 29.07
(E) 29.10

103.
Which of the following statements regarding immunization are true?

I. If long-term interest rates are lower than short-term rates, the need for immunization is reduced.
II. Either Macaulay or modified duration can be used to develop an immunization strategy.
III. Both processes of matching the present values of the flows or the flows themselves will produce exact matching.

(A) I only
(B) II only
(C) III only
(D) I, II and III
(E) The correct answer is not given by (A), (B), (C), or (D).

104.
A company owes 500 and 1000 to be paid at the end of year one and year four, respectively. The company will set up an investment program to match the duration and the present value of the above obligation using an annual effective interest rate of 10%.

The investment program produces asset cash flows of X today and Y in three years.

Calculate X and determine whether the investment program satisfies the conditions for Redington immunization.

(A) $X = 75$ and the Redington immunization conditions are not satisfied.
(B) $X = 75$ and the Redington immunization conditions are satisfied.
(C) $X = 1138$ and the Redington immunization conditions are not satisfied.
(D) $X = 1138$ and the Redington immunization conditions are satisfied.
(E) $X = 1414$ and the Redington immunization conditions are satisfied.

105.
An insurance company has a known liability of 1,000,000 that is due 8 years from now. The technique of full immunization is to be employed. Asset I will provide a cash flow of 300,000 exactly 6 years from now. Asset II will provide a cash flow of X, exactly y years from now, where $y > 8$.

The annual effective interest rate is 4%.

Calculate X.

(A) 697,100
(B) 698,600
(C) 700,000
(D) 701,500
(E) 702,900

106.
A company has liabilities of 573 due at the end of year 2 and 701 due at the end of year 5.

A portfolio comprises two zero-coupon bonds, Bond A and Bond B.

Determine which portfolio produces a Redington immunization of the liabilities using an annual effective interest rate of 7.0%.

(A) Bond A: 1-year, current price 500; Bond B: 6-years, current price 500
(B) Bond A: 1-year, current price 572; Bond B: 6-years, current price 428
(C) Bond A: 3-years, current price 182; Bond B: 4-years, current price 1092
(D) Bond A: 3-years, current price 637; Bond B: 4-years, current price 637
(E) Bond A: 3.5 years, current price 1000; Bond B: Not used

107.
A company has liabilities of 402.11 due at the end of each of the next three years. The company will invest 1000 today to fund these payouts. The only investments available are one-year and three-year zero-coupon bonds, and the yield curve is flat at a 10% annual effective rate. The company wishes to match the duration of its assets to the duration of its liabilities.

Determine how much the company should invest in each bond.

(A) 366 in the one-year bond and 634 in the three-year bond.
(B) 484 in the one-year bond and 516 in the three-year bond.
(C) 500 in the one-year bond and 500 in the three-year bond.
(D) 532 in the one-year bond and 468 in the three-year bond.
(E) 634 in the one-year bond and 366 in the three-year bond.

108.
You are given the following information about a company's liabilities:

- Present value: 9697
- Macaulay duration: 15.24
- Macaulay convexity: 242.47

The company decides to create an investment portfolio by making investments into two of the following three zero-coupon bonds: 5-year, 15-year, and 20-year. The company would like its position to be Redington immunized against small changes in yield rate.

The annual effective yield rate for each of the bonds is 7.5%.

Determine which of the following portfolios the company should create.

(A) Invest 3077 for the 5-year bond and 6620 for the 20-year bond.
(B) Invest 6620 for the 5-year bond and 3077 for the 20-year bond.
(C) Invest 465 for the 15-year bond and 9232 for the 20-year bond.
(D) Invest 4156 for the 15-year bond and 5541 for the 20-year bond.
(E) Invest 9232 for the 15-year bond and 465 for the 20-year bond.

109.
A bank accepts a 20,000 deposit from a customer on which it guarantees to pay an annual effective interest rate of 10% for two years. The customer needs to withdraw half of the accumulated value at the end of the first year. The customer will withdraw the remaining value at the end of the second year.

The bank has the following investment options available, which may be purchased in any quantity:

Bond H:	A one-year zero-coupon bond yielding 10% annually
Bond I:	A two-year zero-coupon bond yielding 11% annually
Bond J:	A two-year bond that sells at par with 12% annual coupons

Any portion of the 20,000 deposit that is not needed to be invested in bonds is retained by the bank as profit.

Determine which of the following investment strategies produces the highest profit for the bank and is guaranteed to meet the customer's withdrawal needs.

(A) 9,091 in Bond H, 8,264 in Bond I, 2,145 in Bond J
(B) 10,000 in Bond H, 10,000 in Bond I
(C) 10,000 in Bond H, 9,821 in Bond I
(D) 8,910 in Bond H, 731 in Bond I, 10,000 in Bond J
(E) 8,821 in Bond H, 10,804 in Bond J

110.
An insurance company wants to match liabilities of 25,000 payable in one year and 20,000 payable in two years with specific assets. The following assets are currently available:

i) One-year bond with an annual coupon of 6.75% at par
ii) Two-year bond with annual coupons of 4.50% at par
iii) Two-year zero-coupon bond yielding 5.00% annual effective

Calculate the smallest amount the company needs to disburse today to purchase assets that will exactly match these liabilities.

(A) 41,220
(B) 41,390
(C) 41,560
(D) 41,660
(E) 41,750

111.
Martha leaves an estate of 500,000. Interest on this estate is paid to John for the first X years at the end of each year. Karen receives annual interest payments from the end of year X+1 forever. At an annual effective interest rate of 5%, the present value of Karen's interest payments is 1.59 times the present value of John's.

Calculate X.

(A) 6
(B) 7
(C) 8
(D) 9
(E) 10

112.
At time 0, Cheryl deposits X into a bank account that credits interest at an annual effective rate of 7%. At time 3, Gomer deposits 1000 into a different bank account that credits simple interest at an annual rate of y%. At time 5, the annual forces of interest on the two accounts are equal, and Gomer's account has accumulated to Z.

Calculate Z.

(A) 1160
(B) 1200
(C) 1390
(D) 1400
(E) 1510

113.
Which of the following is an expression for the present value of a perpetuity with annual payments of 1, 2, 3, …, where the first payment will be made at the end of n years, using an annual effective interest rate of i?

(A) $\dfrac{\ddot{a}_{\overline{n}|} - nv^n}{i}$

(B) $\dfrac{n - a_{\overline{n}|}}{i}$

(C) $\dfrac{v^n}{d}$

(D) $\dfrac{v^n}{d^2}$

(E) $\dfrac{v^n}{di}$

114.
Jennifer establishes an investment account to pay for college expenses for her daughter. She plans to invest X at the beginning of each month for the next 21 years. Beginning at the end of the 18th year, she will withdraw 20,000 annually. The final withdrawal at the end of the 21st year will exhaust the account. She anticipates earning an annual effective yield of 8% on the investment.

Calculate X.

(A) 137.90
(B) 142.80
(C) 146.40
(D) 150.60
(E) 154.30

115.
For a given interest rate $i > 0$, the present value of a 20-year continuous annuity of one dollar per year is equal to 1.5 times the present value of a 10-year continuous annuity of one dollar per year.

Calculate the accumulated value of a 7-year continuous annuity of one dollar per year.

(A) 5.36
(B) 5.55
(C) 8.70
(D) 9.01
(E) 9.33

116.
An annuity having n payments of 1 has a present value of X. The first payment is made at the end of three years and the remaining payments are made at seven-year intervals thereafter.

Determine X.

(A) $\dfrac{a_{\overline{7n+3}|} - a_{\overline{3}|}}{s_{\overline{3}|}}$

(B) $\dfrac{a_{\overline{7n+3}|} - a_{\overline{3}|}}{a_{\overline{7}|}}$

(C) $\dfrac{a_{\overline{7n+3}|} - a_{\overline{7}|}}{a_{\overline{3}|}}$

(D) $\dfrac{a_{\overline{7n+3}|} - a_{\overline{7}|}}{a_{\overline{7}|}}$

(E) $\dfrac{a_{\overline{7n+3}|} - a_{\overline{7}|}}{s_{\overline{3}|}}$

117.
At an annual effective interest rate of 10.9%, each of the following are equal to X:

- The accumulated value at the end of n years of an n-year annuity-immediate paying 21.80 per year.
- The present value of a perpetuity-immediate paying 19,208 at the end of each n-year period.

Calculate X.

(A) 1555
(B) 1750
(C) 1960
(D) 2174
(E) There is not enough information given to calculate X.

118.
An investor decides to purchase a five-year annuity with an annual nominal interest rate of 12% convertible monthly for a price of X.

Under the terms of the annuity, the investor is to receive 2 at the end of the first month. The payments increase by 2 each month thereafter.

Calculate *X*.

(A) 2015
(B) 2386
(C) 2475
(D) 2500
(E) 2524

119.
A perpetuity-due with semi-annual payments consists of two level payments of 300, followed by a series of increasing payments. Beginning with the third payment, each payment is 200 larger than the preceding payment.

Using an annual effective interest rate of *i*, the present value of the perpetuity is 475,000.

Calculate *i*.

(A) 4.05%
(B) 4.09%
(C) 4.13%
(D) 4.17%
(E) 4.21%

120.
At an annual effective interest rate of 6%, the present value of a perpetuity immediate with successive annual payments of 6, 8, 10, 12, ..., is equal to *X*.

Calculate *X*.

(A) 100
(B) 656
(C) 695
(D) 1767
(E) 2222

121.
A company is considering investing in a particular project. The project requires an investment of *X* today. Additional investments are required at the beginning of each of the next five years, with each year's investment 5% greater than the previous year's investment.

The investment is expected to produce an income of 100 per year at the end of each year forever, with the first payment expected at the end of the first year.

At an annual effective interest rate of 10.25%, the project has a net present value of zero.

Calculate *X*.

(A) 183
(B) 192
(C) 205
(D) 215
(E) 225

122.
A perpetuity-due with annual payments consists of ten level payments of X followed by a series of increasing payments. Beginning with the eleventh payment, each payment is 1.5% larger than the preceding payment.

Using an annual effective interest rate of 5%, the present value of the perpetuity is 45,000.

Calculate X.

(A) 1,679
(B) 1,696
(C) 1,737
(D) 1,763
(E) 1,781

123.
A family purchases a perpetuity-immediate that provides annual payments that decrease by 0.4% each year. The price of the perpetuity is 10,000 at an annual force of interest of 0.06.

Calculate the amount of the perpetuity's first payment.

(A) 604
(B) 620
(C) 640
(D) 658
(E) 678

124.
Company *X* received the approval to start no more than two projects in the current calendar year.

Three different projects were recommended, each of which requires an investment of 800 to be made at the beginning of the year.

The cash flows for each of the three projects are as follows:

End of year	Project A	Project B	Project C
1	500	500	500
2	500	300	250
3	–175	–175	–175
4	100	150	200
5	0	200	200

The company uses an annual effective interest rate of 10% to discount its cash flows.

Determine which combination of projects the company should select.

(A) Projects A and B
(B) Projects B and C
(C) Projects A and C
(D) Project A only
(E) Project B only

125.
A borrower took out a loan of 100,000 and promised to repay it with a payment at the end of each year for 30 years.

The amount of each of the first ten payments equals the amount of interest due. The amount of each of the next ten payments equals 150% of the amount of interest due. The amount of each of the last ten payments is *X*.

The lender charges interest at an annual effective rate of 10%.

Calculate *X*.

(A) 3,204
(B) 5,675
(C) 7,073
(D) 9,744
(E) 11,746

126.
A borrower takes out a loan of 4000 at an annual effective interest rate of 6%.

Starting at the end of the fifth year, the loan is repaid by annual payments, each of which equals 600 except for a final balloon payment that is less than 1000.

Calculate the final balloon payment.

(A) 616
(B) 639
(C) 642
(D) 688
(E) 696

127.
A loan of 20,000 is repaid by a payment of X at the end of each year for 10 years. The loan has an annual effective interest rate of 11% for the first five years and 12% thereafter.

Calculate X.

(A) 2739.5
(B) 3078.5
(C) 3427.5
(D) 3467.5
(E) 3484.5

128.
A 16-year loan of L is repaid with a payment at the end of each year. During the first eight years, the payment is 100. During the final eight years, the payment is 300. Interest is charged on the loan at an annual effective rate of i, such that $1/(1+i)^8 > 0.3$.

After the first payment of 100 is made, the outstanding principal is $L + 25$.

Calculate the outstanding balance on the loan immediately after the eighth annual payment of 100 has been made.

(A) 1660
(B) 1760
(C) 1870
(D) 1970
(E) 2080

129.
An entrepreneur takes out a business loan for 60,000 with a nominal annual interest rate compounded monthly. The loan is scheduled to be paid off with level monthly payments, plus a final drop payment. All payments will be made at the end of the month.

The principal portion of the payment is 1,400 for the first month and 1,414 for the second month.

Calculate the drop payment.

(A) 780
(B) 788
(C) 1183
(D) 1676
(E) 1692

130.
A student borrows money to pay for university tuition. He borrows 1000 at the end of each month for four years. No payments are made to repay the loan until the end of five years.

The loan accumulates interest at a 6% nominal interest rate convertible monthly for the first two years and at an 8% nominal interest rate convertible monthly for the following two years.

Calculate the loan balance at the end of four years immediately following the receipt of the final 1000.

(A) 54,098
(B) 55,224
(C) 55,762
(D) 56,134
(E) 56,350

131.
A loan is amortized with level monthly payments at an annual effective interest rate of 10%. The amount of principal repaid in the 6th month is 500.

Calculate the principal repaid in the 30th month.

(A) 500
(B) 555
(C) 605
(D) 705
(E) 805

132.
Seth repays a 30-year loan with a payment at the end of each year. Each of the first 20 payments is 1200, and each of the last 10 payments is 900. Interest on the loan is at an annual effective rate of i, $i > 0$. The interest portion of the 11th payment is twice the interest portion of the 21st payment.

Calculate the interest portion of the 21st payment.

(A) 250
(B) 275
(C) 300
(D) 325
(E) There is not enough information to calculate the interest portion of the 21st payment.

133.
John took out a 20-year loan of 85,000 on July 1, 2005 at an annual nominal interest rate of 6% compounded monthly. The loan was to be paid by level monthly payments at the end of each month with the first payment on July 31, 2005.

Right after the regular monthly payment on June 30, 2009, John refinanced the loan at a new annual nominal rate of 5.40% compounded monthly, and the remaining balance will be paid with monthly payments beginning July 31, 2009. The amount of each payment is 500 except for a final drop payment.

Calculate the date of John’s last payment.

(A) July 31, 2022
(B) April 30, 2030
(C) May 31, 2030
(D) April 30, 2031
(E) May 31, 2031

134.
A 25-year loan is being repaid with annual payments of 1300 at an annual effective rate of interest of 7%. The borrower pays an additional 2600 at the time of the 5th payment and wants to repay the remaining balance over 15 years.

Calculate the revised annual payment.

(A) 1054.58
(B) 1226.65
(C) 1300.00
(D) 1369.38
(E) 1512.12

135.
A 1000-par value 30-year bond has an annual coupon rate of 7% paid semiannually. After an initial 10-year period of call protection, the bond is callable immediately following the payment of any of the 20th through the 59th coupons.

i) If the bond is called before payment of the 40th coupon, the redemption value is 1250.
ii) If the bond is called immediately after the payment of any of the 40th through the 59th coupons, the redemption value is 1125.
iii) If the bond is not called, it will be redeemed at par.

To ensure that the bond will provide at least an annual nominal yield rate of 5% convertible semiannually, it must be assumed that the bond will be called or redeemed immediately after the payment of the *n*th coupon.

Calculate *n*.

(A) 20
(B) 39
(C) 40
(D) 59
(E) 60

136.
Bond X is a 20-year bond with annual coupons and the following characteristics:

i) Par value is 1000.
ii) The annual coupon rate is 10%.
iii) Bond X is callable at par on any of the last five coupon dates.

Calculate the maximum purchase price for Bond X that will guarantee an annual effective yield rate of at least 5%.

(A) 1519
(B) 1542
(C) 1570
(D) 1596
(E) 1623

137.
A life insurance company invests two million in a 10-year zero-coupon bond and four million in a 30-year zero-coupon bond. The annual effective yield rate for both bonds is 8%.

When the 10-year bond matures, the company reinvests the proceeds in another 10-year zero-coupon bond. At that time the bond yield rate is 12% annual effective.

After 20 years from the initial investment, the 30-year bond is sold to yield an annual effective rate of 10% to the buyer. The maturity of the second 10-year bond and the sale of the 30-year bond result in a gain of X on the company's initial investment of six million.

Calculate X.

(A) 23 million
(B) 29 million
(C) 32 million
(D) 34 million
(E) 42 million

138.
You are given the following information about a 20-year bond with face amount 7500:

i) The bond has an annual coupon rate of 7.4% paid semiannually.
ii) The purchase price results in an annual nominal yield rate to the investor of 5.3% convertible semiannually.
iii) The amount for amortization of premium in the fourth coupon payment is 28.31.

Calculate the redemption value of the bond.

(A) 7660
(B) 7733
(C) 7795
(D) 7879
(E) 7953

139.
Claire purchases an eight-year callable bond with a 10% annual coupon rate payable semiannually. The bond has a face value of 3000 and a redemption value of 2800.

The purchase price assumes the bond is called at the end of the fourth year for 2900, and provides an annual effective yield of 10.0%.

Immediately after the first coupon payment is received, the bond is called for 2960. Claire's annual effective yield rate is *i*.

Calculate *i*.

(A) 9.8%
(B) 10.1%
(C) 10.8%
(D) 11.1%
(E) 11.8%

140.
You are given the following information about a 20-year bond with face amount 1000:

i) The bond has an annual coupon rate of *r* payable semiannually and is redeemable at par.
ii) The nominal annual yield rate convertible semiannually is 7.2%.
iii) The amount for accumulation of discount in the seventh coupon payment is 4.36.

Calculate *r*.

(A) 2.1%
(B) 4.0%
(C) 4.3%
(D) 6.0%
(E) 6.9%

141.
Bond A and Bond B are both annual coupon, five-year, 10,000 par value bonds bought to yield an annual effective rate of 4%.

i) Bond A has an annual coupon rate of r%, a redemption value that is 10% below par, and a price of P.
ii) Bond B has an annual coupon rate of $(r+1)$%, a redemption value that is 10% above par, and a price of $1.2P$.

Calculate r %.

(A) 5.85%
(B) 6.85%
(C) 7.85%
(D) 8.85%
(E) 9.85%

142.
You are given the following information about an n-year bond, where $n > 10$:

i) The bond pays 8% semiannual coupons and has face amount 1000.
ii) The bond is redeemable at par.
iii) The bond is callable at par 5 years after issue or 10 years after issue.
iv) P is the price to guarantee a yield of 6.8% convertible semiannually and Q is the price to guarantee a yield of 8.8% convertible semiannually.
v) $|P - Q| = 123.36$.

Calculate n.

(A) 11
(B) 15
(C) 19
(D) 22
(E) 26

143.
An insurer enters into a four-year contract today. The contract requires the insured to deposit 500 into a fund that earns an annual effective rate of 5.0%, and from which all claims will be paid. The insurer expects that 100 in claims will be paid at the end of each year, for the next four years. At the end of the fourth year, after all claims are paid, the insurer is required to return 75% of the remaining fund balance to the insured.

To issue this policy, the insurer incurs 100 in expenses today. It also collects a fee of 125 at the end of two years.

Calculate the insurer's yield rate.

(A) 9%
(B) 24%
(C) 39%
(D) 54%
(E) 69%

144.
An investor buys a perpetuity-immediate providing annual payments of 1, with an annual effective interest rate of i and Macaulay duration of 17.6 years.

Calculate the Macaulay duration in years using an annual effective interest rate of $2i$ instead of i.

(A) 8.8
(B) 9.3
(C) 9.8
(D) 34.2
(E) 35.2

145.
An investor purchases two bonds. The bonds have the same annual effective yield rate i, with $i > 0$.

With respect to the annual effective yield rate, their modified durations are a years and b years, with $0 < a < b$.

One of these two bonds has a Macaulay duration of c years, with $a < c < b$.

Determine which of the following is an expression, in years, for the Macaulay duration of the other bond.

(A) bc/a
(B) ac/b
(C) ab/c
(D) $b + c – a$
(E) $a + c – b$

146.
A 20-year bond priced to have an annual effective yield of 10% has a Macaulay duration of 11. Immediately after the bond is priced, the market yield rate increases by 0.25%. The bond's approximate percentage price change, using a first-order Macaulay approximation, is X.

Calculate X.

(A) –2.22%
(B) –2.47%
(C) –2.50%
(D) –2.62%
(E) –2.75%

147.
Determine which of the following statements regarding asset-liability management techniques is true.

(A) Redington immunization requires that the convexity of the liabilities is greater than the convexity of the assets.
(B) An advantage of the Redington immunization technique over the cash-flow matching technique is that the portfolio manager has more investment choices available.
(C) Both Redington immunization and full immunization are based on the assumption that the yield curve has higher yields for longer term investments.
(D) A fully immunized portfolio ensures that the present value of assets will exceed the present value of liabilities with non-parallel shifts in the yield curve.
(E) A cash-flow matched portfolio requires less rebalancing than a Redington immunized portfolio, but more rebalancing than a fully immunized portfolio.

148.
A company has liabilities that require it to make payments of 1000 at the end of each of the next five years. The only investments available to the company are as follows:

Investment	Price	Subsequent Cash Flows
J	1500	500 at the end of each year for 5 years
K	500	1000 at the end of year 5
L	1000	500 at the end of each year for 4 years
M	4000	1000 at the end of each year for 5 years

The company is able to purchase as many of each investment as it wants, but only in whole units. The company's investment objective is to be fully immunized over the next five years.

Calculate the lowest possible cost to achieve this objective.

(A) 1500
(B) 2000
(C) 2500
(D) 3000
(E) 4000

149.
A railroad company is required to pay 79,860, which is due three years from now. The company invests 15,000 in a bond with modified duration 1.80, and 45,000 in a bond with modified duration *D*mod, to Redington immunize its position against small changes in the yield rate.

The annual effective yield rate for each of the bonds is 10%.

Calculate *D*mod.

(A) 2.73
(B) 3.04
(C) 3.34
(D) 3.40
(E) 3.65

150.
A company must pay liabilities of 1000 at the end of year 1 and *X* at the end of year 2.
The only investments available are:

i) One-year zero-coupon bonds with an annual effective yield of 5%
ii) Two-year bonds with a par value of 1000 and 10% annual coupons, with an annual effective yield of 6%

The company constructed a portfolio that creates an exact cash flow matching strategy for these liabilities. The total purchase price of this portfolio is 1783.76.

Calculate the amount invested in the one-year zero-coupon bonds.

(A) 784
(B) 831
(C) 871
(D) 915
(E) 935

151.
A company must pay liabilities of 4000 and 6000 at the end of years one and two, respectively. The only investments available to the company are one-year zero-coupon bonds with an annual effective yield of 8% and two-year zero-coupon bonds with an annual effective yield of 11%.

Determine how much the company must invest today to exactly match its liabilities.

(A) 8,473
(B) 8,573
(C) 8,848
(D) 9,109
(E) 10,000

152.
A 20-year bond priced to have an annual effective yield of 10% has a Macaulay duration of 11. Immediately after the bond is priced, the market yield rate increases by 0.25%. The bond's approximate percentage price change, using a first-order modified approximation, is *X*.

Calculate *X*.

(A) –2.22%
(B) –2.47%
(C) –2.50%
(D) –2.62%
(E) –2.75%

153.
Krishna buys an n-year 1000 bond at par. The Macaulay duration is 7.959 years using an annual effective interest rate of 7.2%.

Calculate the estimated price of the bond, using the first-order Macaulay approximation, if the interest rate rises to 8.0%.

(A) 940.60
(B) 942.54
(C) 944.56
(D) 947.03
(E) 948.47

154.
A bond has a modified duration of 8 and a price of 112,955 calculated using an annual effective interest rate of 6.4%.

E_{MAC} is the estimated price of this bond at an interest rate of 7.0% using the first-order Macaulay approximation.

E_{MOD} is the estimated price of this bond at an interest rate of 7.0% using the first-order modified approximation.

Calculate $E_{MAC} - E_{MOD}$.

(A) 91
(B) 102
(C) 116
(D) 127
(E) 143

155.
SOA Life Insurance Life Insurance Company has a portfolio of two bonds:

- Bond 1 is a bond with a Macaulay duration of 7.28 and a price of 35,000; and
- Bond 2 is a bond with a Macaulay duration of 12.74 and a price of 65,000.

The price and Macaulay duration for both bonds were calculated using an annual effective interest rate of 4.32%.

Bailey estimates the value of this portfolio at an interest rate of i using the first-order Macaulay approximation to be 105,000.

Determine i.

(A) 3.49%
(B) 3.62%
(C) 3.85%
(D) 3.92%
(E) 4.03%

156.
Graham is the beneficiary of an annuity due. At an annual effective interest rate of 5%, the present value of payments is 123,000 and the modified duration is D_{MOD}.

Tyler uses the first-order Macaulay approximation to estimate the present value of Graham's annuity due at an annual effective interest rate was 5.4%. Tyler estimates the present value to be 121,212.

Calculate D_{MOD}, the modified duration of Graham's annuity at 5%.

(A) 3.67
(B) 3.75
(C) 3.85
(D) 3.95
(E) 4.04

157.
A company owes 1000 one year from now and 1000 two years from now.
Which of the following demonstrates a strategy to use exact cash-flow matching between assets and liabilities?

I. The company purchases a one-year zero-coupon bond and a two-year zero-coupon bond, each with a face amount of 1000.
II. The company deposits 1859.41 into an account that currently earns an annual effective interest rate of 5% that is subject to change in one year.
III. The company purchases an asset that has the same duration as the liabilities and a larger convexity.

(A) I only
(B) II only
(C) III only
(D) I, II, and III
(E) The correct answer is not given by (A), (B), (C) or (D).

158.
Two 15-year par value bonds, X and Y, each pay an annual coupon of 200 at the end of the year. The face amount of Bond X is one-half the face amount of Bond Y.

At an annual effective yield of *i*, the price of Bond X is 2695.39 and the price of Bond Y is 3490.78.

Calculate the coupon rate for Bond X.

(A) 6.3%
(B) 7.4%
(C) 8.8%
(D) 10.0%
(E) 11.4%

159.
Bank P offers a 3-year certificate of deposit that pays an annual effective interest rate of 4%. In addition, a bonus of 2% of the initial investment is paid at the end of the 3-year period.

Bank Q offers a 3-year certificate of deposit without any bonus.

Calculate the annual effective interest rate that Bank Q would have to offer to produce the same annual yield as the certificate from Bank P.

(A) 4.3%
(B) 4.4%
(C) 4.5%
(D) 4.6%
(E) 4.7%

160.
A 20-year arithmetically increasing annuity-due sells for 600,000 and provides annual payments. The first payment is X, and each payment thereafter is X more than the previous payment.

A 25-year arithmetically increasing annuity-due provides annual payments. The first payment is X, and each payment thereafter is X more than the previous payment.

The prices of the two annuities are calculated using a continuously compounded annual interest rate of 6%.

Calculate the price of the 25-year annuity.

(A) 667,026
(B) 668,707
(C) 750,000
(D) 779,336
(E) 782,712

161.
A small business takes out a 10-year loan with level end-of-quarter payments. The payments are based on an annual nominal interest rate of 12% convertible quarterly.

The amount of principal repaid in the 15^{th} payment is 10,030.27.

Calculate the amount of interest paid in the 25^{th} payment.

(A) 7,521
(B) 8,151
(C) 9,467
(D) 10,030
(E) 11,601

162.
Trish had a loan with a balance of 4000 at the beginning of month 1. Starting with month 1, and every month thereafter, she made a payment of X in the middle of the month. At the beginning of month 4, and every 6 months thereafter, she borrowed an additional 800.

Trish's loan balance became 4000 again at the end of month 36.

The annual nominal interest rate for the loan is 26.4%, convertible quarterly.

Determine which of the following is an equation of value that can be used to solve for X.

(A) $$\sum_{n=1}^{6} \frac{800}{\left(1+\frac{0.2640}{4}\right)^{2n-1}} = \sum_{n=1}^{36} \frac{X}{\left(1+\frac{0.2640}{4}\right)^{\frac{n-0.5}{3}}}$$

(B) $$\sum_{n=1}^{6} \frac{800}{\left(1+\frac{0.2640}{12}\right)^{6n-3}} = \sum_{n=1}^{36} \frac{X}{\left(1+\frac{0.2640}{12}\right)^{n-0.5}}$$

(C) $$4000+\sum_{n=1}^{6} \frac{800}{\left(1+\frac{0.2640}{4}\right)^{\frac{6n-2}{3}}} = \frac{4000}{\left(1+\frac{0.2640}{4}\right)^{12}} + \sum_{n=1}^{36} \frac{X}{\left(1+\frac{0.2640}{4}\right)^{\frac{n}{3}}}$$

(D) $$4000+\sum_{n=1}^{6} \frac{800}{\left(1+\frac{0.2640}{12}\right)^{6n-3}} = \frac{4000}{\left(1+\frac{0.2640}{12}\right)^{36}} + \sum_{n=1}^{36} \frac{X}{\left(1+\frac{0.2640}{12}\right)^{n-0.5}}$$

(E) $$4000+\sum_{n=1}^{6} \frac{800}{\left(1+\frac{0.2640}{4}\right)^{2n-1}} = \frac{4000}{\left(1+\frac{0.2640}{4}\right)^{12}} + \sum_{n=1}^{36} \frac{X}{\left(1+\frac{0.2640}{4}\right)^{\frac{n-0.5}{3}}}$$

163.
A bank issues two 20-year bonds, A and B, each with annual coupons, an annual effective yield rate of 10%, and a face amount of 1000. The total combined price of these two bonds is 1600.

Bond B's annual coupon rate is equal to Bond A's annual coupon rate plus 1 percentage point.

Calculate the annual coupon rate of Bond A.

(A) 6.46%
(B) 7.15%
(C) 7.29%
(D) 8.02%
(E) 8.90%

164.
An annuity provides level payments of 1000 every six months for a fixed period.

Using an annual effective interest rate of i, the future value of this annuity at the time of the last payment is 19,549.25 and the present value of this annuity at the time of the first payment is 7,968.89.

Calculate i.

(A) 7.4%
(B) 8.5%
(C) 15.4%
(D) 17.0%
(E) 17.7%

165.
An annuity provides the following payments:

i) X at the beginning of each year for 20 years, starting today
ii) $4X$ at the beginning of each year for 30 years, starting 20 years from today

Calculate the Macaulay duration of this annuity using an annual effective interest rate of 2%.

(A) 27.32
(B) 27.87
(C) 28.30
(D) 33.53
(E) 35.41

166.
An investor deposits 100 into a bank account at time 0. The bank credits interest at an annual nominal interest rate of i, compounded semi-annually.

The total amount of interest credited in the twelfth year is twice the amount of interest credited in the fifth year.

Calculate i.

(A) 10.15%
(B) 10.24%
(C) 10.32%
(D) 10.41%
(E) 10.48%

167.
Let A and B be bonds with semiannual coupons as described in the table below:

Bond	Price	Annual coupon rate	Par	Years to redemption	Annual nominal yield rate convertible semiannually
A	X	8%	1000	5	6%
B	X	y	1000	5	7%

Calculate y.

(A) 8.45%
(B) 8.65%
(C) 8.85%
(D) 9.05%
(E) 9.25%

168.
A borrower takes out a 15-year loan at an annual effective interest rate of i with payments of 50 at the end of each year.

The borrower decides to pay off the loan early by making extra payments of 30 with each of the sixth through tenth regularly scheduled payments. As a result, the loan will be paid off at the end of 10 years (instead of 15).

Determine which of the following equations of value is correct.

(A) $50a_{\overline{15|}} = 50a_{\overline{10|}} + 30a_{\overline{5|}}$

(B) $50s_{\overline{15|}} = 50s_{\overline{10|}} + 30s_{\overline{5|}}$

(C) $50v^5 s_{\overline{15|}} = 50s_{\overline{10|}} + 30s_{\overline{5|}}$

(D) $50s_{\overline{15|}} = 50s_{\overline{10|}} + 30v^5 s_{\overline{5|}}$

(E) $50s_{\overline{15|}} = 50s_{\overline{10|}} + 30(1+i)^5 s_{\overline{5|}}$

169.
A borrower takes out a loan to be repaid over 20 years. The first payment is 1102 payable at the end of the first month. Each subsequent monthly payment is five more than the previous month's payment.

Calculate the accumulated value of the payments at the end of 15 years using an annual effective interest rate of 6.5%.

(A) 442,031
(B) 443,525
(C) 445,578
(D) 447,287
(E) 448,547

170.
Fund J begins with a balance of 20,000 and earns an annual effective rate of 6.5%. At the end of each year, the interest earned and an additional 1000 is withdrawn from the fund so that by the end of the 20th year, the fund is depleted.

The annual withdrawals of interest and principal are deposited into Fund K, which earns an annual effective rate of 8.25%. At the end of the 20^{th} year, the accumulated value of Fund K is x.

Calculate x.

(A) 39,332
(B) 54,818
(C) 84,593
(D) 86,902
(E) 97,631

171.
Company Q invests X at the end of each year for 25 years at an annual effective interest rate of 9%. Company R invests 100 at the end of each year for 25 years at an annual effective interest rate of 9%, but, at the end of each year, the interest earned is reinvested at an annual effective interest rate of 8%.

Immediately after the 25^{th} payment, Company R's total investment, including the reinvested interest, has the same value as Company Q's investment.

Calculate X.

(A) 91.22
(B) 91.93
(C) 92.67
(D) 93.41
(E) 94.03

172.
Fund X receives a deposit of 1000 at time 0. Fund X accumulates at a nominal rate of interest k, compounded semiannually.

Fund Y receives a deposit of 921.90 at time 0. Fund Y accumulates at a nominal rate of discount, also equal to k, compounded semiannually.

At the end of 5 years, the accumulated amount in Fund X and the accumulated amount in Fund Y are both equal to P.

Calculate P.

(A) 1820
(B) 1970
(C) 2100
(D) 2240
(E) 2370

173.
The present value of a perpetuity-due with payments of X at the beginning of each 3-year period with an annual effective interest rate of 7% is 735.

Calculate X.

(A) 135
(B) 138
(C) 141
(D) 144
(E) 147

174.
The present value of a perpetuity-immediate with a first payment of P and successive annual increases of 9 at an annual effective interest rate of 6% is 2600.

Calculate P.

(A) 5.50
(B) 6.00
(C) 6.50
(D) 7.00
(E) 7.50

175.
The following two annuities-immediate have the same present value at an annual effective interest rate of i, $i > 0$.

- i) A ten-year annuity with annual payments of 475.
- ii) A perpetuity with annual payments of 400 in years 1-5, zero in years 6-10, and 400 in years 11 and beyond.

Calculate i.

(A) 10.65%
(B) 10.75%
(C) 10.85%
(D) 10.95%
(E) 11.05%

176.
A two-year loan of 100 is repaid with a payment of X at the end of the first year and $2X$ at the end of the second year. The annual effective interest rate charged by the lender is 8% in the first year and i in the second year. The annual effective yield rate for the lender is 10%.

Calculate i.

(A) 12.8%
(B) 12.9%
(C) 13.0%
(D) 13.1%
(E) 13.2%

177.
A ten-year loan will be repaid with payments at the end of each year. Each of the first five payments is 1000 and each of the next five payments is 2000.

Interest on the loan is charged at an annual effective rate of 10%.

Calculate the total interest paid in the first five payments.

(A) 2418
(B) 2646
(C) 2978
(D) 4083
(E) 4249

178.
An investor will accumulate 10,000 at the end of ten years by making level deposits of *X* at the beginning of each year. The deposits earn 12% simple interest at the end of every year but the interest is reinvested at an annual effective rate of 8%.

Calculate *X*.

(A) 508.79
(B) 541.47
(C) 569.84
(D) 597.73
(E) 608.42

179.
An investment of 10,000 produces a series of 30 annual payments. The first payment of X is made one year after the investment is made. Each successive payment decreases by 5 from the previous payment.

At an annual effective interest rate of 5%, calculate X.

(A) 685
(B) 695
(C) 705
(D) 715
(E) 725

180.
A company is considering a project that will require an initial investment of 600 and additional investments of 100 and 50 at the end of years one and two, respectively. It is expected that revenue from this project will be 150 per year for five years, beginning one year from the initial investment.

Assuming an annual effective rate of 15%, calculate the net present value of this project.

(A) –222
(B) –134
(C) 0
(D) 134
(E) 222

181.
Determine which of the following conditions are necessary for an immunization strategy.

I. The present value of the cash inflow from the assets is equal to the present value of the cash outflow from the liabilities.
II. The price sensitivity to changes in interest rates is greater for assets than for liabilities.
III. The convexity of assets is less than the convexity of liabilities.

(A) I only
(B) II only
(C) III only
(D) I, II, and III
(E) The correct answer is not given by (A), (B), (C), or (D).

182.
Four annual tuition payments of 25,000 are to be paid at a future date.

The payments will be funded by investing 1000 at the beginning of each month. The last deposit will be made six months before the first tuition payment. Interest is payable at a nominal interest rate of 6% convertible monthly.

Calculate the minimum number of monthly deposits required to fund the total tuition.

(A) 70
(B) 71
(C) 73
(D) 74
(E) There is not enough information to calculate the minimum number of monthly deposits.

183.
Consider a 7-year loan to be repaid with equal payments made at the end of each year. The annual effective interest rate is 10%.

Calculate the Macaulay duration of the loan payments.

(A) 3.15
(B) 3.29
(C) 3.40
(D) 3.50
(E) 3.62

184.
A company has liabilities of 402.11 due at the end of each of the next three years.

The company will match the duration of its liabilities by investing a total of 1000 in one-year and three-year zero-coupon bonds. The annual effective yield of both bonds is 10%.

Calculate the amount the company will invest in one-year bonds.

(A) 366
(B) 402
(C) 442
(D) 500
(E) 532

185.
A trucking company with assets and liabilities needs to choose between various ten-year par value bonds each with 8% annual effective yield rate and annual coupons. The bonds have varying face values and varying coupon rates.

The company wants to analyze the effects of face value and coupon rate changes on Macaulay duration of these bonds, in order to choose an investment strategy that immunizes its position.

Determine which of the following statements is true about the separate effects of face value and coupon rate changes on the duration of these bonds.

(A) Macaulay duration increases as face value increases, and increases as coupon rate increases.
(B) Macaulay duration increases as face value increases, and decreases as coupon rate increases.
(C) Macaulay duration remains constant as face value increases, and increases as coupon rate increases.
(D) Macaulay duration remains constant as face value increases, and remains constant as coupon rate increases.
(E) Macaulay duration remains constant as face value increases, and decreases as coupon rate increases.

186.
A corporation makes a payment at the end of each month into a savings account that offers an annual nominal interest rate of 8% compounded quarterly.

Determine the equivalent effective rate of interest per payment period.

(A) $\left(1+\frac{8\%}{4}\right)^{1/3}-1$

(B) $\left(1+\frac{8\%}{12}\right)-1$

(C) $\left(1+\frac{8\%}{12}\right)^{3}-1$

(D) $\left(1+\frac{8\%}{12}\right)^{12}-1$

(E) $\left(1+\frac{8\%}{4}\right)^{4}-1$

187.
A mortgage for 125,000 has level payments at the end of each month and an annual nominal interest rate compounded monthly. The balances owed immediately after the first and second payments were 124,750 and 124,498, respectively.

Calculate the number of payments needed to pay off the mortgage.

(A) 198
(B) 199
(C) 200
(D) 201
(E) 202

188.
A company is required to pay 500,000 ten years from now and 500,000 fifteen years from now. The company needs to create an investment portfolio using 5-year and 20-year zero-coupon bonds, so that, using a 7% annual force of interest, the present value and Macaulay duration of its assets match those of its liabilities.

Calculate the amount invested today in each bond.

(A) 211,631 for the 5-year bond and 211,631 for the 20-year bond
(B) 217,699 for the 5-year bond and 217,699 for the 20-year bond
(C) 223,852 for the 5-year bond and 199,410 for the 20-year bond
(D) 229,857 for the 5-year bond and 205,540 for the 20-year bond
(E) 248,293 for the 5-year bond and 174,969 for the 20-year bond

189.
A bond with a face value of 1000 and a redemption value of 1080 has an annual coupon rate of 8% payable semiannually. The bond is bought to yield an annual nominal rate of 10% convertible semiannually.

At this yield rate, the present value of the redemption value is 601 on the purchase date.

Calculate the purchase price of the bond.

(A) 911
(B) 923
(C) 956
(D) 974
(E) 984

190.
A loan of 10,000 is being repaid by payments of 1,000 at the end of each quarter for as long as necessary, plus a drop payment.

The annual nominal rate of interest on the loan is 16% convertible quarterly.

Calculate the amount of interest in the tenth payment.

(A) 112
(B) 146
(C) 179
(D) 233
(E) 281

191.
A five-year loan has an annual nominal interest rate of 30%, convertible monthly. The loan is scheduled to be repaid with level monthly payments of 500, beginning one month after the date of the loan.

The borrower misses the thirteenth through the eighteenth payments, but increases the next six payments to X so that the final 36 payments of 500 will repay the loan.

Calculate X.

(A) 1070
(B) 1075
(C) 1080
(D) 1150
(E) 1160

192.
A construction firm is facing three liabilities of 1000, due at times 1, 2, and 3 in years. There are three bonds available to match these liabilities, as follows:

Bond I	A bond due at the end of period 1 with a coupon rate of 1% per year, valued at a annual effective yield rate of 14%.
Bond II	A bond due at the end of period 2 with a coupon rate of 2% per year, valued at a annual effective yield rate of 15%.
Bond III	A zero-coupon bond due at time 3 valued at a periodic effective yield rate of 18%.

Calculate the face value of each bond that should be purchased to exactly match the liabilities.

	Bond I	Bond II	Bond III
(A)	970.68	980.39	1000.00
(B)	970.68	1000.00	980.39
(C)	980.39	970.68	1000.00
(D)	1000.00	980.39	970.68
(E)	1000.00	1000.00	1000.00

193.
An institute has provided an early retirement incentive package to a 60-year-old retiree that pays 12,000 per year at the end of each year up to and including age 65, plus a lump sum payment of 150,000 at age 65. All payments are guaranteed whether or not the retiree is alive at age 65.
The institute will create a portfolio of two five-year bonds to exactly match the payments under this package.
The first bond has a face amount of 100,000 and an annual coupon rate of 10%.

Determine which of the following second bonds will exactly match the liability.

(A) A bond with a price of 42,015, an annual coupon rate of 4% and annual effective yield of 8%.
(B) A bond with a price of 50,000, an annual coupon rate of 4% and annual effective yield of 8%.
(C) A bond with a price of 41,588, an annual coupon rate of 8% and annual effective yield of 10%.
(D) A bond with a price of 50,000, an annual coupon rate of 8% and annual effective yield of 10%.
(E) A bond with a price of 55,451, an annual coupon rate of 8% and annual effective yield of 10%.

194.
You are given the following information about a 30-year bond:

i) The par value is 2000.
ii) The redemption value is 2250.
iii) Coupons are paid annually.
iv) The annual coupon rate is twice the annual yield rate.
v) The purchase price is 3609.29.
vi) Based on the yield rate, the Macaulay duration of the bond is 14.41 years.

Calculate the modified duration of the bond, based on the yield rate.

(A) 12.40 years
(B) 13.07 years
(C) 13.71 years
(D) 14.41 years
(E) 15.15 years

195.
A company's preferred stock will pay level annual dividends forever starting five years from now.

Using an annual effective interest rate of 10%, the modified duration of the stock is *D*.

Calculate *D*.

(A) 13.64
(B) 14.55
(C) 15.00
(D) 16.00
(E) 16.50

196.
A three-year bond with a face value of 1000 pays coupons semiannually. The bond is redeemable at face value. It is bought at issue at a price to produce an annual yield rate of 10% convertible semiannually. If the term of the bond is doubled and the yield rate remains the same, the purchase price would decrease by 49.

Calculate the amount of a coupon.

(A) 37
(B) 46
(C) 54
(D) 63
(E) 74

197.
Three asset-liability cash flows are given in the following table where a positive amount is an asset cash flow and a negative amount is a liability due at the corresponding time.

t (in years)	0	1	2	3
X	102,400	−192,000	0	100,000
Y	158,400	−342,000	100,000	100,000
Z	−89,600	288,000	100,000	-300,000

Determine which set of cash flows is Redington immunized for an annual effective interest rate of $i = 25\%$.

(A) X only
(B) Y only
(C) Z only
(D) X, Y, and Z
(E) The correct answer is not given by (A), (B), (C) or (D).

198.
A ten-year 1000 par value bond with coupons paid annually at an annual rate of *r* is callable at par at the end of the 6^{th}, 7^{th}, 8^{th}, or 9^{th} year.
The price of the bond is 1023.

If the bond is called in the worst-case scenario for the bond investor, the resulting annual effective yield rate, *i*, is 96% of *r*.

Calculate *i*.

(A) 4.41%
(B) 7.46%
(C) 8.36%
(D) 10.56%
(E) 14.32%

199.
Determine which of the following statements regarding the Redington immunization technique is false.

(A) This technique assumes that the yield curve is flat.
(B) This technique assumes that only parallel shifts in the yield curve are allowed.
(C) This technique is designed to work only for small changes in the interest rate.
(D) The modified duration of the assets must equal the modified duration of the liabilities.
(E) The convexity of the assets must equal the convexity of the liabilities.

200.
Payments are made to an account at a continuous rate of $100e^{0.5t}$ from time $t = 1$ to time $t = 3$. The force of interest for this account is $\delta = 8\%$.

Calculate the value of the account at time $t = 4$.

(A) 313
(B) 432
(C) 477
(D) 606
(E) 657

201.
You are given the following information about a perpetuity with annual payments:

i) The first 15 payments are each 2500, with the first payment to be made three years from now.
ii) Beginning with the 16th payment, each payment is k% larger than the previous payment.
iii) Using an annual effective interest rate of 3.5%, the present value of the perpetuity is 115,000.

Calculate k.

(A) 1.66
(B) 1.74
(C) 1.78
(D) 1.83
(E) 1.89

202.
An insurance company sells an annuity that provides 20 annual payments, with the first payment beginning one year from today and each subsequent payment 2% greater than the previous payment.

Using an annual effective interest rate of 3%, the present value of the annuity is 200,000.

Calculate the amount of the final payment from this annuity.

(A) 11,282
(B) 16,436
(C) 16,765
(D) 19,784
(E) 24,162

203.
A loan for 10,000 is to be repaid by level annual payments at the end of each year for ten years. The annual effective interest rate for the loan is 10%, which the bank uses to compute the annual payment and the balance on the loan at the end of each year. However, for balances during each year, the bank uses an annual simple interest rate of 10%.

Calculate the balance of the loan halfway through the fourth year.

(A) 7434
(B) 7442
(C) 7885
(D) 8310
(E) 8319

204.
X opens a bank account with 1000 and lets it accumulate at an annual nominal interest rate of 6% convertible monthly. Y also opens a bank account with 1000 at the same time as X, but it grows at an annual nominal interest rate of 3% convertible semiannually.

For each account, interest is credited only at the end of each interest conversion period.

Calculate how many months are required for the amount in X's account to be at least double the amount in Y's account.

(A) 276
(B) 277
(C) 282
(D) 288
(E) 290

205.
A company is required to pay 190,000 in 20.5 years. The company creates an investment portfolio using three bonds with annual coupons, so that its position is Redington immunized based on an annual effective interest rate of 7%. The table below shows the Macaulay duration for each of the bonds.

	Macaulay Duration
Bond A	10 years
Bond B	15 years
Bond C	30 years

The company invests twice as much money in Bond C as in Bond B.

Calculate the amount the company invests in Bond A.

(A) 6,640
(B) 8,630
(C) 11,075
(D) 13,308
(E) 14,240

206.
Bond X and Bond Y are n-year bonds with face amount of 10,000 and semiannual coupons, each yielding an annual nominal interest rate of 7% convertible semiannually. Bond X has an annual coupon rate of 6% and redemption value c. Bond Y has an annual coupon rate of 5% and redemption value $c + 50$. The price of Bond X exceeds the price of Bond Y by 969.52.

Calculate n.

(A) 14
(B) 17
(C) 23
(D) 34
(E) 46

207.
An investor deposited 1000 in each of Bank X and Bank Y. Bank X credits simple interest at an annual rate of 10% for the first five years and 7% thereafter. Bank Y credits interest at an annual nominal rate of 5% compounded quarterly. The interest credited in the eighth year by Bank Y exceeds the interest credited in the eighth year by Bank X by N.

Calculate the absolute value of N.

(A) 0.36
(B) 2.14
(C) 20.00
(D) 56.36
(E) 56.93

208.
A college has a scholarship fund that pays a sum of money twice a year. The scholarship will pay out 500 at the end of six months and another 500 at the end of one year. Every year thereafter, the two semi-annual payments will be increased by 10. For example, in year two, both payments will be 510 and in year three both payments will be 520.

The scholarship fund earns interest at an annual effective interest rate of 7.5%.

Calculate the fund balance needed today to provide this scholarship indefinitely.

(A) 16,000
(B) 16,589
(C) 16,889
(D) 17,134
(E) 17,200

209.
Sam deposits 5000 at the beginning of each year for ten years. The deposits earn an annual effective interest rate of i. All interest is reinvested at an annual effective interest rate of 5%.

Sam has 100,000 at the end of ten years.

Calculate i.

(A) 11.6%
(B) 15.6%
(C) 19.1%
(D) 23.2%
(E) 27.6%

210.
A loan of 10,000 is repaid at an annual effective interest rate of 8%, with equal payments made at the end of each year for ten years. The lender immediately deposits each payment into an account earning an annual effective interest rate of 10%.

Calculate the total amount of interest earned by the lender during the term of the loan.

(A) 11,589
(B) 13,576
(C) 13,751
(D) 14,191
(E) 14,903

211.
A worker is starting an endowment fund for his charity. The fund will accumulate at a nominal annual interest rate of 12% convertible monthly. Beginning today, the worker will deposit 500 monthly for ten years. No deposits or withdrawals will be made for the subsequent ten years. Exactly twenty years from today, monthly payments of X will be made to his charity and continue forever.

Calculate X.

(A) 3270
(B) 3572
(C) 3758
(D) 3796
(E) 3834

212.
Payments of 1000 at the end of each year are paid on a loan of 12,000. The payments are based on an annual effective interest rate of 10%.

Calculate the outstanding loan balance immediately after the 12^{th} payment.

(A) 909
(B) 10,909
(C) 12,853
(D) 16,277
(E) 20,043

213.
A six-year 1000 face amount bond has an annual coupon rate of 8% semiannually. The bond currently sells for 911.37.

Calculate the annual effective yield rate.

(A) 7.29%
(B) 8.00%
(C) 9.72%
(D) 10.00%
(E) 10.25%

214.
Matthew deposits 100 into Fund X, which earns a nominal annual rate of discount of 12% convertible quarterly.

At the same time, Sarah deposits 200 into Fund Y, which earns a constant annual force of interest of 0.08.

After *n* years, the balance of Fund X equals the balance of Fund Y.

Calculate *n*.

(A) 14.5
(B) 16.6
(C) 17.6
(D) 18.1
(E) 18.6

215.
Kevin makes a deposit at the end of each year for 10 years into a fund earning interest at a 4% annual effective rate. The first deposit is equal to *X*, with each subsequent deposit 9.2% greater than the previous year's deposit. The accumulated value of the fund immediately after the 10^{th} deposit is 5000.

Calculate *X*.

(A) 255
(B) 260
(C) 270
(D) 279
(E) 293

216.
An investor pays 962.92 for a ten-year bond with an annual coupon rate of 8% paid semiannually. The annual nominal yield rate is 10% convertible semiannually.

Calculate the discount of this purchase.

(A) 117
(B) 122
(C) 127
(D) 132
(E) 137

217.
An investor makes deposits into an account at the end of each year for ten years. The deposit in year one is 1, year two is 2 and so forth until the final deposit of 10 in year ten. The account pays interest at an annual effective rate of i.

Immediately following the final deposit, the investor uses the entire account balance to purchase a perpetuity-immediate at an annual effective interest rate of i. The perpetuity makes annual payments of 10.

Calculate the purchase price of the perpetuity.

(A) 68.0
(B) 72.4
(C) 76.2
(D) 81.3
(E) 91.3

218.
A 25-year loan is being repaid with payments of 1300 at the end of each year. The loan payments are based on an annual effective interest rate of 8%.

The borrower pays an additional 4000 at the time of the fifth payment and will repay the remaining balance with a payment of X at the end of each of the subsequent ten years.

Calculate X.

(A) 893
(B) 1300
(C) 1306
(D) 1500
(E) 1902

219.
A firm has a liability cash flow of 100 at the end of year two and a second liability cash flow of 200 at the end of year three.

The firm also has asset cash flows of X at the end of years one and five.

Using an annual effective interest rate of 10%, calculate the absolute value of the difference between the Macaulay durations of the asset and liability cash flows.

(A) 0.018
(B) 0.020
(C) 0.022
(D) 0.024
(E) 0.026

220.
A bank lends 100,000 to Sam. The loan is repaid with level payments at the end of each year for 30 years based on an annual effective interest rate of 5%.

The bank reinvests the loan payments at an annual effective interest rate of 4%.

Calculate the bank’s annual effective yield rate over the 30-year period.

(A) 4.0%
(B) 4.1%
(C) 4.2%
(D) 4.4%
(E) 4.5%

221.
A life insurance company sells a two-year immediate annuity with annual payments of 1000 for a price of 1817.

The investment actuary invests the 1817 in two zero-coupon bonds.

- The first bond matures in one year and earns an annual effective interest rate of 6%. The second bond matures in two years and earns an annual effective interest rate of 7%.
- 999.35 is invested in the first bond and 817.65 is invested in the second bond.
- The two bonds are held to maturity.

As long as the effective annual one-year reinvestment rate is at least X% one year from now, the principal and interest earned will be sufficient to make the two annuity payments.

Calculate X.

(A) 6.0
(B) 6.6
(C) 7.0
(D) 7.3
(E) 7.7

222.
You are given the following term structure of interest rates:

Length of investment in years	Spot rate
1	7.50%
2	8.00%
3	8.50%
4	9.00%
5	9.50%

Calculate the one-year annual effective rate for the fifth year implied by this term structure.

(A) 9.0%
(B) 9.5%
(C) 10.0%
(D) 10.5%
(E) 11.5%

223.
Determine which of the following expressions represents the modified duration for a zero-coupon bond that is currently priced at an annual effective yield rate i and an n-year maturity.

(A) n

(B) $n(1+i)$

(C) $n(1+i)^{-1}$

(D) $\dfrac{\sum_{t=1}^{n} t(1+i)^{-t}}{\sum_{t=1}^{n} (1+i)^{t}}$

(E) $\dfrac{\sum_{t=1}^{n} t(1+i)^{-t}}{\sum_{t=1}^{n} (1+i)^{-t}}$

224.
A company has liabilities of 1000 due at the end of each of the next three years. The company will exactly match the cash flows of assets and liabilities. The following zero-coupon bonds are available:

Maturity	Yield
1-Year	8%
2-Year	9%
3-Year	10%

Calculate how much more will be invested in the 1-year bond than in the 3-year bond.

(A) 0
(B) 84
(C) 132
(D) 158
(E) 175

225.
The table below shows the cash flows for a particular investment and the prevailing spot rates.

n	Cash flow (at end of year n)	n- year Spot rate
1	10	4.0%
2	12	4.5%
3	15	5.5%
4	20	7.0%

Calculate the present value of this investment at the start of year 1.

(A) 47.33
(B) 48.64
(C) 49.50
(D) 50.04
(E) 51.14

226.
The price of a 36-year zero-coupon bond is 80% of its face value.

A second bond, with the same price, same face value, and same annual effective yield rate, offers annual coupons with the coupon rate equal to $\frac{4}{9}$ of the annual effective yield rate.

Calculate the number of years until maturity for the second bond.

(A) 45
(B) 54
(C) 63
(D) 72
(E) 81

227.
An investor purchases a 1200 face amount zero-coupon bond for a price of 1000. With respect to the bond's annual effective yield rate, the Macaulay duration is four years and the modified duration is *d* years.

Calculate *d*.

(A) 3.33
(B) 3.82
(C) 3.86
(D) 4.00
(E) 4.19

228.
An insurer has a liability that is expected to result in the following cash outflows.

End of year	Cash Outflow
1	10
2	12
3	15
4	20
5	30

The insurer uses an 8% annual effective interest rate to discount future cash flows.

Calculate the Macaulay duration of this liability.

(A) 3.1 years
(B) 3.2 years
(C) 3.4 years
(D) 3.5 years
(E) 3.6 years

229.
A company created a portfolio in order to protect its position using Redington immunization.

Under which of the following changes in the yield rate is the immunization strategy guaranteed to be effective?

(A) Only a small decrease in the yield rate
(B) Only a small increase in the yield rate
(C) Only a small change in the yield rate
(D) Any decrease in the yield rate
(E) Any change in the yield rate

230.
A bank issues a loan to be repaid by level end-of-year payments for ten years at an annual effective interest rate of 10%.

The bank invests these payments at an annual effective interest rate of 10% for the first four years and 7% for the next six years.

Calculate the bank's annual effective yield rate on this loan over the ten-year period.

(A) 7.90%
(B) 8.20%
(C) 8.33%
(D) 8.67%
(E) 9.10%

231.
Two borrowers obtain loans at the same time. Each loan is for the same amount and is repaid with level end-of month payments.

The first borrower is charged a monthly effective interest rate of i and makes payments of P for k months to pay off the loan, where k is a positive integer.

The second borrower is charged a monthly effective interest rate of j and makes payments of 120 for $5k$ months to pay off the loan.

You are given that $0 < j < 0.04$ and that $i = (1+j)^5 - 1$.

Determine which statement about P is true.

(A) $400 < P \leq 450$
(B) $450 < P \leq 500$
(C) $500 < P \leq 550$
(D) $550 < P \leq 600$
(E) $600 < P \leq 650$

232.
A loan of 4000 has an annual effective interest rate of 5%. The loan is repaid by payments of 250 at the end of each year for ten years along with a final balloon payment at the end of the eleventh year.

Calculate the outstanding loan balance at the beginning of the seventh year.

(A) 3593
(B) 3660
(C) 3790
(D) 3856
(E) 3910

233.
An investor's new savings account earns an annual effective interest rate of 3% for each of the first ten years and an annual effective interest rate of 2% for each year thereafter.

The investor deposits an amount X at the beginning of each year, starting with year 1, so that the account balance just after the deposit in the beginning of year 26 is 100,000.

Determine which of the following is an equation of value that can be used to solve for X.

(A) $$\frac{100,000}{(1.03)^{10}(1.02)^{15}} = X\sum_{k=1}^{11}\frac{1}{(1.03)^{k-1}} + X\sum_{k=12}^{26}\frac{1}{(1.02)^{k-1}}$$

(B) $$\frac{100,000}{(1.03)^{10}(1.02)^{16}} = X\sum_{k=1}^{10}\frac{1}{(1.03)^{k}} + X\sum_{k=11}^{26}\frac{1}{(1.02)^{k}}$$

(C) $$\frac{100,000}{(1.03)^{10}(1.02)^{16}} = X\sum_{k=1}^{11}\frac{1}{(1.03)^{k-1}} + X\sum_{k=12}^{26}\frac{1}{(1.03)^{10}(1.02)^{k-11}}$$

(D) $$\frac{100,000}{(1.03)^{10}(1.02)^{15}} = X\sum_{k=1}^{11}\frac{1}{(1.03)^{k-1}} + X\sum_{k=12}^{26}\frac{1}{(1.03)^{10}(1.02)^{k-11}}$$

(E) $$\frac{100,000}{(1.03)^{10}(1.02)^{16}} = X\sum_{k=1}^{10}\frac{1}{(1.03)^{k}} + X\sum_{k=11}^{26}\frac{1}{(1.03)^{10}(1.02)^{k-10}}$$

234.
A bank issues two 20-year par-value bonds providing annual coupons. Each bond sells for the same price and provides an annual effective yield rate of 6.5%.

The first bond has a redemption value of 6000 and a coupon of 7.6% paid annually. The second bond has a redemption value of 7500 and a coupon of r% paid annually.

Calculate r.

(A) 5.6
(B) 5.9
(C) 6.1
(D) 6.7
(E) 7.2

235.
You are given the following information about Bond X and Bond Y:

i) Both bonds are 20-year bonds.

ii) Both bonds have face amount 1500.

iii) Both bonds have an annual nominal yield rate of 7% compounded semiannually.

iv) Bond X has an annual coupon rate of 10% paid semiannually and a redemption value C.

v) Bond Y has an annual coupon rate of 8% paid semiannually and a redemption value $C+K$.

vi) The price of Bond X exceeds the price of Bond Y by 257.18.

Calculate K.

(A) –380
(B) –88
(C) 0
(D) 235
(E) 250

236.
A perpetuity-immediate with annual payments is priced at *X* based on an annual effective interest rate of 10%. The amount of the first payment is 14,000. Each payment, from the second through the twentieth, is 4% larger than the previous payment. The 21^{st} payment and each subsequent payment will be 1% larger than the previous payment.

Calculate *X*.

(A) 185,542
(B) 191,834
(C) 206,540
(D) 208,508
(E) 212,823

237.
An annuity writer sells an annual perpetuity-immediate for 16,000. The first payment is 1,000 and the payments decrease by *r*% each year.

The annual effective yield rate is 5.7%.

Calculate *r*.

(A) 0.52
(B) 0.55
(C) 0.62
(D) 0.73
(E) 0.91

238.
A perpetuity makes payments every five years with a first payment of 2 to be paid five years from now. Each subsequent payment is 10 more than the previous payment. The annual effective interest rate is 9%.

Calculate the present value of the perpetuity.

(A) 34.47
(B) 35.80
(C) 36.33
(D) 37.12
(E) 38.18

239.
A four-year 1000 face amount bond, with an annual coupon rate of 5% paid semiannually, has redemption value of C. It is bought at a price to yield an annual nominal rate of 6% convertible semiannually. If the term of the bond had been two years, the purchase price would have been 7% less.

Calculate C.

(A) 455
(B) 469
(C) 541
(D) 611
(E) 626

240.
You are given the following information about a fully immunized portfolio:

i) The liability is a single payment of 600,000 due in two years.

ii) The asset portfolio consists of a one-year zero-coupon bond maturing for x and a four-year zero-coupon bond maturing for y.

iii) The annual effective interest rate is 4.6%.

Calculate x.

(A) 218,800
(B) 325,400
(C) 365,600
(D) 382,400
(E) 402,800

241.
An investor pays 4000 today for a three-year investment that returns cash flows of 1400 at the end of each year. The cash flows can be reinvested at the positive annual effective interest rate of i. Using an annual effective rate of interest of 4%, the net present value of this investment is 0.

Calculate i.

(A) 2.5%
(B) 3.0%
(C) 3.5%
(D) 4.0%
(E) 7.0%

242.
Susan won the lottery today which will pay an annual perpetuity of X, with the first payment occurring five years from today.

The perpetuity has a present value of 100,000 based on an annual effective interest rate of 1% for the first ten years and 5% for all years thereafter.

Calculate X.

(A) 4100
(B) 4224
(C) 4357
(D) 4401
(E) 5696

243.
A 15-year loan of 60,000 is to be repaid with payments of X at the end of each month based on an annual nominal interest rate of 7.5%, convertible monthly.

When the loan balance is 49,893, the loan is refinanced at an annual nominal interest rate of 6.0%, convertible monthly. Payments remain at X and are paid at the end of each month for as long as necessary, with a smaller final payment.

Calculate the total number of payments, including the smaller final payment.

(A) 162
(B) 164
(C) 166
(D) 168
(E) 170

244.
Greg buys a 20-year increasing annuity-immediate with annual payments. The first payment is 110 and each succeeding payment is equal to the previous payment plus X. The annuity is priced at 2000 based on an annual effective interest rate of 10%.

Calculate X.

(A) 16.64
(B) 17.45
(C) 19.19
(D) 21.00
(E) 22.68

245.
At an annual effective interest rate of 5%, a 10-year annuity-immediate starting with an annual payment of 100 increases each year by 15% of the previous year's payment and has a present value of X.

Calculate X.

(A) 597
(B) 772
(C) 1040
(D) 1247
(E) 1484

246.
A loan of X is repaid with level payments of R payable at the end of each year for n years.

You are given:

i) The interest paid in year 1 is 797.50.

ii) The principal repaid in year $n - 4$ is 865.

iii) The principal outstanding at the end of year $n - 1$ is 1144.50.

Determine X.

(A) 9,500
(B) 10,000
(C) 10,500
(D) 11,000
(E) 11,500

247.
The amount that must be invested at $i\%$ ($0\% < i\% < 10\%$) to accumulate to Y at the end of three years at an annual rate of:

i) simple interest of $i\%$ is Q

ii) compound interest of $i\%$ is R

iii) simple discount of $i\%$ is S

iv) compound discount of $i\%$ is T

Determine which of the following is true.

(A) $Q < R < S < T$
(B) $R < Q < S < T$
(C) $S < T < R < Q$
(D) $T < S < Q < R$
(E) $T < S < R < Q$

248.
Kate buys a five-year 1000 face amount bond today with a 100 discount. The annual nominal coupon rate is 5% convertible semiannually.

One year later, Wallace buys a four-year bond. It has the same face amount and coupon values as Kate's and is priced to yield an annual nominal interest rate of 10% convertible semiannually. The discount on Wallace's bond is D.

The book value of Kate's bond at the time Wallace buys his bond is B.

Calculate $B - D$.

(A) 724
(B) 738
(C) 756
(D) 838
(E) 917

249.
A company has liabilities of 1000 and 300 due at the end of years two and four, respectively. The company develops an investment program that produces asset cash flows of X and Y at the end of years one and three, respectively. The investment portfolio is constructed to match the present value and duration of the company's payment obligations based on an annual effective rate of interest of 5%.

Calculate $\frac{Y}{X}$.

(A) 2.14
(B) 2.75
(C) 3.42
(D) 4.05
(E) 4.91

250.
A homeowner borrows 1000 to be repaid with payments at the end of each year for 20 years. There are two repayment options. The first option is equal annual payments based on an annual effective interest rate of 3%. The second option is payments of 50 each year plus interest on the unpaid balance at an annual effective interest rate of i. The total payments under the two options are the same.

Calculate i.

(A) 2.86%
(B) 3.00%
(C) 3.28%
(D) 3.44%
(E) 4.76%

251.

A debt is amortized with 120 level payments at the end of each month based on an annual effective interest rate of 8%. The amount of principal in the sixth payment is 600.

Calculate the amount of principal in the 24th payment.

(A) 532
(B) 673
(C) 700
(D) 704
(E) 784

252.

Fund X accumulates at a force of interest of $\delta_t = \dfrac{2}{1+2t}$, where t is measured in years, $0 \le t \le 20$. Fund Y accumulates at an annual effective interest rate of i. An amount of 1 is invested in each fund at time t = 0. After 20 years, Fund X has the same value as Fund Y.

Calculate the value of Fund Y after five years.

(A) 2.46
(B) 2.53
(C) 2.60
(D) 2.67
(E) 2.74

253.

You have the option of purchasing one of the following two annuities-immediate:

i) The first annuity makes annual payments of 1000 for 20 years.

ii) The second annuity is a perpetuity that also has annual payments. The payment in each of the first 10 years is 600. Beginning in year 11, the payments increase to 1200, and remain at 1200 forever.

At an annual effective interest rate of $i > 0$, both annuities have a present value of X.

Calculate X.

(A) 8700
(B) 8750
(C) 8800
(D) 8850
(E) 8900

254.
S perpetuity-immediate is purchased at a price of 5000 based on an annual effective rate of *i*, where $i > 0$. The perpetuity pays 150 in the first year and increases by 10 each year thereafter.

Calculate *i*.

(A) 5.7%
(B) 6.2%
(C) 6.7%
(D) 7.2%
(E) 7.7%

255.
Erin borrows *X* at an annual effective rate of 5%. If the principal and all accumulated interest is paid in one lump sum at the end of 20 years, 1000 more in interest would be paid than if the loan was repaid with level payments at the end of each of the first ten years.

Calculate *X*.

(A) 389
(B) 540
(C) 704
(D) 736
(E) 954

256.
A 20-year loan of 500 based on a 5% annual effective interest rate is repaid with level installments at the end of each year.

Determine the installment in which the principal and interest portions are most nearly equal to each other.

(A) 1^{st}
(B) 7^{th}
(C) 10^{th}
(D) 13^{th}
(E) 20^{th}

257.
The total present value of 10,000 now and 10,815 two years from now is the same as the present value of 20,800 one year from now at either of two different annual effective interest rates, x and y.

Calculate the absolute value of the difference between x and y.

(A) 1.8%
(B) 2.0%
(C) 3.0%
(D) 4.0%
(E) 5.0%

258.
Bond A is a 15-year 1000 face amount bond with an annual coupon rate of 9% paid semiannually. Bond A will be redeemed at 1200 and is bought to yield 8.4% convertible semiannually.

Bond B is an n-year 1000 face amount bond with an annual coupon rate of 8% paid quarterly. Bond B will be redeemed at 1376.69 and is bought to yield 8.4% convertible quarterly.

The two bonds have the same purchase price.

Calculate n.

(A) 12
(B) 14
(C) 15
(D) 16
(E) 18

259.
At the end of each year for 60 years, Marilyn makes a deposit to a bank account that credits interest at an annual effective interest rate of i. She deposits 2 at the end of the first year, and each subsequent year her deposit increases by 2. At the end of 60 years, Marilyn uses the accumulated amount to purchase a 5-year annuity-immediate paying annually at the same annual interest rate of i, with a first payment of X and each subsequent payment increasing by 5%.

Which of the following expressions represents a correct equation of value?

I. $$\frac{2\left(s_{\overline{60}|}-60\right)}{i}=\frac{X\left(1-(1.05v)^5\right)}{i-0.05}$$

II. $$2(Ia)_{\overline{60}|}=Xv^{60}\left(1+1.05v+\ldots+(1.05v)^4\right)$$

III. $$2\sum_{t=0}^{59} s_{\overline{60-t}|}=X\left(v+1.05v^2+\ldots+1.05^4v^5\right)$$

(A) I only
(B) II only
(C) III only
(D) I, II, and III
(E) The correct answer is not given by (A), (B), (C), or (D).

260.
The table below defines available zero-coupon bonds and their prices:

Years to Maturity	Bond Price Per Bond	Redemption Value Per Bond
1	961.54	1,000
2	966.14	1,050
3	878.41	1,000

A company chooses to purchase 15 one-year zero-coupon bonds; 20 two-year zero- coupon bonds; and 30 three-year zero-coupon bonds.

Calculate the Macaulay duration of this portfolio.

(A) 1.97
(B) 2.11
(C) 2.16
(D) 2.20
(E) 2.23

261.
On his 65th birthday, an investor withdrew an amount *P* from a fund of 1,000,000 and withdrew the same amount on each successive birthday. On the date of his 82nd birthday, the fund was again equal to 1,000,000 after the withdrawal.

The fund earns an annual effective interest rate of 10%.

Calculate *P*.

(A) 81,655
(B) 88,915
(C) 90,909
(D) 98,879
(E) 109,729

262.
The first payment of a five-year annuity is due in five years in the amount of 1000. The subsequent four annual payments increase by 500 each year. The annual effective interest rate is *i*.

Determine which of the following formulas gives the present value of the annuity.

(A) $v^6\left[500a_{\overline{5}|\,i}+500(Ia)_{\overline{5}|\,i}\right]$
(B) $v^6\left[500\ddot{a}_{\overline{5}|\,i}+500(I\ddot{a})_{\overline{5}|\,i}\right]$
(C) $v^5\left[500a_{\overline{5}|\,i}+500(I\ddot{a})_{\overline{5}|\,i}\right]$
(D) $v^5\left[500\ddot{a}_{\overline{5}|\,i}+500(I\ddot{a})_{\overline{5}|\,i}\right]$
(E) $v^5\left[1000\ddot{a}_{\overline{5}|\,i}+500(I\ddot{a})_{\overline{5}|\,i}\right]$

263.
An 18-year bond, with a price 61% higher than its face value, offers annual coupons with the coupon rate equal to 2.25 times the annual effective yield rate.

An *n*-year bond, with the same face value, coupon rate, and yield rate, sells for a price that is 45% higher than its face value.

Calculate *n*.

(A) 10
(B) 12
(C) 14
(D) 17
(E) 20

264.
A ten-year bond paying annual coupons of X has a face amount of 1000, a price of 450, and an annual effective yield rate of 10%.

A second ten-year bond has the same face amount and annual effective yield rate as the first bond. This second bond pays semi-annual coupons of $\frac{X}{2}$. The price of the second bond is P.

Calculate P.

(A) 439
(B) 442
(C) 452
(D) 457
(E) 461

265.

Consider two loans. Loan A has an initial principal of P_0 and an annual nominal interest rate of i, convertible monthly. Loan B also has an annual nominal interest rate of i, but the interest is convertible daily.

At the end of the first month, a payment of m is made on Loan A, which includes one month of interest. The remaining balance on Loan A is then P_1.

Let j be the monthly effective interest rate of Loan B, assuming there are 12 equal months in a year and 365 days in a year.

Determine which of the following represents j.

(A) $\left[1+\frac{1}{12}\left(\frac{P_1-P_0+m}{P_0}\right)\right]^{12}-1$

(B) $\left[1+12\left(\frac{P_1-P_0+m}{P_0}\right)\right]^{1/12}-1$

(C) $\left[1+\frac{12}{365}\left(\frac{P_1-P_0+m}{P_0}\right)\right]^{12}-1$

(D) $\left[1+\frac{365}{12}\left(\frac{P_1-P_0+m}{P_0}\right)\right]^{12/365}-1$

(E) $\left[1+\frac{12}{365}\left(\frac{P_1-P_0+m}{P_0}\right)\right]^{365/12}-1$

266.
Let $P(0, t)$ be the current price of a zero-coupon bond that will pay 1 at time t.

Let X be the value at time n of an investment of 1 made at time m, where $m < n$.

Assume all investments earn the same interest rate.

Determine X.

(A) $\dfrac{P(0,m)}{P(0,n)} - 1$

(B) $\dfrac{P(0,n)}{P(0,m)} + 1$

(C) $\dfrac{P(0,m)}{P(0,n)} + 1$

(D) $\dfrac{P(0,m)}{P(0,n)}$

(E) $\dfrac{P(0,n)}{P(0,m)}$

267.
A borrower planned to repay a loan of L with level payments at the end of each month for 30 years. The loan had an annual nominal interest rate of 6%, convertible monthly. Starting with the 181st payment, the borrower increased the monthly payment to 2000, which enabled the borrower to pay off the loan five years earlier than planned.

Calculate L.

(A) 253,554
(B) 277,772
(C) 310,414
(D) 330,347
(E) 333,583

268.
An insurance company aims to structure a portfolio of assets and liabilities so that:

i) Its assets and liabilities have equal present values and equal modified durations.

ii) The convexity of its assets exceeds the convexity of its liabilities.

Determine which term precisely describes the company's position.

(A) Diversified
(B) Fully immunized
(C) Fully leveraged
(D) Exactly matched
(E) Redington immunized

269.
A three-year bond with face amount X and a coupon of 4 paid at the end of every six months is priced at 90.17. A three-year bond with face value of $1.6X$ and a coupon of 4 paid at the end of every six months is priced at 132.47. Both have the same yield rate.

Calculate the annual nominal yield rate, convertible semiannually.

(A) 6%
(B) 8%
(C) 9%
(D) 11%
(E) 12%

270.
Prosperity Insurance sells a two-year annuity with equal payments at the end of each year. The price of this annuity is 9297. Prosperity Insurance creates a portfolio made up of a one-year zero-coupon bond and a two-year zero-coupon bond to immunize this annuity.

The company uses an annual effective interest rate of 5% to vaue its assets and liabilities.

Calculate the amount invested today in the two-year zero-coupon bond.

(A) 3769
(B) 4535
(C) 4762
(D) 5000
(E) 9297

271.
Bond A and Bond B are each five-year 1000 face amount bonds. In addition:

i) Bond A has an annual coupon rate of 5% paid semiannually.

ii) Bond B has an annual coupon rate of 3% paid annually.

iii) The price of Bond B is 100 less than the price of Bond A.

iv) The annual effective yield rate for Bond A is 4%.

Calculate the annual effective yield rate for Bond B.

(A) 4.15%
(B) 4.20%
(C) 4.25%
(D) 4.30%
(E) 4.35%

272.
A 100,000 loan has an annual nominal interest rate of 8% convertible quarterly. The loan will be repaid with quarterly payments and the first payment is due three months from the date of the loan.

For the first five years each payment will be 2500. All payments thereafter will be 5000 except for a final balloon payment, which will be less than 10,000.

Calculate the balloon payment.

(A) 7920
(B) 8078
(C) 9056
(D) 9154
(E) 9237

273.
An investor purchases a 20-year, 1000 face amount bond with semiannual coupons at a price equal to the redemption value of 900. The bond yields an annual nominal rate of 10% convertible semiannually.

After ten years, the investor sells the bond, yielding to the buyer an annual nominal rate of 8% convertible semiannually. The investor uses the proceeds to purchase a 10-year, 1000 face amount bond with redemption value 1100 and semiannual coupons. The yield rate on the new bond is an annual nominal rate of 8% convertible semiannually.

Calculate the semiannual coupon payment for the new bond.

(A) 34
(B) 38
(C) 41
(D) 45
(E) 50

274.
An insurance company has liabilities due at the end of each of the next two years. The liability due at the end of the second year is twice that for the first year.

The company uses a combination of the following two bonds to match the liabilities. The second bond has annual coupons.

Term to maturity (in years)	Annual coupon rate	Annual effective yield
1	0%	6%
2	5%	8%

Assume both bonds are redeemed at par and the company invests 9465 in the two-year bond.

Calculate the amount the company should invest in the one-year bond to construct an exactly matched portfolio.

(A) 4245
(B) 4398
(C) 4481
(D) 4717
(E) 4953

275.
An insurance company has a single liability due in three years. The company fully immunizes its position by purchasing one-year and four-year zero-coupon bonds. The face value of the one-year bond is 20,000 and the face value of the four-year bond is 50,000.

Assume that the yield curve is flat.

Calculate the amount of the liability.

(A) 40,000
(B) 55,699
(C) 69,624
(D) 73,333
(E) 97,500