

SOCIETY OF ACTUARIES  
EXAM FM FINANCIAL MATHEMATICS

**EXAM FM SAMPLE SOLUTIONS**

This set of sample questions includes those published on the interest theory topic for use with previous versions of this examination. Questions from previous versions of this document that are not relevant for the syllabus effective with the October 2022 administration have been deleted. The questions have been renumbered.

Some of the questions in this study note are taken from past SOA examinations.

These questions are representative of the types of questions that might be asked of candidates sitting for the Financial Mathematics (FM) Exam. These questions are intended to represent the depth of understanding required of candidates. The distribution of questions by topic is not intended to represent the distribution of questions on future exams.

The following model solutions are presented for educational purposes. Alternative methods of solution are acceptable.

In these solutions,  $s_m$  is the  $m$ -year spot rate and  ${}_m f_t$  is the  $m$ -year forward rate, deferred  $t$  years.

Update history:

October 2022: Questions 208-275 were added

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1. Solution: C

Given the same principal invested for the same period of time yields the same accumulated value, the two measures of interest  $i^{(2)} = 0.04$  and  $\delta$  must be equivalent, which means:

$\left(1 + \frac{i^{(2)}}{2}\right)^2 = e^\delta$  over a one-year period. Thus,

$$e^\delta = \left(1 + \frac{i^{(2)}}{2}\right)^2 = 1.02^2 = 1.0404$$

$$\delta = \ln(1.0404) = 0.0396.$$

2. Solution: E

From basic principles, the accumulated values after 20 and 40 years are

$$100[(1+i)^{20} + (1+i)^{16} + \dots + (1+i)^4] = 100 \frac{(1+i)^4 - (1+i)^{24}}{1 - (1+i)^4}$$

$$100[(1+i)^{40} + (1+i)^{36} + \dots + (1+i)^4] = 100 \frac{(1+i)^4 - (1+i)^{44}}{1 - (1+i)^4}.$$

The ratio is 5, and thus (setting  $x = (1+i)^4$ )

$$5 = \frac{(1+i)^4 - (1+i)^{44}}{(1+i)^4 - (1+i)^{24}} = \frac{x - x^{11}}{x - x^6}$$

$$5x - 5x^6 = x - x^{11}$$

$$5 - 5x^5 = 1 - x^{10}$$

$$x^{10} - 5x^5 + 4 = 0$$

$$(x^5 - 1)(x^5 - 4) = 0.$$

Only the second root gives a positive solution. Thus

$$x^5 = 4$$

$$x = 1.31951$$

$$X = 100 \frac{1.31951 - 1.31951^{11}}{1 - 1.31951} = 6195.$$

Annuity symbols can also be used. Using the annual interest rate, the equation is

$$100 \frac{s_{\overline{40}|}}{a_{\overline{4}|}} = 5(100) \frac{s_{\overline{20}|}}{a_{\overline{4}|}}$$

$$\frac{(1+i)^{40} - 1}{i} = 5 \frac{(1+i)^{20} - 1}{i}$$

$$(1+i)^{40} - 5(1+i)^{20} + 4 = 0$$

$$(1+i)^{20} = 4$$

and the solution proceeds as above.

3. Solution: C

Eric's (compound) interest in the last 6 months of the 8th year is  $100 \left(1 + \frac{i}{2}\right)^{15} \frac{i}{2}$ .

Mike's (simple) interest for the same period is  $200 \frac{i}{2}$ .

Thus,

$$100 \left(1 + \frac{i}{2}\right)^{15} \frac{i}{2} = 200 \frac{i}{2}$$

$$\left(1 + \frac{i}{2}\right)^{15} = 2$$

$$1 + \frac{i}{2} = 1.047294$$

$$i = 0.09459 = 9.46\%.$$

4. Solution: C

$$\begin{aligned}
 77.1 &= v(Ia)_{\overline{n}|} + \frac{nv^{n+1}}{i} \\
 &= v \left[ \frac{\ddot{a}_{\overline{n}|} - nv^n}{i} \right] + \frac{nv^{n+1}}{i} \\
 &= \frac{a_{\overline{n}|}}{i} - \frac{nv^{n+1}}{i} + \frac{nv^{n+1}}{i} \\
 &= \frac{a_{\overline{n}|}}{i} = \frac{1-v^n}{i^2} = \frac{1-v^n}{0.011025}
 \end{aligned}$$

$$0.85003 = 1 - v^n$$

$$1.105^{-n} = 0.14997$$

$$n = -\frac{\ln(0.14997)}{\ln(1.105)} = 19.$$

To obtain the present value without remembering the formula for an increasing annuity, consider the payments as a perpetuity of 1 starting at time 2, a perpetuity of 1 starting at time 3, up to a perpetuity of 1 starting at time  $n + 1$ . The present value one period before the start of each perpetuity is  $1/i$ . The total present value is  $(1/i)(v + v^2 + \dots + v^n) = (1/i)a_{\overline{n}|}$ .

5. Solution: C

The interest earned is a decreasing annuity of 6, 5.4, etc. Combined with the annual deposits of 100, the accumulated value in fund Y is

$$\begin{aligned}
 &6(Ds)_{\overline{10}|0.09} + 100s_{\overline{10}|0.09} \\
 &= 6 \left( \frac{10(1.09)^{10} - s_{\overline{10}|0.09}}{0.09} \right) + 100(15.19293) \\
 &= 565.38 + 1519.29 \\
 &= 2084.67.
 \end{aligned}$$

6. Solution: D

For the first 10 years, each payment equals 150% of interest due. The lender charges 10%, therefore 5% of the principal outstanding will be used to reduce the principal.

At the end of 10 years, the amount outstanding is  $1000(1 - 0.05)^{10} = 598.74$ .

Thus, the equation of value for the last 10 years using a comparison date of the end of year 10 is

$$598.74 = Xa_{\overline{10}|10\%} = 6.1446X$$

$$X = 97.44.$$

7. Solution: B

The book value at time 6 is the present value of future payments:

$$BV_6 = 10,000v^4 + 800a_{\overline{4}|0.06} = 7920.94 + 2772.08 = 10,693.$$

The interest portion is  $10,693(0.06) = 641.58$ .

8. Solution: A

The value of the perpetuity after the fifth payment is  $100/0.08 = 1250$ . The equation to solve is:

$$1250 = X(v + 1.08v^2 + \dots + 1.08^{24}v^{25})$$

$$= X(v + v + \dots + v) = X(25)/1.08$$

$$X = 50(1.08) = 54.$$

9. Solution: C

Equation of value at end of 30 years:

$$10(1 - d/4)^{-40}(1.03)^{40} + 20(1.03)^{30} = 100$$

$$10(1 - d/4)^{-40} = [100 - 20(1.03)^{30}] / 1.03^{40} = 15.7738$$

$$1 - d/4 = 1.57738^{-1/40} = 0.98867$$

$$d = 4(1 - 0.98867) = 0.0453 = 4.53\%.$$

10. Solution: E

The accumulation function is  $a(t) = \exp\left[\int_0^t (s^2/100)ds\right] = \exp(t^3/300)$ .

The accumulated value of 100 at time 3 is  $100 \exp(3^3/300) = 109.41743$ .

The amount of interest earned from time 3 to time 6 equals the accumulated value at time 6 minus the accumulated value at time 3. Thus

$$(109.41743 + X)[a(6)/a(3) - 1] = X$$

$$(109.41743 + X)(2.0544332/1.0941743 - 1) = X$$

$$(109.41743 + X)0.877613 = X$$

$$96.026159 = 0.122387X$$

$$X = 784.61.$$

11. Solution: A

$$167.50 = 10a_{\overline{5}|9.2\%} + 10(1.092)^{-5} \sum_{t=1}^{\infty} \left[ \frac{(1+k)}{1.092} \right]^t$$

$$167.50 = 38.6955 + 6.44001 \frac{(1+k)/1.092}{1 - (1+k)/1.092}$$

$$(167.50 - 38.6955)[1 - (1+k)/1.092] = 6.44001(1+k)/1.092$$

$$128.8045 = 135.24451(1+k)/1.092$$

$$1+k = 1.0400$$

$$k = 0.0400 \Rightarrow K = 4.0\%.$$

12. Solution: B

$$\text{Option 1: } 2000 = Pa_{\overline{10}|0.0807}$$

$$P = 299 \Rightarrow \text{Total payments} = 2990$$

Option 2: Interest needs to be  $2990 - 2000 = 990$

$$990 = i[2000 + 1800 + 1600 + \dots + 200]$$

$$= 11,000i$$

$$i = 0.09 = 9.00\%$$

13. Solution: B

Monthly payment at time  $t$  is  $1000(0.98)^{t-1}$ .

Because the loan amount is unknown, the outstanding balance must be calculated prospectively.

The value at time 40 months is the present value of payments from time 41 to time 60:

$$OB_{40} = 1000[0.98^{40}v^1 + \dots + 0.98^{59}v^{20}]$$

$$= 1000 \frac{0.98^{40}v^1 - 0.98^{60}v^{21}}{1 - 0.98v}, v = 1/(1.0075)$$

$$= 1000 \frac{0.44238 - 0.25434}{1 - 0.97270} = 6888.$$

14. Solution: C

The equation of value is

$$98S_{\overline{3n}|} + 98S_{\overline{2n}|} = 8000$$

$$\frac{(1+i)^{3n} - 1}{i} + \frac{(1+i)^{2n} - 1}{i} = 81.63$$

$$(1+i)^n = 2$$

$$\frac{8-1}{i} + \frac{4-1}{i} = 81.63$$

$$\frac{10}{i} = 81.63$$

$$i = 12.25\%$$

15. Solution: B

Convert 9% convertible quarterly to an effective rate of  $j$  per month:

$$(1+j)^3 = \left(1 + \frac{0.09}{4}\right) \text{ or } j = 0.00744.$$

Then

$$2(Ia)_{\overline{60}|0.00744} = 2 \frac{\ddot{a}_{\overline{60}|0.00744} - 60v^{60}}{0.00744} = 2 \frac{48.6136 - 38.4592}{0.00744} = 2729.7.$$

16. Solution: A

Equating present values:

$$100 + 200v^n + 300v^{2n} = 600v^{10}$$

$$100 + 200(0.76) + 300(0.76)^2 = 600v^{10}$$

$$425.28 = 600v^{10}$$

$$0.7088 = v^{10}$$

$$0.96617 = v$$

$$1.03501 = 1+i$$

$$i = 0.035 = 3.5\%.$$

17. Solution: A

The accumulation function is:

$$a(t) = e^{\int_0^t \frac{1}{8+r} dr} = e^{\ln(8+r)|_0^t} = \frac{8+t}{8}.$$

Using the equation of value at end of 10 years:

$$\begin{aligned} 20,000 &= \int_0^{10} (8k + tk) \frac{a(10)}{a(t)} dt = k \int_0^{10} (8+t) \frac{18/8}{(8+t)/8} dt = k \int_0^{10} 18 dt \\ &= 180k \Rightarrow k = \frac{20,000}{180} = 111. \end{aligned}$$

18. Solution: D

Let  $C$  be the redemption value and  $v = 1/(1+i)$ . Then

$$\begin{aligned} X &= 1000ra_{\overline{2n}|i} + Cv^{2n} \\ &= 1000r \frac{1-v^{2n}}{i} + 381.50 \\ &= 1000(1.03125)(1-0.5889^2) + 381.50 \\ &= 1055.11. \end{aligned}$$

19. Solution: D

Equate net present values:

$$-4000 + 2000v + 4000v^2 = 2000 + 4000v - Xv^2$$

$$\frac{4000 + X}{1.21} = 6000 + \frac{2000}{1.1}$$

$$X = 5460.$$

20. Solution: D

The present value of the perpetuity =  $X/i$ . Let  $B$  be the present value of Brian's payments.

$$B = Xa_{\overline{n}|i} = 0.4 \frac{X}{i}$$

$$a_{\overline{n}|i} = \frac{0.4}{i} \Rightarrow 0.4 = 1 - v^n \Rightarrow v^n = 0.6$$

$$K = v^{2n} \frac{X}{i}$$

$$K = 0.36 \frac{X}{i},$$

Thus the charity's share is 36% of the perpetuity's present value.



21. Solution: D

The given information yields the following amounts of interest paid:

$$\text{Seth} = 5000 \left( \left( 1 + \frac{0.12}{2} \right)^{10} - 1 \right) = 8954.24 - 5000 = 3954.24$$

$$\text{Janice} = 5000(0.06)(10) = 3000.00$$

$$\text{Lori} = P(10) - 5000 = 1793.40 \text{ where } P = \frac{5000}{a_{\overline{10}|6\%}} = 679.35$$

The sum is 8747.64.

22. Solution: E

For Bruce,  $X = 100[(1+i)^{11} - (1+i)^{10}] = 100(1+i)^{10}i$ . Similarly, for Robbie,  $X = 50(1+i)^{16}i$

.Dividing the second equation by the first gives  $1 = 0.5(1+i)^6$  which implies

$$i = 2^{1/6} - 1 = 0.122462. \text{ Thus } X = 100(1.122462)^{10}(0.122462) = 38.879.$$

23. Solution: D

Year  $t$  interest is  $ia_{\overline{n-t+1}|i} = 1 - v^{n-t+1}$ .

Year  $t+1$  principal repaid is  $1 - (1 - v^{n-t}) = v^{n-t}$ .

$$X = 1 - v^{n-t+1} + v^{n-t} = 1 + v^{n-t}(1 - v) = 1 + v^{n-t}d.$$

24. Solution: B

For the first perpetuity,

$$32 = 10(v^3 + v^6 + \dots) = 10v^3 / (1 - v^3)$$

$$32 - 32v^3 = 10v^3$$

$$v^3 = 32 / 42.$$

For the second perpetuity,

$$X = v^{1/3} + v^{2/3} + \dots = v^{1/3} / (1 - v^{1/3}) = (32 / 42)^{1/9} / [1 - (32 / 42)^{1/9}] = 32.599.$$

25 Solution: D

Under either scenario, the company will have  $822,703(0.05) = 41,135$  to invest at the end of each of the four years. Under Scenario A these payments will be invested at 4.5% and accumulate to  $41,135s_{\overline{4}|0.045} = 41,135(4.2782) = 175,984$ . Adding the maturity value produces 998,687 for a loss of 1,313. Note that only answer D has this value.

The Scenario B calculation is

$$41,135s_{\overline{4}|0.055} = 41,135(4.3423) = 178,621 + 822,703 - 1,000,000 = 1,324.$$

26. Solution: D.

The present value is

$$5000[1.07v + 1.07^2v^2 + \dots + 1.07^{20}v^{20}] \\ = 5000 \frac{1.07v - 1.07^{21}v^{21}}{1 - 1.07v} = 5000 \frac{1.01905 - 1.48622}{1 - 1.01905} = 122,617.$$

27. Solution: C.

The first cash flow of 60,000 at time 3 earns 2400 in interest for a time 4 receipt of 62,400.

Combined with the final payment, the investment returns 122,400 at time 4. The present value is  $122,400(1.05)^{-4} = 100,699$ . The net present value is 699.

28. Solution: B.

Using spot rates, the value of the bond is:

$$60/1.07 + 60/1.08^2 + 1060/1.09^3 = 926.03.$$

29. Solution: E.

Using spot rates, the value of the bond is:

$$60/1.07 + 60/1.08^2 + 1060/1.09^3 = 926.03. \text{ The annual effective rate is the solution to } \\ 926.03 = 60a_{\overline{3}|i} + 1000(1+i)^{-3}. \text{ Using a calculator, the solution is } 8.9\%.$$

30. Solution: C.

Duration is the negative derivative of the price multiplied by one plus the interest rate and divided by the price. Hence, the duration is  $-(-700)(1.08)/100 = 7.56$ .

31. Solution: C

The size of the dividend does not matter, so assume it is 1. Then the duration is

$$\frac{\sum_{t=1}^{\infty} tv^t}{\sum_{t=1}^{\infty} v^t} = \frac{(Ia)_{\overline{\infty}|}}{a_{\overline{\infty}|}} = \frac{\ddot{a}_{\overline{\infty}|}/i}{1/i} = \frac{1/(di)}{1/i} = \frac{1}{d} = \frac{1.1}{0.1} = 11.$$

32. Solution: B

$$\frac{\sum_{t=1}^{\infty} tv^t R_t}{\sum_{t=1}^{\infty} v^t R_t} = \frac{\sum_{t=1}^{\infty} tv^t 1.02^t}{\sum_{t=1}^{\infty} v^t 1.02^t} = \frac{(Ia)_{\infty|j}}{a_{\infty|j}} = \frac{\ddot{a}_{\infty|j} / j}{1/j} = \frac{1}{d}.$$

Duration =

The interest rate  $j$  is such that  $(1+j)^{-1} = 1.02v = 1.02/1.05 \Rightarrow j = 0.03/1.02$ . Then the duration is  $1/d = (1+j)/j = (1.05/1.02)/(0.03/1.02) = 1.05/0.03 = 35$ .

33. Solution: A

The outstanding balance is the present value of future payments. With only one future payment, that payment must be  $559.12(1.08) = 603.85$ . The amount borrowed is  $603.85a_{\overline{4}|0.08} = 2000$ . The first payment has  $2000(0.08) = 160$  in interest, thus the principal repaid is  $603.85 - 160 = 443.85$ .

Alternatively, observe that the principal repaid in the final payment is the outstanding loan balance at the previous payment, or 559.12. Principal repayments form a geometrically decreasing sequence, so the principal repaid in the first payment is  $559.12/1.08^3 = 443.85$ .

34. Solution: B

Because the yield rate equals the coupon rate, Bill paid 1000 for the bond. In return he receives 30 every six months, which accumulates to  $30s_{\overline{20}|j}$  where  $j$  is the semi-annual interest rate. The equation of value is  $1000(1.07)^{10} = 30s_{\overline{20}|j} + 1000 \Rightarrow s_{\overline{20}|j} = 32.238$ . Using a calculator to solve for the interest rate produces  $j = 0.0476$  and so  $i = 1.0476^2 - 1 = 0.0975 = 9.75\%$ .

35. Solution: A

To receive 3000 per month at age 65 the fund must accumulate to  $3,000(1,000/9.65) = 310,880.83$ . The equation of value is  $310,880.83 = X\ddot{s}_{\overline{300}|0.08/12} = 957.36657X \Rightarrow 324.72$ .

36. Solution: D

(A) The left-hand side evaluates the deposits at age 0, while the right-hand side evaluates the withdrawals at age 17.

(B) The left-hand side has 16 deposits, not 17.

(C) The left-hand side has 18 deposits, not 17.

(D) The left-hand side evaluates the deposits at age 18 and the right-hand side evaluates the withdrawals at age 18.

(E) The left-hand side has 18 deposits, not 17 and 5 withdrawals, not 4.

37. Solution: D

Because only Bond II provides a cash flow at time 1, it must be considered first. The bond provides 1025 at time 1 and thus  $1000/1025 = 0.97561$  units of this bond provides the required cash. This bond then also provides  $0.97561(25) = 24.39025$  at time 0.5. Thus Bond I must provide  $1000 - 24.39025 = 975.60975$  at time 0.5. The bond provides 1040 and thus  $975.60975/1040 = 0.93809$  units must be purchased.

38. Solution: C

Because only Mortgage II provides a cash flow at time two, it must be considered first. The mortgage provides  $Y / a_{\overline{2}|0.07} = 0.553092Y$  at times one and two. Therefore,  $0.553092Y = 1000$  for  $Y = 1808.02$ . Mortgage I must provide  $2000 - 1000 = 1000$  at time one and thus  $X = 1000/1.06 = 943.40$ . The sum is 2751.42.

39. Solution: A

Bond I provides the cash flow at time one. Because 1000 is needed, one unit of the bond should be purchased, at a cost of  $1000/1.06 = 943.40$ .

Bond II must provide 2000 at time three. Therefore, the amount to be reinvested at time two is  $2000/1.065 = 1877.93$ . The purchase price of the two-year bond is  $1877.93/1.07^2 = 1640.26$ . The total price is 2583.66.

40. Solution: C

Given the coupon rate is greater than the yield rate, the bond sells at a premium. Thus, the minimum yield rate for this callable bond is calculated based on a call at the earliest possible date because that is most disadvantageous to the bond holder (earliest time at which a loss occurs). Thus,  $X$ , the par value, which equals the redemption value because the bond is a par value bond, must satisfy

$$\text{Price} = 1722.25 = 0.04Xa_{\overline{30}|0.03} + Xv_{0.03}^{30} = 1.196X \Rightarrow X = 1440.$$

41. Solution: B

Because  $40/1200$  is greater than  $0.03$ , for early redemption the earliest redemption should be evaluated. If redeemed after 15 years, the price is  $40a_{\overline{30}|0.03} + 1200/1.03^{30} = 1278.40$ . If the bond is redeemed at maturity, the price is  $40a_{\overline{40}|0.03} + 1100/1.03^{40} = 1261.80$ . The smallest value should be selected, which is 1261.80. (When working with callable bonds, the maximum a buyer will pay is the smallest price over the various call dates. Paying more may not earn the desired yield.)

42. Solution: E

Given the coupon rate is less than the yield rate, the bond sells at a discount. Thus, the minimum yield rate for this callable bond is calculated based on a call at the latest possible date because that is most disadvantageous to the bond holder (latest time at which a gain occurs). Thus,  $X$ , the par value, which equals the redemption value because the bond is a par value bond, must satisfy

$$\text{Price} = 1021.50 = 0.02Xa_{\overline{20}|0.03} + Xv_{0.03}^{20} = 0.851225X \Rightarrow X = 1200.$$

43. Solution: B

Given the price is less than the amount paid for an early call, the minimum yield rate for this callable bond is calculated based on a call at the latest possible date. Thus, for an early call, the effective yield rate per coupon period,  $j$ , must satisfy  $\text{Price} = 1021.50 = 22a_{\overline{19}|j} + 1200v_j^{19}$ . Using the calculator,  $j = 2.86\%$ . We also must check the yield if the bond is redeemed at maturity. The equation is  $1021.50 = 22a_{\overline{20}|j} + 1100v_j^{20}$ . The solution is  $j = 2.46\%$ . Thus, the yield, expressed as a nominal annual rate of interest convertible semiannually, is twice the smaller of the two values, or  $4.92\%$ .

44. Solution: C

First, the present value of the liability is  $PV = 35,000a_{\overline{15}|6.2\%} = 335,530.30$ .

The duration of the liability is:

$$\bar{d} = \frac{\sum tv^t R_t}{\sum v^t R_t} = \frac{35,000v + 2(35,000)v^2 + \dots + 15(35,000)v^{15}}{335,530.30} = \frac{2,312,521.95}{335,530.30} = 6.89214.$$

Let  $X$  denote the amount invested in the 5 year bond.

$$\text{Then, } \frac{X}{335,530.30}(5) + \left(1 - \frac{X}{335,530.30}\right)(10) = 6.89214 \Rightarrow X = 208,556.$$

45. Solution: A

The present value of the first eight payments is:

$$PV = 2000v + 2000(1.03)v^2 + \dots + 2000(1.03)^7 v^8 = \frac{2000v - 2000(1.03)^8 v^9}{1 - 1.03v} = 13,136.41.$$

The present value of the last eight payments is:

$$PV = 2000(1.03)^7 0.97v^9 + 2000(1.03)^7 (0.97)^2 v^{10} + \dots + 2000(1.03)^7 (0.97^8)v^{16} \\ = \frac{2000(1.03)^7 0.97v^9 - 2000(1.03)^7 (0.97)^9 v^{17}}{1 - 0.97v} = 7,552.22.$$

Therefore, the total loan amount is  $L = 20,688.63$ .

46. Solution: E

$$2000 = 500 \exp \left( \int_0^t \frac{\frac{r^2}{100}}{3 + \frac{r^3}{150}} dr \right)$$

$$4 = \exp \left( 0.5 \int_0^t \frac{\frac{r^2}{50}}{3 + \frac{r^3}{150}} dr \right) = \exp \left[ 0.5 \ln \left( 3 + \frac{r^3}{150} \right) \right]_0^t$$

$$4 = \exp \left[ 0.5 \ln \left( 1 + \frac{t^3}{450} \right) \right] = \left( 1 + \frac{t^3}{450} \right)^{\frac{1}{2}}$$

$$16 = \left( 1 + \frac{t^3}{450} \right)$$

$$t = 18.8988$$

47. Solution: E

Let  $F$ ,  $C$ ,  $r$ , and  $i$  have their usual interpretations. The discount is  $(Ci - Fr)a_{\overline{n}|}$  and the discount in the coupon at time  $t$  is  $(Ci - Fr)v^{n-t+1}$ . Then,

$$194.82 = (Ci - Fr)v^{26}$$

$$306.69 = (Ci - Fr)v^{21}$$

$$0.63523 = v^5 \Rightarrow v = 0.91324 \Rightarrow i = 0.095$$

$$(Ci - Fr) = 194.82(1.095)^{26} = 2062.53$$

$$\text{Discount} = 2062.53 a_{\overline{40}|0.095} = 21,135$$

48. Solution: A

$$699.68 = Pv^{8-5+1}$$

$$P = 842.39 \text{ (annual payment)}$$

$$P_1 = \frac{699.68}{1.0475^4} = 581.14$$

$$I_1 = 842.39 - 581.14 = 261.25$$

$$L = \frac{261.25}{0.0475} = 5500 \text{ (loan amount)}$$

$$\text{Total interest} = 842.39(8) - 5500 = 1239.12$$

49. Solution: D

$$OB_{18} = 22,000(1.007)^{18} - 450.30s_{\overline{18}|0.007} = 16,337.10$$

$$16,337.10 = Pa_{\overline{24}|0.004}$$

$$P = 715.27$$

50. Solution: C

If the bond has no premium or discount, it was bought at par so the yield rate equals the coupon rate, 0.038.

$$d = \frac{\frac{1}{2} \left( 1(190)v + 2(190)v^2 + \dots + 14(190)v^{14} + 14(5000)v^{14} \right)}{190v + 190v^2 + \dots + 190v^{14} + 5000v^{14}}$$

$$d = \frac{95(Ia)_{\overline{14}|} + 7(5000)v^{14}}{190a_{\overline{14}|} + 5000v^{14}}$$

$$d = 5.5554$$

Or, taking advantage of a shortcut:

$$d = \ddot{a}_{\overline{14}|0.038} = 11.1107. \text{ This is in half years, so dividing by two, } d = \frac{11.1107}{2} = 5.5554.$$

51. Solution: A

$$\bar{v} = \frac{7.959}{1.072} = 7.425$$

$$P(0.08) = P(0.072)[1 - (\Delta i)\bar{v}]$$

$$P(0.08) = 1000[1 - (0.008)(7.425)] = 940.60$$

52. Solution: E

$$(1 + s_3)^3 = (1 + s_2)^2(1 + {}_1f_2)$$

$$0.85892 = \frac{1}{(1 + s_3)^3}, s_3 = 0.052$$

$$0.90703 = \frac{1}{(1 + s_2)^2}, s_2 = 0.050$$

$$1.052^3 = 1.050^2(1 + {}_1f_2)$$

$${}_1f_2 = 0.056$$

53. Solution: C

Let  $d_0$  be the Macaulay duration at time 0.

$$d_0 = \ddot{a}_{\overline{8}|0.05} = 6.7864$$

$$d_1 = d_0 - 1 = 5.7864$$

$$d_2 = \ddot{a}_{\overline{7}|0.05} = 6.0757$$

$$\frac{d_1}{d_2} = \frac{5.7864}{6.0757} = 0.9524$$

This solution employs the fact that when a coupon bond sells at par the duration equals the present value of an annuity-due. For the duration just before the first coupon the cash flows are the same as for the original bond, but all occur one year sooner. Hence the duration is one year less.

Alternatively, note that the numerators for  $d_1$  and  $d_2$  are identical. That is because they differ only with respect to the coupon at time 1 (which is time 0 for this calculation) and so the payment does not add anything. The denominator for  $d_2$  is the present value of the same bond, but with 7 years, which is 5000. The denominator for  $d_1$  has the extra coupon of 250 and so is 5250. The desired ratio is then  $5000/5250 = 0.9524$ .

54. Solution: A

Let  $N$  be the number of shares bought of the bond as indicated by the subscript.

$$N_C(105) = 100, N_C = 0.9524$$

$$N_B(100) = 102 - 0.9524(5), N_B = 0.9724$$

$$N_A(107) = 99 - 0.9524(5), N_A = 0.8807$$

55. Solution: B

All are true except B. Immunization requires frequent rebalancing.



56. Solution: D

Set up the following two equations in the two unknowns:

$$A(1.05)^2 + B(1.05)^{-2} = 6000$$

$$2A(1.05)^1 - 2B(1.05)^{-3} = 0.$$

Solving simultaneously gives:

$$A = 2721.09$$

$$B = 3307.50$$

$$|A - B| = 586.41.$$

57. Solution: A

Set up the following two equations in the two unknowns.

$$(1) \quad 5000(1.03)^3 + B(1.03)^{-b} = 12,000 \Rightarrow$$

$$5463.635 + B(1.03)^{-b} = 12,000 \Rightarrow B(1.03)^{-b} = 6536.365$$

$$(2) \quad 3(5000)(1.03)^3 - bB(1.03)^{-b} = 0 \Rightarrow 16,390.905 - b6536.365 = 0$$

$$b = 2.5076$$

$$B = 7039.27$$

$$\frac{B}{b} = 2807.12$$

58. Solution: D

$$P_A = A(1+i)^{-2} + B(1+i)^{-9}$$

$$P_L = 95,000(1+i)^{-5}$$

$$P'_A = -2A(1+i)^{-3} - 9B(1+i)^{-10}$$

$$P'_L = -5(95,000)(1+i)^{-6}$$

Set the present values and derivatives equal and solve simultaneously.

$$0.92456A + 0.70259B = 78,083$$

$$-1.7780A - 6.0801B = -375,400$$

$$B = \frac{78,083(1.7780 / 0.92456) - 375,400}{0.70259(1.7780 / 0.92456) - 6.0801} = 47,630$$

$$A = [78,083 - 0.70259(47,630)] / 0.92456 = 48,259$$

$$\frac{A}{B} = 1.0132$$

59. Solution: D

Throughout the solution, let  $j = i/2$ .

For bond A, the coupon rate is  $(i + 0.04)/2 = j + 0.02$ .

For bond B, the coupon rate is  $(i - 0.04)/2 = j - 0.02$ .

The price of bond A is  $P_A = 10,000(j + 0.02)a_{\overline{20}|j} + 10,000(1 + j)^{-20}$ .

The price of bond B is  $P_B = 10,000(j - 0.02)a_{\overline{20}|j} + 10,000(1 + j)^{-20}$ .

Thus,

$$P_A - P_B = 5,341.12 = [200 - (-200)]a_{\overline{20}|j} = 400a_{\overline{20}|j}$$

$$a_{\overline{20}|j} = 5,341.12 / 400 = 13.3528.$$

Using the financial calculator,  $j = 0.042$  and  $i = 2(0.042) = 0.084$ .

60. Solution: D

The initial level monthly payment is

$$R = \frac{400,000}{a_{\overline{15 \times 12}|0.09/12}} = \frac{400,000}{a_{\overline{180}|0.0075}} = 4,057.07.$$

The outstanding loan balance after the 36th payment is

$$B_{36} = Ra_{\overline{180-36}|0.0075} = 4,057.07a_{\overline{144}|0.0075} = 4,057.07(87.8711) = 356,499.17.$$

The revised payment is  $4,057.07 - 409.88 = 3,647.19$ .

Thus,

$$356,499.17 = 3,647.19a_{\overline{144}|j/12}$$

$$a_{\overline{144}|j/12} = 356,499.17 / 3,647.19 = 97.7463.$$

Using the financial calculator,  $j/12 = 0.575\%$ , for  $j = 6.9\%$ .

61. Solution: D

The price of the first bond is

$$1000(0.05/2)a_{\overline{30 \times 2}|0.05/2} + 1200(1 + 0.05/2)^{-30 \times 2} = 25a_{\overline{60}|0.025} + 1200(1.025)^{-60} \\ = 772.72 + 272.74 = 1,045.46.$$

The price of the second bond is also 1,045.46. The equation to solve is

$$1,045.46 = 25a_{\overline{60}|j/2} + 800(1 + j/2)^{-60}.$$

The financial calculator can be used to solve for  $j/2 = 2.2\%$  for  $j = 4.4\%$ .

62. Solution: E

Let  $n$  = years. The equation to solve is

$$1000(1.03)^{2n} = 2(1000)(1.0025)^{12n}$$

$$2n \ln 1.03 + \ln 1000 = 12n \ln 1.0025 + \ln 2000$$

$$0.029155n = 0.69315$$

$$n = 23.775.$$

This is 285.3 months. The next interest payment to Lucas is at a multiple of 6, which is 288 months.

63. Solution: A

Equating the accumulated values after 4 years provides an equation in  $K$ .

$$10 \left( 1 + \frac{K}{25} \right)^4 = 10 \exp \left( \int_0^4 \frac{1}{K + 0.25t} dt \right)$$

$$4 \ln \left( 1 + \frac{K}{25} \right) = \int_0^4 \frac{1}{K + 0.25t} dt = 4 \ln(K + 0.25t) \Big|_0^4 = 4 \ln(K + 1) - 4 \ln(K) = 4 \ln \frac{K + 1}{K}$$

$$1 + 0.04K = \frac{K + 1}{K}$$

$$0.04K^2 = 1$$

$$K = 5.$$

Therefore,  $X = 10(1 + 5/25)^4 = 20.74$ .

64. Solution: D

The outstanding balance at time 25 is  $100(Da)_{\overline{25}|} = 100 \frac{25 - a_{\overline{25}|}}{i}$ . The principle repaid in the 26th

payment is  $X = 2500 - i(100) \frac{25 - a_{\overline{25}|}}{i} = 2500 - 2500 + 100a_{\overline{25}|} = 100a_{\overline{25}|}$ . The amount borrowed is

the present value of all 50 payments,  $2500a_{\overline{25}|} + v^{25}100(Da)_{\overline{25}|}$ . Interest paid in the first payment is then

$$\begin{aligned} & i \left[ 2500a_{\overline{25}|} + v^{25}100(Da)_{\overline{25}|} \right] \\ &= 2500(1 - v^{25}) + 100v^{25}(25 - a_{\overline{25}|}) \\ &= 2500 - 2500v^{25} + 2500v^{25} - v^{25}100a_{\overline{25}|} \\ &= 2500 - Xv^{25}. \end{aligned}$$

65. Solution: C

The accumulated value is  $1000\ddot{s}_{\overline{20}|0.816} = 50,382.16$ . This must provide a semi-annual annuity-due of 3000. Let  $n$  be the number of payments. Then solve  $3000\ddot{a}_{\overline{n}|0.04} = 50,382.16$  for  $n = 26.47$ .

Therefore, there will be 26 full payments plus one final, smaller, payment. The equation is  $50,382.16 = 3000\ddot{a}_{\overline{26}|0.04} + X(1.04)^{-26}$  with solution  $X = 1430$ . Note that the while the final payment is the 27th payment, because this is an annuity-due, it takes place 26 periods after the annuity begins.

66. Solution: D

For the first perpetuity,

$$\frac{1}{(1+i)^2 - 1} + 1 = 7.21$$

$$\frac{1}{6.21} = (1+i)^2 - 1$$

$$i = 0.0775.$$

For the second perpetuity,

$$R \left[ \frac{1}{(1.0775 + 0.01)^3 - 1} + 1 \right] (1.0875)^{-1} = 7.21$$

$$1.286139R = 7.21(1.0875)(0.286139)$$

$$R = 1.74.$$

67. Solution: E

$$10,000 = 100(Ia)_{\overline{5}|} + Xv^5 a_{\overline{15}|} = 100 \left( \frac{\ddot{a}_{\overline{5}|} - 5v^5}{0.05} \right) + Xv^5 a_{\overline{15}|}$$

$$10,000 = 1256.64 + 8.13273X$$

$$1075 = X$$

68. Solution: C

$$5000 = Xs_{\overline{10}|0.06}(1.05)^5$$

$$X = \frac{5000}{13.1808(1.2763)} = 297.22$$

69. Solution: E

The monthly payment on the original loan is  $\frac{65,000}{a_{\overline{180}|8/12\%}} = 621.17$ . After 12 payments the

outstanding balance is  $621.17a_{\overline{168}|8/12\%} = 62,661.40$ . The revised payment is  $\frac{62,661.40}{a_{\overline{168}|6/12\%}} = 552.19$ .

70. Solution: E

At the time of the final deposit the fund has  $750s_{\overline{18}|0.07} = 25,499.27$ . This is an immediate annuity because the evaluation is done at the time the last payments is made (which is the end of the final year). A tuition payment of  $6000(1.05)^{17} = 13,752.11$  is made, leaving 11,747.16. It earns 7%, so a year later the fund has  $11,747.16(1.07) = 12,569.46$ . Tuition has grown to  $13,752.11(1.05) = 14,439.72$ . The amount needed is  $14,439.72 - 12,569.46 = 1,870.26$

71. Solution: B

The coupons are  $1000(0.09)/2 = 45$ . The present value of the coupons and redemption value at 5% per semiannual period is  $P = 45a_{\overline{40}|0.05} + 1200(1.05)^{-40} = 942.61$ .

72. Solution: A

For a bond bought at discount, the minimum price will occur at the latest possible redemption date.  $P = 50a_{\overline{20}|0.06} + 1000(1.06)^{-20} = 885.30$ . (When working with callable bonds, the maximum a buyer will pay is the smallest price over the various call dates. Paying more may not earn the desired yield.)

73. Solution: C

$$\frac{1.095^5}{1.090^4} - 1 = 11.5\%$$

74. Solution: D

The accumulated value of the first year of payments is  $2000s_{\overline{12}|0.005} = 24,671.12$ . This amount increases at 2% per year. The effective annual interest rate is  $1.005^{12} - 1 = 0.061678$ . The present value is then

$$P = 24,671.12 \sum_{k=1}^{25} 1.02^{k-1} (1.061678)^{-k} = 24,671.12 \frac{1}{1.02} \sum_{k=1}^{25} \left( \frac{1.02}{1.061678} \right)^k$$
$$= 24,187.37 \frac{0.960743 - 0.960743^{26}}{1 - 0.960743} = 374,444.$$

This is 56 less than the lump sum amount.

75. Solution: A

The monthly interest rate is  $0.072/12 = 0.006$ . 6500 five years from today has value  $6500(1.006)^{-60} = 4539.77$ . The equation of value is

$$4539.77 = 1700(1.006)^{-n} + 3400(1.006)^{-2n}.$$

Let  $x = 1.006^{-n}$ . Then, solve the quadratic equation

$$3400x^2 + 1700x - 4539.77 = 0$$

$$x = \frac{-1700 + \sqrt{1700^2 - 4(3400)(-4539.77)}}{2(3400)} = 0.93225.$$

Then,

$$1.006^{-n} = 0.93225 \Rightarrow -n \ln(1.006) = \ln(0.93225) \Rightarrow n = 11.73.$$

To ensure there is 6500 in five years, the deposits must be made earlier and thus the maximum integral value is 11.

76. Solution: C

$$\frac{(1-d/2)^{-4}}{(1-d/4)^{-4}} = \left( \frac{39}{38} \right)^4 \Rightarrow \frac{1-d/2}{1-d/4} = \frac{38}{39} \Rightarrow 39 - 39(d/2) = 38 - 38(d/4)$$

$$d(39/2 - 38/4) = 39 - 38$$

$$d = 1 / (19.5 - 9.5) = 0.1$$

$$1+i = (1-d/2)^{-2} = .95^{-2} = 1.108 \Rightarrow i = 10.8\%.$$

77. Solution: C

The monthly interest rate is  $0.042/12 = 0.0035$ . The quarterly interest rate is  $1.0035^3 - 1 = 0.0105$ . The investor makes 41 quarterly deposits and the ending date is 124 months from the start. Using January 1 of year  $y$  as the comparison date produces the following equation:

$$X + \sum_{k=1}^{41} \frac{100}{1.0105^k} = \frac{1.9X}{1.0035^{124}}$$

Substituting  $1.0105 = 1.0035^3$  gives answer (C).

78. Solution: D

Convert the two annual rates, 4% and 5%, to two-year rates as  $1.04^2 - 1 = 0.0816$  and  $1.05^2 - 1 = 0.1025$ .

The accumulated value is

$$100\ddot{s}_{\overline{3}|0.0816} (1.05)^4 + 100\ddot{s}_{\overline{2}|0.1025} = 100(3.51678)(1.21551) + 100(2.31801) = 659.269$$

With only five payments, an alternative approach is to accumulate each one to time ten and add them up.

The two-year yield rate is the solution to  $100\ddot{s}_{\overline{5}|i} = 659.269$ . Using the calculator, the two-year rate is 0.093637. The annual rate is  $1.093637^{0.5} - 1 = 0.04577$  which is 4.58%.

79. Solution: C

$$(1.08)^{1/12} - 1 = 0.006434$$

$$\frac{1}{1.08^{15}} 25,000\ddot{a}_{\overline{4}|8\%} = X\ddot{a}_{\overline{216}|0.6434\%}$$

$$X = \frac{25,000(3.57710)}{3.17217(117.2790)} = 240.38$$

80. Solution: B

$$PV_{\text{perp.}} = \left[ \frac{1}{0.1} + \frac{\frac{1}{0.08} - \frac{1}{0.1}}{1.1^{10}} \right] (15,000) + 15,000$$

$$= 164,457.87 + 15,000 = 179,457.87$$

$$X \left( \ddot{a}_{\overline{10}|0.10} + \frac{\ddot{a}_{\overline{15}|0.08}}{1.1^{10}} \right) = 179,458$$

$$X \left( 6.759 + \frac{9.244}{1.1^{10}} \right) = 179,458$$

$$X = 17,384$$

81. Solution: A

$$1050.50 = (22.50 + X)a_{\overline{14}|0.03} + X \left( \frac{a_{\overline{14}|0.03} - 14(1.03)^{-14}}{0.03} \right) + 300(1.03)^{-14}$$

$$1050.50 = (22.50 + X)11.2961 + X \left( \frac{11.2961 - 9.25565}{0.03} \right) + 198.335 \Rightarrow 79.3111X = 598 \Rightarrow X = 7.54$$

82. Solution: D

The amount of the loan is the present value of the deferred increasing annuity:

$$(1.05)^{-10} \left[ 500\ddot{a}_{\overline{30}|0.05} + 500(I\ddot{a})_{\overline{30}|0.05} \right] = (1.05^{-10})(500) \left[ \ddot{a}_{\overline{30}|0.05} + \frac{\ddot{a}_{\overline{30}|0.05} - 30(1.05)^{-30}}{0.05/1.05} \right] = 64,257.$$

83. Solution: C

$$50,000 \left[ \frac{(1+i)^{30} - (1.03)^{30}}{(1+i)^{30}(i-0.03)} \right] (1+i) = 5,000 \left[ \frac{(1+i)^{30} - (1.03)^{30}}{i-0.03} \right]$$

$$50,000 / (1+i)^{29} = 5,000$$

$$(1+i)^{29} = 10$$

$$i = 10^{1/29} - 1 = 0.082637$$

The accumulated amount is

$$50,000 \left[ \frac{(1.082637)^{30} - (1.03)^{30}}{(1.082637)^{30}(0.082637 - 0.03)} \right] (1.082637) = 797,836.82$$



84. Solution: D

The first payment is 2,000, and the second payment of 2,010 is 1.005 times the first payment. Since we are given that the series of quarterly payments is geometric, the payments multiply by 1.005 every quarter.

Based on the quarterly interest rate, the equation of value is

$$100,000 = 2,000 + 2,000(1.005)v + 2,000(1.005)^2 v^2 + 2,000(1.005)^3 v^3 + \dots = \frac{2,000}{1-1.005v}$$

$$1-1.005v = 2,000/100,000 \Rightarrow v = 0.98/1.005.$$

The annual effective rate is  $v^{-4} - 1 = (0.98/1.005)^{-4} - 1 = 0.10601 = 10.6\%$ .

85. Solution: A

Present value for the first 10 years is  $\frac{1-(1.06)^{-10}}{\ln(1.06)} = 7.58$

Present value of the payments after 10 years is

$$(1.06)^{-10} \int_0^{\infty} (1.03)^s (1.06)^{-s} ds = \frac{0.5584}{\ln(1.06) - \ln(1.03)} = 19.45$$

Total present value = 27.03

86. Solution: C

$$\left[ 10,000(1.06)^5 + X(1.06)^2 \right] e^{\int_5^{10} \frac{1}{t+1} dt} = 75,000$$

$$(13,382.26 + 1.1236X) \frac{11}{6} = 75,000$$

$$1.1236X = 27,526.83$$

$$X = 24,498.78$$

87. Solution: D

The effective annual interest rate is  $i = (1-d)^{-1} - 1 = (1-0.055)^{-1} - 1 = 5.82\%$

The balance on the loan at time 2 is  $15,000,000(1.0582)^2 = 16,796,809$ .

The number of payments is given by  $1,200,000a_{\overline{n}|} = 16,796,809$  which gives  $n = 29.795 \Rightarrow 29$  payments of 1,200,000. The final equation of value is

$$1,200,000a_{\overline{29}|} + X(1.0582)^{-30} = 16,796,809$$

$$X = (16,796,809 - 16,621,012)(5.45799) = 959,490.$$

88. Solution: C

$$1 - v^2 = 0.525(1 - v^4) \Rightarrow 1 = 0.525(1 + v^2) \Rightarrow v^2 = 0.90476 \Rightarrow v = 0.95119$$

$$1 - v^2 = 0.1427(1 - v^n) \Rightarrow 1 - v^n = (1 - 0.90476) / 0.1427 = 0.667414 \Rightarrow v^n = 0.332596$$

$$n = \ln(0.332596) / \ln(0.95119) = 22$$

89. Solution: C

The monthly payment is  $200,000 / a_{\overline{360}|0.005} = 1199.10$ . Using the equivalent annual effective rate of 6.17%, the present value (at time 0) of the five extra payments is 41,929.54 which reduces the original loan amount to  $200,000 - 41,929.54 = 158,070.46$ . The number of months required is the solution to  $158,070.46 = 1199.10 a_{\overline{n}|0.005}$ . Using calculator,  $n = 215.78$  months are needed to pay off this amount. So there are 215 full payments plus one fractional payment at the end of the 216th month, which is December 31, 2020.

90. Solution: D

The annual effective interest rate is  $0.08 / (1 - 0.08) = 0.08696$ . The level payments are  $500,000 / a_{\overline{5}|0.08696} = 500,000 / 3.9205 = 127,535$ . This rounds up to 128,000. The equation of value for  $X$  is

$$128,000 a_{\overline{4}|0.08696} + X(1.08696)^{-5} = 500,000$$

$$X = (500,000 - 417,466.36)(1.51729) = 125,227.$$

91. Solution: B

The accumulated value is the reciprocal of the price. The equation is  $X[(1/0.94) + (1/0.95) + (1/0.96) + (1/0.97) + (1/0.98) + (1/0.99)] = 100,000$ .

$$X = 16,078$$

92. Solution: D

Let  $P$  be the annual payment. The fifth line is obtained by solving a quadratic equation.

$$P(1 - v^{10}) = 3600$$

$$Pv^{10-6+1} = 4871$$

$$\frac{1 - v^{10}}{v^5} = \frac{3600}{4871}$$

$$1 - v^{10} = 0.739068v^5$$

$$v^5 = 0.69656$$

$$v^{10} = 0.485195$$

$$i = 0.485195^{-1/10} - 1 = 0.075$$

$$X = P \frac{1 - v^{10}}{i} = \frac{3600}{0.075} = 48,000$$

93. Solution: A

Let  $j$  = periodic yield rate,  $r$  = periodic coupon rate,  $F$  = redemption (face) value,  $P$  = price,  $n$  = number of time periods, and  $v_j = \frac{1}{1+j}$ . In this problem,  $j = (1.0705)^{\frac{1}{2}} - 1 = 0.03465$ ,  $r = 0.035$ ,  $P = 10,000$ , and  $n = 50$ .

The present value equation for a bond is  $P = Fv_j^n + Fra_{\overline{n}|j}$ ; solving for the redemption value  $F$  yields

$$F = \frac{P}{v_j^n + ra_{\overline{n}|j}} = \frac{10,000}{(1.03465)^{-50} + 0.035a_{\overline{50}|0.03465}} = \frac{10,000}{0.18211 + 0.035(23.6044)} = 9,918.$$

94. Solution: B

Jeff's monthly cash flows are coupons of  $10,000(0.09)/12 = 75$  less loan payments of  $2000(0.08)/12 = 13.33$  for a net income of 61.67. At the end of the ten years (in addition to the 61.67) he receives 10,000 for the bond less a 2,000 loan repayment. The equation is

$$8000 = 61.67a_{\overline{120}|i^{(12)}/12} + 8000(1 + i^{(12)}/12)^{-120}$$

$$i^{(12)}/12 = 0.00770875$$

$$i = 1.00770875^{12} - 1 = 0.0965 = 9.65\%.$$

95. Solution: B

The present value equation for a par-valued annual coupon bond is  $P = Fv_i^n + Fra_{\overline{n}|i}$ ; solving for

the coupon rate  $r$  yields  $r = \frac{P - Fv_i^n}{Fa_{\overline{n}|i}} = \frac{P}{a_{\overline{n}|i}} \left( \frac{1}{F} \right) - \frac{v_i^n}{a_{\overline{n}|i}}$ .

All three bonds have the same values except for  $F$ . We can write  $r = x(1/F) + y$ . From the first two bonds:

$0.0528 = x/1000 + y$  and  $0.0440 = x/1100 + y$ . Then,

$0.0528 - 0.044 = x(1/1000 - 1/1100)$  for  $x = 96.8$  and  $y = 0.0528 - 96.8/1000 = -0.044$ . For the third bond,  $r = 96.8/1320 - 0.044 = 0.2933 = 2.93\%$ .

96. Solution: A

The effective semi-annual yield rate is  $1.04 = \left( 1 + \frac{i^{(2)}}{2} \right)^2 \Rightarrow \frac{i^{(2)}}{2} = 1.9804\%$ . Then,

$$582.53 = c(1.02)v + c(1.02v)^2 + \dots + c(1.02v)^{12} + 250v^{12}$$

$$= c \frac{1.02v - (1.02v)^{13}}{1 - 1.02v} + 250v^{12} = 12.015c + 197.579 \Rightarrow c = 32.04.$$

$$582.53 = c \frac{1.02v - (1.02v)^{13}}{1 - 1.02v} + 250v^{12} = 12.015c + 197.579 \Rightarrow c = 32.04$$

97. Solution: E

Book values are linked by  $BV3(1+i) - Fr = BV4$ . Thus  $1254.87(1.06) - Fr = 1277.38$ .

Therefore, the coupon is  $Fr = 52.7822$ . The prospective formula for the book value at time 3 is

$$1254.87 = 52.7822 \frac{1 - 1.06^{-(n-3)}}{0.06} + 1890(1.06)^{-(n-3)}$$

$$375.1667 = 1010.297(1.06)^{-(n-3)}$$

$$n - 3 = \frac{\ln(375.1667 / 1010.297)}{-\ln(1.06)} = 17.$$

Thus,  $n = 20$ . Note that the financial calculator can be used to solve for  $n - 3$ .

98. Solution: A

Book values are linked by  $BV_3(1+i) - Fr = BV_4$ . Thus  $BV_3(1.04) - 2500(0.035) = BV_3 + 8.44$ . Therefore,  $BV_3 = [2500(0.035) + 8.44]/0.04 = 2398.5$ . The prospective formula for the book value at time 3 is, where  $m$  is the number of six-month periods.

$$2398.5 = 2500(0.035) \frac{1 - 1.04^{-(m-3)}}{0.04} + 2500(1.04)^{-(m-3)}$$

$$211 = 312.5(1.04)^{-(m-3)}$$

$$m-3 = \frac{\ln(211/312.5)}{-\ln(1.04)} = 10.$$

Thus,  $m = 13$  and  $n = m/2 = 6.5$ . Note that the financial calculator can be used to solve for  $m - 3$ .

99. Solution: C

$$s_1 = {}_1f_0 = 0.04$$

$${}_1f_1 = 0.06 = \frac{(1+s_2)^2}{(1+s_1)} - 1 \Rightarrow s_2 = \sqrt{(1.06)(1.04)} - 1 = 0.04995$$

$${}_1f_2 = 0.08 = \frac{(1+s_3)^3}{(1+s_2)^2} - 1 \Rightarrow s_3 = [(1.08)(1.04995)^2]^{1/3} - 1 = 0.05987 = 6\%.$$

100. Solution: B

The Macaulay duration of Annuity A is  $0.93 = \frac{0(1) + 1(v) + 2(v^2)}{1 + v + v^2} = \frac{v + 2v^2}{1 + v + v^2}$ , which leads to the quadratic equation  $1.07v^2 + 0.07v - 0.93 = 0$ . The unique positive solution is  $v = 0.9$ .

The Macaulay duration of Annuity B is  $\frac{0(1) + 1(v) + 2(v^2) + 3(v^3)}{1 + v + v^2 + v^3} = 1.369$ .

101. Solution: D

With  $v = 1/1.07$ ,

$$D = \frac{2(40,000)v^2 + 3(25,000)v^3 + 4(100,000)v^4}{40,000v^2 + 25,000v^3 + 100,000v^4} = 3.314.$$

102. Solution: C

$$30 = MacD = \frac{\sum_{n=0}^{\infty} nv^n}{\sum_{n=0}^{\infty} v^n} = \frac{Ia_{\infty|}]{d}}{\ddot{a}_{\infty|}} = \frac{1/(di)}{1/d} = \frac{(1+i)/i^2}{(1+i)/i} = \frac{1}{i} \text{ and so } i = 1/30.$$

$$ModD = \frac{MacD}{1+i} = \frac{30}{1 + \frac{1}{30}} = 29.032.$$

Then,

103. Solution: B

I) False. The yield curve structure is not relevant.

II) True.

III) False. Matching the present values is not sufficient when interest rates change.

104. Solution: A

The present value function and its derivatives are

$$P(i) = X + Y(1+i)^{-3} - 500(1+i)^{-1} - 1000(1+i)^{-4}$$

$$P'(i) = -3Y(1+i)^{-4} + 500(1+i)^{-2} + 4000(1+i)^{-5}$$

$$P''(i) = 12Y(1+i)^{-5} - 1000(1+i)^{-3} - 20,000(1+i)^{-6}.$$

The equations to solve for matching present values and duration (at  $i = 0.10$ ) and their solution are

$$P(0.1) = X + 0.7513Y - 1137.56 = 0$$

$$P'(0.1) = -2.0490Y + 2896.91 = 0$$

$$Y = 2896.91 / 2.0490 = 1413.82$$

$$X = 1137.56 - 0.7513(1413.82) = 75.36.$$

The second derivative is

$$P''(0.1) = 12(1413.82)(1.1)^{-5} - 1000(1.1)^{-3} - 20,000(1.1)^{-6} = -1506.34.$$

Redington immunization requires a positive value for the second derivative, so the condition is not satisfied.

105. Solution: D

This solution uses time 8 as the valuation time. The two equations to solve are

$$P(i) = 300,000(1+i)^2 + X(1+i)^{8-y} - 1,000,000 = 0$$

$$P'(i) = 600,000(1+i) + (8-y)X(1+i)^{7-y} = 0.$$

Inserting the interest rate of 4% and solving:

$$300,000(1.04)^2 + X(1.04)^{8-y} - 1,000,000 = 0$$

$$600,000(1.04) + (8-y)X(1.04)^{7-y} = 0$$

$$X(1.04)^{-y} = [1,000,000 - 300,000(1.04)^2] / 1.04^8 = 493,595.85$$

$$624,000 + (8-y)(1.04)^7(493,595.85) = 0$$

$$y = 8 + 624,000 / [493,595.85(1.04)^7] = 8.9607$$

$$X = 493,595.85(1.04)^{8.9607} = 701,459.$$

106. Solution: A

This solution uses Macaulay duration and convexity. The same conclusion would result had modified duration and convexity been used.

The liabilities have present value  $573/1.07^2 + 701/1.07^5 = 1000$ . Only portfolios A, B, and E have a present value of 1000.

The duration of the liabilities is  $[2(573)/1.07^2 + 5(701)/1.07^5] / 1000 = 3.5$ . The duration of a zero coupon bond is its term. The portfolio duration is the weighted average of the terms. For portfolio A the duration is  $[500(1) + 500(6)] / 1000 = 3.5$ . For portfolio B it is  $[572(1) + 428(6)] / 1000 = 3.14$ . For portfolio E it is 3.5. This eliminates portfolio B.

The convexity of the liabilities is  $[4(573)/1.07^2 + 25(701)/1.07^5] / 1000 = 14.5$ . The convexity of a zero-coupon bond is the square of its term. For portfolio A the convexity is  $[500(1) + 500(36)] / 1000 = 18.5$  which is greater than the convexity of the liabilities. Hence portfolio A provides Redington immunization. As a check, the convexity of portfolio E is 12.25, which is less than the liability convexity.

107. Solution: D

The present value of the liabilities is 1000, so that requirement is met. The duration of the liabilities is  $402.11[1.1^{-1} + 2(1.1)^{-2} + 3(1.1)^{-3}] / 1000 = 1.9365$ . Let  $X$  be the investment in the one-year bond. The duration of a zero-coupon is its term. The duration of the two bonds is then  $[X + (1000 - X)(3)] / 1000 = 3 - 0.002X$ . Setting this equal to 1.9365 and solving yields  $X = 531.75$ .

108. Solution: A

Let  $x$ ,  $y$ , and  $z$  represent the amounts invested in the 5-year, 15-year, and 20-year zero-coupon bonds, respectively. Note that in this problem, one of these three variables is 0.

The present value, Macaulay duration, and Macaulay convexity of the assets are, respectively,

$$x + y + z, \frac{5x + 15y + 20z}{x + y + z}, \frac{5^2x + 15^2y + 20^2z}{x + y + z}$$

We are given that the present value, Macaulay duration, and Macaulay convexity of the liabilities are, respectively, 9697, 15.24, and 242.47.

Since present values and Macaulay durations need to match for the assets and liabilities, we have the two equations

$$x + y + z = 9697, \quad \frac{5x + 15y + 20z}{x + y + z} = 15.24$$

Note that 5 and 15 are both less than the desired Macaulay duration 15.24, so  $z$  cannot be zero. So try either the 5-year and 20-year bonds (i.e.  $y = 0$ ), or the 15-year and 20-year bonds (i.e.  $x = 0$ ).

In the former case, substituting  $y = 0$  and solving for  $x$  and  $z$  yields

$$x = \frac{(20 - 15.24)9697}{20 - 5} = 3077.18 \quad \text{and} \quad z = \frac{(15.24 - 5)9697}{20 - 5} = 6619.82.$$

We need to check if the Macaulay convexity of the assets exceeds that of the liabilities.

The Macaulay convexity of the assets is  $\frac{5^2(3077.18) + 20^2(6619.82)}{9697} = 281.00$ , which exceeds

the Macaulay convexity of the liabilities, 242.47. The company should invest 3077 for the 5-year bond and 6620 for the 20-year bond.

Note that setting  $x = 0$  produces  $y = 9231.54$  and  $z = 465.46$  and the convexity is 233.40, which is less than that of the liabilities.

109. Solution: E

The correct answer is the lowest cost portfolio that provides for \$11,000 at the end of year one and provides for \$12,100 at the end of year two. Let  $H$ ,  $I$ , and  $J$  represent the face amount of each purchased bond. The time one payment can be exactly matched with  $H + 0.12J = 11,000$ . The time two payment can be matched with  $I + 1.12J = 12,100$ . The cost of the three bonds is  $H/1.1 + I/1.2321 + J$ . This function is to be minimized under the two constraints. Substituting for  $H$  and  $I$  gives  $(11,000 - 0.12J)/1.1 + (12,100 - 1.12J)/1.2321 + J = 19,820 - 0.0181J$ . This is minimized by purchasing the largest possible amount of  $J$ . This is  $12,100/1.12 = 10,803.57$ . Then,  $H = 11,000 - 0.12(10,803.57) = 9703.57$ . The cost of Bond H is  $9703.57/1.1 = 8,821.43$ .



110. Solution: C

The strategy is to use the two highest yielding assets: the one-year bond and the two-year zero-coupon bond. The cost of these bonds is  $25,000/1.0675 + 20,000/1.05^2 = 41,560$ .

111. Solution: E

Let  $P$  be the annual interest paid. The present value of John's payments is  $Pa_{\overline{X}|0.05}$ . The present value of Karen's payments is  $P(1.05)^{-X} a_{\infty|0.05} = P(1.05)^{-X} / 0.05$ . Then,

$$P(1.05)^{-X} / 0.05 = 1.59Pa_{\overline{X}|0.05}$$

$$\frac{1.05^{-X}}{0.05} = 1.59 \frac{1 - 1.05^{-X}}{0.05}$$

$$1.59 = 2.59(1.05)^{-X}$$

$$\ln 1.59 = \ln 2.59 - X \ln 1.05$$

$$X = 10.$$

112. Solution: A

Cheryl's force of interest at all times is  $\ln(1.07) = 0.06766$ . Gomer's accumulation function is from time 3 is  $1 + yt$  and the force of interest is  $y/(1 + yt)$ . To be equal at time 2, the equation is  $0.06766 = y/(1 + 2y)$ , which implies  $0.06766 + 0.13532y = y$  for  $y = 0.07825$ . Gomer's account value is  $1000(1 + 2 \times 0.07825) = 1156.5$ .

113. Solution: D

One way to view these payments is as a sequence of level immediate perpetuities of 1 that are deferred  $n-1, n, n+1, \dots$  years. The present value is then

$$v^{n-1}/i + v^n/i + v^{n+1}/i + \dots = (v^{n-2}/i)(v + v^2 + v^3 + \dots) = v^{n-2}/i^2.$$

Noting that only answers C, D, and E have this form and all have the same numerator,

$$v^{n-2}/i^2 = v^n/(vi)^2 = v^n/d^2.$$

114. Solution: B

The monthly interest rate is  $j = (1.08)^{1/12} - 1 = 0.643\%$ . Then,

$$20,000s_{\overline{4}|0.08} = X\ddot{s}_{\overline{252}|0.00643}, \quad 90,122.24 = 630.99X, \quad X = 142.83.$$

115. Solution: D

$\bar{a}_{\overline{20}|} = 1.5\bar{a}_{\overline{10}|}$ ,  $\frac{1 - e^{-20\delta}}{\delta} = 1.5 \frac{1 - e^{-10\delta}}{\delta}$ ,  $e^{-20\delta} - 1.5e^{-10\delta} + 0.5 = 0$ . Let  $X = e^{-10\delta}$ . We then have the quadratic equation  $X^2 - 1.5X + 0.5 = 0$  with solution  $X = 0.5$  for  $\delta = \ln 0.5 / (-10) = 0.069315$ .

Then, the accumulated value of a 7-year continuous annuity of 1 is  $\bar{s}_{\overline{7}|} = \frac{e^{7(0.069315)} - 1}{0.069315} = 9.01$ .

116. Solution: B

The present value is

$$v^3 + v^{10} + v^{17} + \dots + v^{-4+7n}$$

$$= \frac{v^3 - v^{3+7n}}{1 - v^7} = \frac{(1 - v^{3+7n}) - (1 - v^3)}{1 - v^7} = \frac{a_{\overline{3+7n}|} - a_{\overline{3}|}}{a_{\overline{7}|}}.$$

117. Solution: C

From the first annuity,  $X = 21.8s_{\overline{n}|0.109} = 21.8 \cdot \frac{1.109^n - 1}{0.109} = 200[1.109^n - 1]$ .

From the second annuity,  $X = 19,208(v^n + v^{2n} + \dots) = 19,208 \frac{v^n}{1 - v^n} = 19,208 \frac{1}{1.109^n - 1}$ .

Hence,

$$200[1.109^n - 1] = 19,208 \frac{1}{1.109^n - 1}$$

$$[1.109^n - 1]^2 = 19,208 / 200 = 96.04$$

$$1.109^n - 1 = 9.8$$

$$X = 200(9.8) = 1960.$$

118. Solution: C

$$2(Ia)_{\overline{60}|1\%} = 2 \frac{\ddot{a}_{\overline{60}|} - 60v^{60}}{0.01} = 2 \frac{45.4 - 33.03}{0.01} = 2,474.60.$$

119. Solution: E

Let  $j$  be the semi-annual interest rate. Then,

$$475,000 = 300 + 300a_{\infty|j} + (1+j)^{-1}200(Ia)_{\infty|j} = 300 + 300/j + 200/j^2$$

$$474,700j^2 - 300j - 200 = 0$$

$$j = \frac{300 + \sqrt{300^2 - 4(474,700)(-200)}}{2(474,700)} = 0.02084$$

$$i = (1+j)^2 - 1 = 0.04212 = 4.21\%.$$

120. Solution: B

The present value is

$$4a_{\infty|0.06} + 2(Ia)_{\infty|0.06} = 4/0.06 + 2(1.06)/0.06^2 = 655.56.$$

121. Solution: A

The present value of the income is  $100a_{\infty|0.1025} = 100/0.1025 = 975.61$ . The present value of the investment is

$$X \left[ 1 + 1.05/1.1025 + (1.05/1.1025)^2 + (1.05/1.1025)^3 + (1.05/1.1025)^4 + (1.05/1.1025)^5 \right]$$

$$= X [1 + 1.05^{-1} + 1.05^{-2} + 1.05^{-3} + 1.05^{-4} + 1.05^{-5}] = X \frac{1 - 1.05^{-6}}{1 - 1.05^{-1}} = 5.3295X.$$

Then  $975.61 = 5.3295X$  for  $X = 183.06$ .

122. Solution: A

The present value of the ten level payments is  $X\ddot{a}_{10|0.05} = 8.10782X$ . The present value of the remaining payments is

$$X(v^{10}1.015 + v^{11}1.015^2 + \dots) = X \frac{v^{10}1.015}{1 - v1.015} = X \frac{1.015/1.05^{10}}{1 - 1.015/1.05} = 18.69366X.$$

Then,  $45,000 = 8.10782X + 18.69366X = 26.80148X$  for  $X = 1679$ .

123. Solution: D

The equation of value is

$$10,000 = X(v + v^2 0.996 + v^3 0.996^2 + \dots) = X \frac{v}{1 - v 0.996} = X \frac{e^{-0.06}}{1 - e^{-0.06} 0.996} = 15.189X. \text{ The}$$

solution is  $X = 10,000/15.189 = 658.37$ .

124. Solution: D

Discounting at 10%, the net present values are 4.59, -2.36, and -9.54 for Projects A, B, and C respectively. Hence, only Project A should be funded. Note that Project C's net present value need not be calculated. Its cash flows are the same as Project B except being 50 less at time 2 and 50 more at time 4. This indicates Project C must have a lower net present value and therefore be negative.

125. Solution: D

The loan balance after 10 years is still 100,000. For the next 10 payments, the interest paid is 10% of the outstanding balance and therefore the principal repaid is 5% of the outstanding balance. After 10 years the outstanding balance is  $100,000(0.95)^{10} = 59,874$ . Then,  
 $X = 59,874 / a_{\overline{10}|0.1} = 59,874 / 6.14457 = 9,744$ .

126. Solution: B

First determine number of regular payments:

$4000 = 600v^4 a_{\overline{n}|0.06}$ ,  $a_{\overline{n}|0.06} = (4000 / 600)1.06^4 = 8.4165$ . Using the calculator,  $n = 12.07$  and thus there are 11 regular payments. The equation for the balloon payment,  $X$ , is:

$$4000 = 600v^4 a_{\overline{11}|0.06} + Xv^{16} = 3748.29 + 0.39365X, X = 639.43.$$

127. Solution: C

$$20,000 = X \left( a_{\overline{5}|0.11} + 1.11^{-5} a_{\overline{5}|0.12} \right) = X (3.69590 + 3.60478 / 1.68506) = 5.83516 X$$

$$X = 20,000 / 5.83516 = 3427.50.$$

128. Solution: A

The principal repaid in the first payment is  $100 - iL$ . The outstanding principal is  $L - 100 + iL = L + 25$ . Hence,  $iL = 125$ . Also,

$$L = 300a_{\overline{16}|} - 200a_{\overline{8}|} = \frac{300(1-v^{16}) - 200(1-v^8)}{i}$$

$$125 = iL = 100 + 200v^8 - 300v^{16}$$

$$300v^{16} - 200v^8 + 25 = 0$$

$$v^8 = \frac{200 \pm \sqrt{200^2 - 4(300)(25)}}{600} = \frac{200 \pm 100}{600} = 0.5.$$

The larger of the two values is used due to the value being known to exceed 0.3. The outstanding balance at time eight is the present value of the remaining payments:

$$300a_{\overline{8}|} = 300 \frac{1-0.5}{2^{1/8} - 1} = 1657.$$

129. Solution: E

Let  $j$  be the monthly rate and  $X$  be the level monthly payment. The principal repaid in the first payment is  $1400 = X - 60,000j$ . The principal repaid in the second payment is  $1414 = X - (60,000 - 1400)j$ . Substituting  $X = 1400 + 60,000j$  from the first equation gives  $1414 = 1400 + 60,000j - 58,600j$  or  $14 = 1400j$  and thus  $j = 0.01$  and  $X = 2000$ . Let  $n$  be the number of payments. Then  $60,000 = 2000a_{\overline{n}|0.01}$  and the calculator (or algebra) gives  $n = 35.8455$ . The

equation for the drop payment,  $P$ , is  $60,000 = 2000a_{\overline{35}|0.01} + Pv^{36} = 58,817.16 + 0.698925P$  for  $P = 1692$ .

130. Solution: C

The accumulated value is

$$1000 \left( s_{\overline{24}|0.06/12} (1 + 0.08/12)^{24} + s_{\overline{24}|0.08/12} \right) = 1000(25.4320(1.1729) + 25.9332) = 55,762.$$

131. Solution: C

Each month the principal paid increases by  $1.1^{1/12}$ . Thus, the amount of principal paid increases to  $500(1.1^{1/12})^{30-6} = 500(1.1)^2 = 605$ .

132. Solution: C

$$\text{Int}_{11} = i \cdot \left[ 900 \cdot a_{\overline{20}|i} + 300a_{\overline{10}|i} \right] = 900(1 - v^{20}) + 300(1 - v^{10}) = 1200 - 300v^{10} - 900v^{20}$$

$$\text{Int}_{21} = i \left[ 900 \cdot a_{\overline{10}|i} \right] = 900(1 - v^{10})$$

$$\text{Int}_{11} = 2\text{Int}_{21} \Rightarrow 1200 - 300v^{10} - 900v^{20} = 1800 - 1800v^{10}$$

$$\Rightarrow 9v^{20} - 15v^{10} + 6 = 0 \Rightarrow v^{10} = 2/3$$

$$\text{Int}_{21} = 900(1 - v^{10}) = 300$$

133. Solution: C

The original monthly payment is  $85,000 / a_{\overline{240}|0.005} = 85,000 / 139.5808 = 608.97$ . On July 1, 2009 there has been 4 years of payments, hence  $16 \times 12 = 192$  remaining payments. The outstanding balance is  $608.97 a_{\overline{192}|0.005} = 608.97(123.2380) = 75,048.24$ . The number of remaining payments after refinancing is determined as

$$75,048.24 = 500 a_{\overline{n}|0.0045} = 500 \frac{1 - 1.0045^{-n}}{0.0045}$$

$$0.67543 = 1 - 1.0045^{-n}$$

$$n = -\ln(0.32457) / \ln(1.0045) = 250.62.$$

Thus the final payment will be 251 months from June 30, 2009. This is 20 years and 11 months and so the final payment is May 31, 2030.

134. Solution: B

Just prior to the extra payment at time 5, the outstanding balance is

$1300 a_{\overline{20}|0.07} = 1300(10.5940) = 13,772.20$ . After the extra payment it is 11,172.20. Paying this off in 15 years requires annual payments of  $11,172.20 / a_{\overline{15}|0.07} = 11,172.20 / 9.1079 = 1226.65$ .

135. Solution: C

During the first redemption period the modified coupon rate is  $1000(0.035)/1250 = 2.80\%$  which is larger than the desired yield rate. If redeemed during this period, bond sells at a premium and so the worst case for the buyer is the earliest redemption. The price if called at that time is

$$35a_{\overline{20}|0.025} + 1250(1.025)^{-20} = 35(15.5892) + 762.84 = 1308.46.$$

During the second redemption period the modified coupon rate is  $1000(0.035)/1125 = 3.11\%$  which is also larger than the desired yield rate and the worst case for the buyer is again the earliest redemption. The price if called at that time is

$$35a_{\overline{40}|0.025} + 1125(1.025)^{-40} = 35(25.1028) + 418.98 = 1297.58.$$

Finally, if the bond is not called, its value is

$$35a_{\overline{60}|0.025} + 1000(1.025)^{-60} = 35(30.9087) + 227.28 = 1309.08.$$

The appropriate price is the lowest of these three, which relates to the bond being called after the 40th coupon is paid.

136. Solution: B

Because the yield is less than the coupon rate, the bond sells at a premium and the worst case for the buyer is an early call. Hence the price should be calculated based on the bond being called at time 16. The price is  $100a_{\overline{16}|0.05} + 1000(1.05)^{-16} = 100(10.0378) + 458.11 = 1542$ . (When working with callable bonds, the maximum a buyer will pay is the smallest price over the various call dates. Paying more may not earn the desired yield.)

137. Solution: A

All calculations are in millions. For the ten-year bond, at time ten it is redeemed for

$$2(1.08)^{10} = 4.31785.$$

After being reinvested at 12% it matures at time twenty for

$$4.31785(1.12)^{10} = 13.4106.$$

The thirty-year bond has a redemption value of  $4(1.08)^{30} = 40.2506$ .

For the buyer to earn 10%, it is sold for  $40.2506(1.1)^{-10} = 15.5184$ . The gain is  $13.4106 +$

$$15.5184 - 6 = 22.9290.$$

138. Solution: A

The book value after the third coupon is

$$7500(0.037)a_{\overline{37}|0.0265} + C(1.0265)^{-37} = 6493.05 + 0.379943C$$

$$\text{and after the fourth coupon it is } 7500(0.037)a_{\overline{36}|0.0265} + C(1.0265)^{-36} = 6387.61 + 0.390012C.$$

$$\text{Then, } 6493.05 + 0.379943C - (6387.61 + 0.390012C) = 28.31$$

$$105.44 - 0.010069C = 28.31$$

$$C = 7660.15.$$

139. Solution: C

The semiannual yield rate is  $1.1^{1/2} - 1 = 0.0488$ . Assuming the bond is called for 2900 after four years, the purchase price is  $150a_{\overline{8}|0.0488} + 2900(1.0488)^{-8} = 150(6.4947) + 1980.87 = 2955.08$ . With a call after the first coupon, the equation to solve for the semi-annual yield rate ( $j$ ) and then the annual effective rate ( $i$ ) is

$$2955.08 = (150 + 2960) / (1 + j)$$

$$1 + j = 1.05242$$

$$i = 1.05242^2 - 1 = 0.10759.$$

140. Solution: C

The book value after the sixth coupon is  $1000(r/2)a_{\overline{34}|0.036} + 1000(1.036)^{-34} = 9716.01r + 300.45$ .

After the seventh coupon it is  $1000(r/2)a_{\overline{33}|0.036} + 1000(1.036)^{-33} = 9565.79r + 311.26$ . Then,

$$4.36 = 9565.79r + 311.26 - (9716.01r + 300.45) = 10.81 - 150.22r$$

$$r = (10.81 - 4.36) / 150.22 = 0.0429.$$

141. Solution: B

The two equations are:

$$P = (10,000r)a_{\overline{5}|0.04} + 9,000(1.04)^{-5} = 44,518.22r + 7,397.34$$

$$1.2P = [10,000(r + 0.01)]a_{\overline{5}|0.04} + 11,000(1.04)^{-5} = 44,518.22r + 9,486.38.$$

Subtracting the first equation from the second gives  $0.2P = 2089.04$  for  $P = 10,445.20$ . Inserting this in the first equation gives  $r = (10,445.20 - 7,397.34) / 44,518.22 = 0.0685$ .

142. Solution: C

When the yield is  $6.8\% < 8\%$ , the bond is sold at a premium and hence an early call is most disadvantageous. Therefore,  $P = 40a_{\overline{10}|0.034} + 1000(1.034)^{-10} = 1050.15$ . When the yield is  $8.8\% > 8\%$ , the bond is sold at discount. Hence,  $Q < 1000 < P$ , and thus  $Q = 1050.15 - 123.36 = 926.79$ . Also, because the bond is sold at a discount, the latest call is the most disadvantageous. Thus,

$$926.79 = 40a_{\overline{2n}|0.044} + 1000(1.044)^{-2n} = \frac{40}{0.044} + (1.044)^{-2n} \left( 1000 - \frac{40}{.044} \right) = 909.09 + 90.90(1.044)^{-2n}$$

$$17.70 = 90.90(1.044)^{-2n}$$

$$2n = -\ln(17.70 / 90.90) / \ln(1.044) = 38$$

$$n = 19.$$



143. Solution: B

The fund will have  $500(1.05)^4 - 100s_{\overline{4}|0.05} = 176.74$  after four years. After returning 75% to the insured, the insurer receives  $0.25(176.74) = 44.19$ . So the insurer's cash flows are to pay 100 at time 0, receive 125 at time 2, and receive 44.19 at time four. The equation of value and the solution are:

$$100(1+i)^4 - 125(1+i)^2 - 44.19 = 0$$

$$(1+i)^2 = \frac{125 \pm \sqrt{(-125)^2 - 4(100)(-44.19)}}{200} = 1.5374$$

$$1+i = 1.2399$$

$$i = 24\%.$$

144. Solution: B

The Macaulay duration of the perpetuity is  $\frac{\sum_{n=1}^{\infty} nv^n}{\sum_{n=1}^{\infty} v^n} = \frac{(Ia)_{\infty|}}{a_{\infty|}} = \frac{(1+i)/i^2}{1/i} = \frac{1+i}{i} = 1 + 1/i = 17.6$ .

This implies that  $i = 1/16.6$ . With  $i = 2i = 2/16.6$ , the duration is  $1 + 16.6/2 = 9.3$ .

145. Solution: A

Because the interest rate is greater than zero, the Macaulay duration of each bond is greater than its modified duration. Therefore, the bond with a Macaulay duration of  $c$  must be the bond with a modified duration of  $a$  and  $a = c/(1+i)$  which implies  $1+i = c/a$ . The Macaulay duration of the other bond is  $b(1+i) = bc/a$ .

146. Solution: B

$P(0.1025) \approx P(0.10) \left( \frac{1.10}{1.1025} \right)^{11} = 0.97534P(0.10)$ . Therefore, the approximate percentage price change is  $100(0.97534 - 1) = -2.47\%$ .

147. Solution: B

Cash-flow matching limits the number of investment choices available to the portfolio manager to a subset of the choices available for immunization.

148. Solution: C

Options for full immunization are:

2J (cost is 3000), K+2L (cost is 2500), and M (cost is 4000). The lowest possible cost is 2500. Another way to view this is that the prices divided by total cash flows are 0.6, 0.5, 0.5, and 0.8. The cheapest option will be to use K and L, if possible.

149. Solution: B

The present value of the assets is  $15,000 + 45,000 = 60,000$  which is also the present value of the liability. The modified duration of the assets is the weighted average, or  $0.25(1.80) + 0.75D_{\text{mod}}$ . The modified duration of the liability is  $3/1.1$  and so  $D_{\text{mod}} = (3/1.1 - 0.45)/0.75 = 3.04$ .

150. Solution: C

Let  $A$  be the redemption value of the zero-coupon bonds purchased and  $B$  the number of two-year bonds purchased. The total present value is:

$$1783.76 = A/1.05 + B(100/1.06 + 1100/1.06^2) = 0.95238A + 1073.3357B.$$

To exactly match the cash flow at time one,  $A + 100B = 1000$ . Substituting  $B = 10 - 0.01A$  in the first equation gives  $1783.76 = 0.95238A + 10733.357 - 10.733357A$  for  $A = 8949.597/9.780977 = 915$ . The amount invested is then  $915/1.05 = 871$ .

151. Solution: B

The company must purchase 4000 in one-year bonds and 6000 in two-year bonds. The total purchase price is  $4000/1.08 + 6000/1.11^2 = 8573$ .

152. Solution: C

The modified duration is  $11/1.10 = 10$ . Then,

$P(0.1025) \approx P(0.10)[1 - (0.1025 - 0.10)10] = 0.975P(0.10)$ . Therefore, the approximate percentage price change is  $100(0.975 - 1) = -2.50\%$ .

153. Solution: B

$$P(0.08) \approx 1000 \left( \frac{1.072}{1.08} \right)^{7.959} = 942.54.$$

154. Solution: E

Modified duration = (Macaulay duration)/(1 + i) and so Macaulay duration = 8(1.064) = 8.512.

$$E_{MAC} = 112,955 \left( \frac{1.064}{1.07} \right)^{8.512} = 107,676 \text{ and } E_{MOD} = 112,955 [1 - (0.07 - 0.064)(8)] = 107,533.$$

Then,  $E_{MAC} - E_{MOD} = 107,676 - 107,533 = 143$ .

155. Solution: C

The Macaulay duration of the portfolio is  $\frac{35,000(7.28) + 65,000(12.74)}{35,000 + 65,000} = 10.829$ . Then,

$$105,000 = 100,000 \left( \frac{1.0432}{1+i} \right)^{10.829} \Rightarrow \frac{1.0432}{1+i} = \left( \frac{105,000}{100,000} \right)^{1/10.829} = 1.004516 \Rightarrow i = 0.0385.$$

156. Solution: A

$$121,212 = 123,000 \left( \frac{1.05}{1.054} \right)^{D_{MAC}} \Rightarrow D_{MAC} = \frac{\ln(121,212/123,000)}{\ln(1.05/1.054)} = 3.8512. \quad \text{Then,}$$

$$D_{MOD} = 3.8512 / 1.05 = 3.67.$$

157. Solution: A

I provides cash flows that exactly matches the liabilities. II only has  $PV(A) = PV(B)$ , which is not sufficient for exact matching. III describes Reddington immunization, not exact matching.

158. Solution: D

Let  $F$  be the face amount of Bond X. Then,

$$2695.39 = 200a_{\overline{15}|} + Fv^{15} \text{ and } 3490.78 = 200a_{\overline{15}|} + Fv^{15}.$$

Subtract the first equation from the second to obtain  $795.39 = Fv^{15}$ .

Then for bond X,  $2695.39 = 200a_{\overline{15}|} + 795.39 \Rightarrow a_{\overline{15}|} = (2695.39 - 795.39) / 200 = 9.5$ . This

implies  $i = 0.0634$ . Then  $9.5 = (1 - v^{15}) / 0.0634 \Rightarrow v^{15} = 1 - 0.0634(9.5) = 0.3977$  and

$F = 795.39 / 0.3977 = 2000$ . The coupon rate is  $200/2000 = 10.0\%$ .

159. Solution: D

The value of 1 invested with bank P after three years is  $1.04^3 + 0.02 = 1.144864$ . The yield from Bank Q satisfies  $1.144864 = (1+i)^3 \Rightarrow i = 1.144864^{1/3} - 1 = 0.04613 = 4.6\%$ .

160. Solution: D

With a continuously compounded annual interest rate of 6%,  $v = e^{-0.06}$ . The value of the first annuity is

$$600,000 = X(I\ddot{a})_{\overline{20}|} = X \frac{\ddot{a}_{\overline{20}|} - 20v^{20}}{d} = X \frac{\frac{1 - e^{-1.2}}{1 - e^{-0.06}} - 20e^{-1.2}}{1 - e^{-0.06}} = 102.614X. \text{ Hence,}$$

$X = 600,000/102.614 = 5847.155$ . Then the value of the second annuity is

$$5847.155 \frac{\ddot{a}_{\overline{25}|} - 25v^{25}}{d} = 5847.155 \frac{\frac{1 - e^{-1.5}}{1 - e^{-0.06}} - 25e^{-1.5}}{1 - e^{-0.06}} = 779,366.$$

161. Solution: B

The amount of principal repaid at payment 15 is (where  $R$  is the quarterly payment)

$$10,030.27 = Rv^{40-15+1} = R(1.03)^{-26} \Rightarrow R = 10,030.27(1.03)^{26} = 21,631.19.$$

The amount of interest in payment 25 is

$$21,631.19(1 - v^{40-25+1}) = 21,631.19(1 - 1.03^{-16}) = 8,151.35.$$

162. Solution: E

The present value of the payments (4000 at month 36 plus the payments of  $X$ ) must match the present value of the present value of the amounts borrowed (4000 at month 0 plus the payments of 800).

The quarterly interest rate is  $0.264/4$  and all payment times should be in quarters of a year. On that time scale, the 4000 at month 36 is at time 12. The payments of 4000 are at times  $1/6, 3/6, 5/6, \dots, 71/6$  and there are 36 such payments. One way to write the present value of these payments is

$$\frac{4000}{\left(1 + \frac{0.264}{4}\right)^{12}} + \sum_{n=1}^{36} \frac{X}{\left(1 + \frac{0.264}{4}\right)^{\frac{n-0.5}{3}}}.$$

The payments of 800 are at times 1, 3, 5, 7, 9, and 11, in quarters. One way to write the present value of these payments plus the initial debt of 4000 is

$$4000 + \sum_{n=1}^6 \frac{800}{\left(1 + \frac{0.264}{12}\right)^{2n-1}}.$$

These are the two sides of equation in answer choice E.

163. Solution: B

Let  $r$  be the coupon rate for Bond A. The coupon rate for Bond B is then  $r + 0.01$ . Then,

$$1600 = 1000 \left[ \frac{1}{(1.1)^{20}} + ra_{\overline{20}|0.1} + \frac{1}{(1.1)^{20}} + (r + 0.01)a_{\overline{20}|0.1} \right]$$

$$1.6 = \frac{2}{(1.1)^{20}} + 2ra_{\overline{20}|0.1} + 0.01a_{\overline{20}|0.1} = 0.29729 + 17.02713r + 0.08514$$

$$r = \frac{1.6 - 0.29729 - 0.08514}{17.02713} = 0.0715 = 7.15\%.$$

164. Solution: E

Let  $n$  be the number of payments and let  $j$  be the interest rate per half-year. Because the given values are  $n - 1$  half-years apart,  $7,968.89(1 + j)^{n-1} = 19,549.25$ . Also,

$$7,968.89 = 1,000\ddot{a}_{\overline{n}|} = 1,000(a_{\overline{n-1}|} + 1) = 1,000 \left( \frac{1 - v^{n-1}}{j} + 1 \right) = 1,000 \left( \frac{1 - 7,968.89/19,549.25}{j} + 1 \right)$$

Then,

$$j = \frac{1 - 7,968.89/19,549.25}{\frac{7,968.89}{1,000} - 1} = 0.085$$

$$\text{for } i = (1.085)^2 - 1 = 0.1772 = 17.7\%.$$

165. Solution: B

The denominator of the duration is the present value of the annuity:

$$X\ddot{a}_{\overline{20}|0.02} + 4Xv^{20}\ddot{a}_{\overline{30}|0.02} = 78.1729X.$$

The numerator is the time-weighted present value of the annuity. In units of  $X$  we need the present value of 0, 1, ..., 19, 80, 84, ..., 196. One way to view this is as four times a 49-year increasing immediate annuity (so payments of 4, 8, ..., 76, 80, 84, ..., 196) less three times a 19-year increasing immediate annuity (so payments of 3, 6, ..., 57). The present value is:

$$4X(Ia)_{\overline{49}|0.02} - 3X(Ia)_{\overline{19}|0.02} = [4(655.2078) - 3(147.4923)]X = 2,178.3542X.$$

The duration is the ratio,  $2,178.3542/78.1729 = 27.87$ .

166. Solution: A

$$100\left(1+\frac{i}{2}\right)^{22}\left[\left(1+\frac{i}{2}\right)^2-1\right]=2(100)\left(1+\frac{i}{2}\right)^8\left[\left(1+\frac{i}{2}\right)^2-1\right]$$

$$2=\left(1+\frac{i}{2}\right)^{14}$$

$$i=(2^{1/14}-1)2=0.1015=10.15\%.$$

167. Solution: D

Let  $C$  be the amount of the semiannual coupon for bond B.

$$X=40a_{\overline{10}|0.03}+1000(1.03)^{-10}=1085.30$$

$$X=1085.30=Ca_{\overline{10}|0.035}+1000(1.035)^{-10}=8.3166C+708.9188$$

$$C=(1085.30-708.9188)/8.3166=45.2566$$

$$y=\frac{45.2566 \times 2}{1000}=0.0905=9.05\%.$$

168. Solution: C

Let  $X$  be the original loan value. From the original loan terms,  $X=50a_{\overline{15}|}$ . Under the revised repayment plan,  $X=50a_{\overline{10}|}+30v^5a_{\overline{5}|}$ . Equating the two gives  $50a_{\overline{15}|}=50a_{\overline{10}|}+30v^5a_{\overline{5}|}$  which does not match answer A. All the other choices use  $s$ . Multiplying both sides by  $(1+i)^{10}$  gives  $50v^5s_{\overline{15}|}=50s_{\overline{10}|}+30s_{\overline{5}|}$ , which is answer C. This can also be obtained by equating the values of the two payment streams at time 10 rather than time 0.

169. Solution: A

The effective monthly rate is  $1.065^{1/12}-1=0.0052617$ . The accumulated value is

$$\begin{aligned} &1097s_{\overline{180}|0.0052617}+5(Is)_{\overline{180}|0.0052617} \\ &=1097(298.733)+5\frac{\ddot{s}_{\overline{180}|0.0052617}-180}{0.0052617} \\ &=327,710+5\frac{300.3049-180}{0.0052617}=442,031. \end{aligned}$$

170. Solution D

Fund K receives 1000 at the end of each year and also receives interest payments of 1300, 1235, 1170, ..., 65. The accumulated value is

$$\begin{aligned} & 1000s_{\overline{20}|0.0825} + 65(Ds)_{\overline{20}|0.0825} \\ &= 1000(47.0491) + 65 \frac{20(1.0825)^{20} - s_{\overline{20}|0.0825}}{0.0825} \\ &= 47,049.1 + 65 \frac{97.6311 - 47.0491}{0.0825} = 86,902. \end{aligned}$$

171. Solution: D

For Q the accumulated value is  $Xs_{\overline{25}|0.09} = 84.7009X$ . For R the accumulated value is

$$\begin{aligned} 100(25) + 9(Is)_{\overline{24}|0.08} &= 2500 + 9 \frac{\ddot{s}_{\overline{24}|0.08} - 24}{0.08} = 2500 + 9 \frac{72.1059 - 24}{0.08} = 7911.91. \text{ Then} \\ X &= 7911.91/84.7009 = 93.41. \end{aligned}$$

172. Solution: E

The accumulated values for Funds X and Y are  $1000\left(1 + \frac{k}{2}\right)^{10}$  and  $921.90\left(1 - \frac{k}{2}\right)^{-10}$

respectively. Equating them and solving for  $k$ :

$$\begin{aligned} 1000\left(1 + \frac{k}{2}\right)^{10} &= 921.90\left(1 - \frac{k}{2}\right)^{-10} \\ 0.9219 &= \left[\left(1 + \frac{k}{2}\right)\left(1 - \frac{k}{2}\right)\right]^{10} = \left(1 - \frac{k^2}{4}\right)^{10} \\ 1 - \frac{k^2}{4} &= 0.9919 \\ k^2 &= 0.0324 \\ k &= 0.18. \end{aligned}$$

$$\text{Then } P = 1000\left(1 + \frac{0.18}{2}\right)^{10} = 2367.36.$$

173. Solution: A

The 3-year interest rate is  $1.07^3 - 1 = 0.225043$ . Then,

$$735 = X \ddot{a}_{\infty|0.225043} = X \frac{1.225043}{0.225043} = 5.443595X \text{ and } X = 735/5.443595 = 135.02.$$

174. Solution: B

$$2600 = Pa_{\infty|0.06} + v^9(Ia)_{\infty|0.06}$$

$$= P \frac{1}{0.06} + \frac{1}{1.06} 9 \frac{1.06}{0.06^2}$$

$$= \frac{P}{0.06} + 2500$$

$$P = (2600 - 2500)(0.06) = 6.$$

175. Solution: B

$$475a_{\overline{10}|i} = 400(a_{\overline{5}|i} + v^{10}a_{\infty|i})$$

$$475 \frac{1-v^{10}}{i} = 400 \frac{1-v^5 + v^{10}}{i}$$

$$475(1-v^{10}) = 400(1-v^5 + v^{10})$$

$$875v^{10} - 400v^5 - 75 = 0$$

$$v^5 = \frac{400 \pm \sqrt{400^2 + 4(875)(75)}}{2(875)} = 0.6$$

$$i = (1/0.6)^{1/5} - 1 = 0.1076 = 10.76\%$$

176. Solution: E

Based on the effective yield rate,  $100 = X/1.1 + 2X/1.1^2 \Rightarrow X = 39.03$ . After one year, the outstanding loan balance is  $100 + 8 - 39.03 = 68.97$ . For the balance to be zero after two years,  $68.97(1+i) - 2(39.03) = 0 \Rightarrow i = 78.06/68.97 - 1 = 0.1318 = 13.2\%$ .

177. Solution: D

The amount borrowed is  $1000a_{\overline{5}|0.1} + 2000v^5a_{\overline{5}|0.1} = 8498.35$ . The outstanding balance after five years is  $2000a_{\overline{5}|0.1} = 7581.57$ . The principal repaid is  $8498.35 - 7581.57 = 916.78$ . The interest paid is  $5000 - 916.78 = 4083.22$ .



178. Solution: B

The accumulation is

$$10X + 0.12X(Is)_{\overline{10}|0.08} = 10X + 0.12X \frac{\ddot{s}_{\overline{10}|0.08} - 10}{0.08} = 10X + 0.12X \frac{15.6455 - 10}{0.08} = 18.4683X.$$

Then,  $X = 10,000/18.4683 = 541.47$

179. Solution: C

The equation of value is

$$10,000 = (X + 5)a_{\overline{30}|0.05} - 5(Ia)_{\overline{30}|0.05} = (X + 5)a_{\overline{30}|0.05} - 5 \frac{\ddot{a}_{\overline{30}|0.05} - 30v^{30}}{0.05}$$
$$= (X + 5)(15.37245) - 5 \frac{16.14107 - 6.94132}{0.05} = 15.37245X - 843.11275$$

$$X = (10,000 + 843.11275) / 15.37245 = 705.36.$$

180. Solution: A

$$NPV = -600 + \frac{150 - 100}{1.15} + \frac{150 - 50}{1.15^2} + \frac{150}{1.15^3} + \frac{150}{1.15^4} + \frac{150}{1.15^5} = -221.94$$

181. Solution: A

I is true.

II is false, the price sensitivity of assets and liabilities must be equal.

III is false, the convexity of assets should be greater than the convexity of liabilities.

182. Solution: D

The effective annual rate of interest is  $(1.005)^{12} - 1 = 0.06168$ . The present value of the tuition payments six months before the first payment is

$25,000(1.005)^{-6} \ddot{a}_{\overline{40}|0.06168} = 24,262.95(3.66473) = 88,917.16$ . The accumulated value of the

deposits at that time is  $1000s_{\overline{n}|0.005}$ . Equating the two amounts:

$$88,917.16 = 1000 \frac{1.005^n - 1}{0.005}$$

$$1.44459 = 1.005^n$$

$$n = \ln(1.44459) / \ln(1.005) = 73.75.$$

Therefore, at least 74 payments will be required.

183. Solution: E

Let  $x$  be the annual payment amount. Macaulay duration is

$$\frac{\frac{1x}{1.1} + \frac{2x}{1.1^2} + \frac{3x}{1.1^3} + \dots + \frac{7x}{1.1^7}}{\frac{x}{1.1} + \frac{x}{1.1^2} + \frac{x}{1.1^3} + \dots + \frac{x}{1.1^7}} = \frac{17.6315}{4.8684} = 3.62$$

Alternatively, the duration can be calculated as  $(Ia)_{\overline{7}|0.1} / a_{\overline{7}|0.1}$ .

184. Solution: E

PV of liabilities is  $402.11(1/1.1 + 1/1.1^2 + 1/1.1^3) = 1000$ . Duration of liabilities is

$402.11(1/1.1 + 2/1.1^2 + 3/1.1^3) / 1000 = 1.93653$ . Let  $X$  be the investment in one-year bonds. To match duration, since zero-coupon bonds have duration = maturity,  $1.93653 = [X + 3(1000 - X)] / 1000$ . Then,  $2X = 3000 - 1936.53 = 1063.47$  and  $X = 532$ .

185. Solution: E

A change in face value multiplies all cash flows by the same amount. Therefore, there is no change in the duration. If the coupon rate increases, the coupons become larger, but the redemption value stays the same. This causes payments prior to redemption to receive more weight relative to the payment at redemption and thus the duration will decrease.

186. Solution: A

We are given  $i^{(4)} = 8\%$  and want to determine  $i^{(12)} / 12$ . The equation that links the two and its solution is:

$$\left(1 + \frac{i^{(12)}}{12}\right)^{12} = \left(1 + \frac{i^{(4)}}{4}\right)^4 = \left(1 + \frac{8\%}{4}\right)^4$$

$$\left(1 + \frac{i^{(12)}}{12}\right) = \left(1 + \frac{8\%}{4}\right)^{4/12}$$

$$\frac{i^{(12)}}{12} = \left(1 + \frac{8\%}{4}\right)^{1/3} - 1.$$

187. Solution: E

Let  $m$  be the monthly payment and  $i$  be the monthly interest rate. The interest in the first payment is  $125,000i$  and the principal repaid is  $125,000 - 124,750 = 250$ . Thus  $m = 125,000i + 250$ .

Similarly, for the second payment,  $m = 124,750i + 252$ . Thus,  $250i = 2$  for  $i = 2/250 = 0.008$  and then  $m = 1250$ . To obtain the number of payments, the equation to solve is

$$125,000 = 1250a_{\overline{n}|0.008}$$

$$100 = \frac{1 - 1.008^{-n}}{0.008}$$

$$0.2 = 1.008^{-n}$$

$$n = -\ln(0.2) / \ln(1.008) = 202.$$

188. Solution: C

Let  $x$  and  $y$  be the amount invested in the five and twenty year bonds respectively. To match the present values:  $x + y = 500,000e^{-0.07(10)} + 500,000e^{-0.07(15)} = 423,262$ . To match the durations, noting that the denominators of the durations for assets and liabilities are the same,

$5x + 20y = 500,000(10)e^{-0.07(10)} + 500,000(15)e^{-0.07(15)} = 5,107,460$ . Subtracting five times the first equation from the second one gives  $15y = 2,991,150$  for  $y = 199,410$  and  $x = 423,262 - 199,410 = 223,852$ .

189. Solution: C

Let  $n$  be the term of the bond in half-years. We know that  $601 = 1080v^n$  and thus  $v^n = 601/1080$

. Then  $a_{\overline{n}|0.05} = \frac{1 - v^n}{0.05} = \frac{1 - 601/1080}{0.05} = 8.87037$ . The purchase price of the bond is

$$40a_{\overline{n}|0.05} + 1080v^n = 40(8.87037) + 610 = 956.$$

190. Solution: B

Principal repaid in the first payment is  $1000 - 10,000(0.04) = 600$ . Therefore, the principle repaid in the tenth payment is  $600(1.04)^9 = 854$  and the interest paid is  $1000 - 854 = 146$ .

191. Solution: C

After one year the outstanding balance is  $500a_{\overline{48}|0.025} = 13,886.58$ . This must match the present value of the revised payments:

$$13,886.58 = Xv^6 a_{\overline{6}|0.025} + 500v^{12} a_{\overline{36}|0.025} = 4.74964X + 8,757.69$$

$$X = (13,886.58 - 8,757.69) / 4.74964 = 1,079.85.$$

Alternatively, each missing payment is being replaced with a larger payment six months later. The larger payment should be the payment due plus the missed payment with interest, or  $500 + 500(1.025)^6 = 1,079.85$ .

192. Solution: A

Only Bond III can match the liability at time 3. The bond must mature for 1000. Only Bond II can match the liability at time 2. The face value and coupon must total 1000. If  $X$  is the face value, then  $X + 0.02X = 1000$  and thus  $X = 980.39$ . Only Answer A has these two values. To check, Bond II also provides a coupon of  $0.02(980.39) = 19.61$  at time 1. Therefore, Bond I must provide the remaining  $980.39$  from its coupon and redemption value. If  $Y$  is the face value, then  $Y + 0.01Y = 980.39$  for  $Y = 970.68$ .

193. Solution: A

Cash flows (in thousands) are 12, 12, 12, 12, and 162. The first bond provides payments of 10, 10, 10, 10, and 110. Therefore, the second bond must provide 2, 2, 2, 2, and 52. This implies a coupon rate of  $2/50 = 4\%$  and a face amount of 50. Only Answers A and B provide these. At an 8% yield, the price of this bond is 42.015 (or 42,015).

194. Solution: C

Let  $i$  be the yield rate. Then,

$$3609.29 = 2000(2i)a_{\overline{30}|i} + 2250(1+i)^{-30}$$

$$= 4000[1 - (1+i)^{-30}] + 2250(1+i)^{-30}$$

$$(1+i)^{-30} = (4000 - 3609.29) / (4000 - 2250) = 0.22326$$

$$i = 0.22326^{-1/30} - 1 = 0.051251.$$

Modified duration is Macaulay duration divided by one plus the yield rate:  $14.14 / 1.051251 = 13.71$ .

195. Solution: A

The amount of the dividends does not matter, so they will be assumed to be 1. First, calculate the Macaulay duration. The present value of the dividends is  $v^4 a_{\infty} = 1.1^{-4}(1/0.1) = 6.83013$ . The numerator is the present value of “payments” of 5, 6, 7, ... starting five years from now. This can be decomposed as a level of annuity of 4 and an increasing annuity of 1, 2, 3, ... . The present value is  $v^4[4a_{\infty} + (Ia)_{\infty}] = v^4 \left[ \frac{4}{i} + \frac{1+i}{i^2} \right] = \frac{1}{1.1^4} \left[ \frac{4}{0.1} + \frac{1.1}{0.1^2} \right] = 102.452$ . The Macaulay duration is  $102.452/6.83013 = 15$ . The modified duration is  $15/1.1 = 13.64$ .

196. Solution: A

Let  $r$  be the semiannual coupon rate. For the original bond,

$$P = 1000ra_{\overline{6}|0.05} + 1000v^6 = 5075.692r + 746.215. \text{ For the modified bond,}$$

$P - 49 = 1000ra_{\overline{12}|0.05} + 1000v^{12} = 8863.252r + 556.837$ . Subtracting the second equation from the first gives  $49 = -3787.56r + 189.378$ . The solution is  $r = 0.037$  and the coupon is 37.

197. Solution: A

Let  $h(i)$  be the present value of the cash flows. For Redington immunization, the value of the function and its first derivative at 25% must be zero and the second derivative must be positive.

X is immunized because:

$$h(0.25) = 102,400 - 192,000/1.25 + 100,000/1.25^3 = 0$$

$$h'(0.25) = 192,000/1.25^2 - 100,000(3)/1.25^4 = 0$$

$$h''(0.25) = -192,000(2)/1.25^3 + 100,000(3)(4)/1.25^5 = 196,608 > 0$$

Y is not immunized because:

$$h(0.25) = 158,400 - 342,000/1.25 + 100,000/1.25^2 + 100,000/1.25^3 = 0$$

$$h'(0.25) = 342,000/1.25^2 - 100,000(2)/1.25^3 - 100,000(3)/1.25^4 = -6,400 \neq 0$$

Z is not immunized because

$$h(0.25) = -89,600 + 288,000/1.25 + 100,000/1.25^2 - 300,000/1.25^3 = 51,200 \neq 0$$

198. Solution: E

The bond sells at a premium, so the worst-case scenario is redemption at time six. Then,

$$1023 = 1000 \frac{i}{0.96} a_{\overline{6}|i} + 1000(1+i)^{-6}$$

$$= \frac{1000}{0.96} [1 - (1+i)^{-6}] + 1000(1+i)^{-6}$$

$$(1+i)^{-6} = 0.448$$

$$i = 0.448^{-1/6} - 1 = 0.1432$$

14.32%.

199. Solution: E

Answer E is false because the convexity of the assets must be greater than the convexity of the liabilities.

200. Solution: E

The accumulated value to time 4 is

$$\int_1^3 100e^{0.5t} e^{0.08(4-t)} dt = 100e^{0.32} \int_1^3 e^{0.42t} dt = 100e^{0.32} \frac{e^{0.42t}}{0.42} \Big|_1^3 = \frac{100e^{0.32}(e^{1.26} - e^{0.42})}{0.42} = 657.$$

201. Solution: E

The value at time 17 of the payments beginning at time 18 is

$$2500 \left( \frac{1+k}{1.035} + \frac{(1+k)^2}{0.035^2} + \dots \right) = 2500 \frac{\frac{1+k}{1.035}}{1 - \frac{1+k}{1.035}} = 2500 \frac{1+k}{0.035-k}. \text{ The total present value is}$$

$$115,000 = 2500(1.035^{-2})a_{\overline{15}|0.035} + 2500v^{17} \frac{1+k}{0.035-k}$$

$$46(0.035-k) = 10.7516(0.035-k) + 0.55720(1+k)$$

$$k = \frac{1.61 - 0.37631 - 0.55720}{46 - 10.7516 + 0.55720} = 0.01889 = 1.89\%.$$

202. Solution: B

The initial payment,  $X$  is

$$200,000 = X \left( \frac{1}{1.03} + \frac{1.02}{1.03^2} + \cdots + \frac{1.02^{19}}{1.03^{20}} \right) = X \frac{1/1.03 - 1.02^{20}/1.03^{21}}{1 - 1.02/1.03} = 17.7267$$

$$X = 11,282.42.$$

The final payment is  $11,282.42(1.02)^{19} = 16,436.36$ .

203. Solution: E

The annual payment is  $10,000 / a_{\overline{10}|0.1} = 10,000 / 6.14457 = 1627.45$ . The balance at time 3 is  $1627.45 a_{\overline{7}|0.1} = 1627.45(4.8684) = 7923.08$ . With one-half year at simple interest, the balance at time 3.5 is  $7923.08(1.05) = 8319.23$ .

204. Solution: B

Let  $m$  be the required number of months. The equation to solve is

$$1000(1.005)^m = 2(1000)(1.015)^{m/6}$$

$$m \ln(1.005) = \ln(2) + (m/6) \ln(1.015)$$

$$m = \ln(2) / [\ln(1.005) - \ln(1.015) / 6] = 276.58.$$

Because interest is credited at the end of each period, both 276 and 277 should be checked. In both cases  $Y$  has 46 periods and an accumulation of 1983.53. At time 276  $X$  has only 3961.26, which is not double, but at time 277 it is 3981.06, which is more than double.

205. Solution: E

Let  $x$  be the amount invested in Bond A and  $y$  the amount invested in Bond B. Then  $2y$  is invested in Bond C. To match the present value of the assets and liabilities:

$$x + y + 2y = 190,000(1.07)^{-20.5}$$

$$x + 3y = 47,466.39.$$

To match the Macauley durations,  $20.5 = \frac{10x + 15y + 30(2y)}{47,466.39}$ . Then,

$$20.5(47,466.39) = 10(47,466.39 - 3y) + 75y$$

$$y = \frac{20.5(47,466.39) - 10(47,466.39)}{75 - 30} = 11,075.49 \quad \text{and}$$

$$X = 47,466.39 - 3(11,075.49) = 14,239.92.$$

206. Solution: B

Let  $X$  be the price of Bond X. Then, for the two bonds:

$$X = 10,000(0.03)a_{\overline{2n}|0.035} + c(1.035)^{-2n}$$

$$X - 969.52 = 10,000(0.025)a_{\overline{2n}|0.035} + (c + 50)(1.035)^{-2n}.$$

Subtracting the second equation from the first gives

$$969.52 = 50a_{\overline{2n}|0.035} - 50(1.035)^{-2n}$$

$$969.52 = \frac{50}{0.035}[1 - (1.035)^{-2n}] - 50(1.035)^{-2n}$$

$$1.035^{-2n} = 459.05 / 1478.57 = 0.310469$$

$$n = -(0.5) \ln(0.310469) / \ln(1.035) = 17.$$

207. Solution: B

With simple interest, the deposit in Bank X earns  $1000(0.07) = 70$  in year 8.

With compound interest, the earning in Bank Y in year 8 is

$$1000(1.0125)^{32} - 1000(1.0125)^{28} = 72.14. \text{ The absolute difference is } 2.14.$$

208. Solution: E

Split this into two perpetuities. One starts at time 0.5 at 500 increasing by 10 every year. The other starts at time 1 at 500 with payments increasing by 10 every year. The semiannual interest rate is  $1.075^{0.5} - 1 = 0.0368221$ . The present value of an increasing perpetuity immediate is found

using the formula:  $\frac{P}{i} + \frac{Q}{i^2}$ , where  $P$  is the initial amount and  $Q$  is the increase amount.

$$\text{The first perpetuity, valued at time 0: } \left( \frac{500}{0.075} + \frac{10}{0.075^2} \right) (1.0368221) = 8755.39$$

$$\text{The second perpetuity, valued at time 0: } \left( \frac{500}{0.075} + \frac{10}{0.075^2} \right) = 8444.44$$

$$\text{The total is } 8755.39 + 8444.44 = 17,199.83.$$

209. Solution: B

$$5000(10) + 5000i(Is)_{\overline{10}|0.05} = 100,000$$

$$5000i \left\{ \frac{\ddot{s}_{\overline{10}|0.05} - 10}{0.05} \right\} = 50,000$$

$$i \left\{ \frac{13.206787 - 10}{0.05} \right\} = 10$$

$$i = 0.15592$$



210. Solution: C

$$\text{Payment equals: } \left(10,000 / a_{\overline{10}|0.08}\right) = 1,490.29$$

$$\text{Accumulated total equals: } 1,490.29 s_{\overline{10}|0.10} = 23,751.46$$

$$23,751.46 - 10,000 = 13,751.46$$

211. Solution: D

$$\text{Value of fund after 20 years: } 500 \ddot{s}_{\overline{120}|0.01} (1.01)^{120} = 383,404.42$$

$$383,404.42 = \frac{X}{d} = \frac{X}{i} (1+i) = \frac{X}{0.01} (1.01)$$

$$X = 3796.08$$

212. Solution: D

$$\text{Using the retrospective method, } OB_{12} = 12,000(1.10)^{12} - 1000s_{\overline{12}|0.10}$$

$$= 37,661.14 - 1000(21.38428) = 16,276.86$$

213. Solution: E

Using the BAII Plus calculator:

$$n = 12$$

$$PV = 911.37$$

$$PMT = -40$$

$$FV = -1000$$

CPT I/Y and you get 5.0% is the half-year rate:  $\frac{i^{(2)}}{2}$ .  $(1+i) = (1+0.05)^2$ ;  $i = 0.1025$ .

214. Solution: B

$$100 \left(1 - \frac{0.12}{4}\right)^{-4n} = 200e^{0.08n}$$

$$(-4n) \ln 0.97 = \ln(200/100) + (0.08n)$$

$$0.0418n = \ln 2$$

$$n = 16.57$$

215. Solution: D

Equating present values gives

$$\frac{5000}{1.04^{10}} = X \left[ \frac{1 - \left( \frac{1.092}{1.04} \right)^{10}}{.04 - .092} \right]$$

$$3377.82 = X(12.094127)$$

$$X = 279.29$$

216. Solution: E

$$P = C + (Fr - Ci)a_{\overline{n}|}$$

$$962.92 = C + (0.04C - 0.05C)a_{\overline{20}|0.05}$$

$$962.92 = C + (-0.01C)12.46221$$

$$962.92 = 0.875378C$$

$$C = 1100$$

The discount is  $1100 - 962.92 = 137.08$ .

217. Solution: D

Equate the accumulated value of the deposits to the present value of the perpetuity:

$$(Is)_{\overline{10}|} = \frac{\ddot{s}_{\overline{10}|} - 10}{i} = \frac{10}{i}$$

$$\ddot{s}_{\overline{10}|} - 10 = 10$$

$$\ddot{s}_{\overline{10}|} = 20 \text{ (using the BAII Plus)} \Rightarrow i = 12.3\%$$

The PV of the perpetuity is  $10/0.123 = 81.30$ .

218. Solution: C

$$1300a_{\overline{20}|0.08} - 4000 = Xa_{\overline{10}|0.08}$$

$$12,763.59 - 4000 = X(6.71008) = 1306.03$$

219. Solution: C

$$\overline{d}_L = \frac{2v^3 + 1v^2}{2v^3 + 1v^2} = \frac{6.16078}{2.32908} = 2.6452$$

$$\overline{d}_A = \frac{Xv^3 + Xv^2}{Xv^3 + Xv^2} = \frac{4.01370}{1.53001} = 2.6233$$

$$2.6452 - 2.6233 = 0.0219$$

220. Solution: D

The bank's accumulated value at the end of 30 years is:

$$\frac{100,000}{a_{\overline{30}|0.05}} s_{\overline{30}|0.04} = 364,841$$

$$100,000(1+i)^{30} = 364,841$$

$$i = 0.044$$

221. Solution: E

$999.35 \times 1.06 = 1059.31$  will be available to make the first payment of 1000, leaving 59.31 to be reinvested at  $X\%$ .

$817.65 \times 1.07^2 = 936.13$  will be available from the second bond to make the second payment of 1000, leaving 63.87 to come from the reinvestment of 59.31.

$$X = 100(63.87 / 59.31 - 1) = 7.69.$$

222. Solution: E

$$(1 + s_4)^4 (1 + {}_1f_4) = (1 + s_5)^5$$

$$(1.09)^4 (1 + {}_1f_4) = (1.095)^5$$

$${}_1f_4 = 0.1152$$

223. Solution: C

$$D_{\text{mod}} = -\frac{\frac{d}{di}(1+i)^{-n}}{(1+i)^{-n}} = \frac{n(1+i)^{-n-1}}{(1+i)^{-n}} = n(1+i)^{-1}$$

224. Solution: E

The amount invested in three-year bond is equal to the PV of the third year's payout,

$$1000 / (1.1^3) = 751.31$$

The amount invested in one-year bond is equal to the PV of the first year's payout,

$$1000 / 1.08 = 925.93$$

$$925.93 - 751.31 = 174.61$$

225. Solution: B

$$PV = \frac{10}{1.04} + \frac{12}{(1.045)^2} + \frac{15}{(1.055)^3} + \frac{20}{(1.07)^4} = 9.615 + 10.989 + 12.774 + 15.258 = 48.64$$

226. Solution: D

Let  $i$  = yield rate,  $r$  = coupon rate (if any),  $F$  = face value,  $P$  = price,  $n$  = # of years.

For the first bond:

$$P = 0.8F = Fv^{36}$$

$$0.8 = v^{36}$$

$$i = 0.006218$$

For the second bond:

$$P = 0.8F = Fv^n + \frac{4}{9}(0.006218)Fa_{\overline{n}|0.006218}$$

$$0.8 = v^n + (0.0027634)a_{\overline{n}|0.006218}$$

Using the BAII Plus, where PV=0.8, I/Y=.6218, PMT=0.0027634, FV=1  
CPT N results in n=72.

227. Solution: B

Since the bond has no coupons, the Macaulay duration is the same as the amount of time until maturity, namely 4 years.

Thus, the effective annual yield rate,  $y$ , is  $\left(\frac{1200}{1000}\right)^{1/4} - 1 = 0.046635$ .

The modified duration equals the Macaulay duration divided by  $(1 + y)$ . Thus the modified duration is  $\frac{4}{1.046635} = 3.82177$  years.

228. Solution: C

Using the general Macaulay duration formula:  $\frac{\sum R_t v^t t}{\sum R_t v^t}$  where  $R$  is the cashflow:

Period	Cashflow	PV at 8%	Period $\times$ PV
1	10	9.26	9.26
2	12	10.29	20.58
3	15	11.91	35.73
4	20	14.70	58.80
5	30	20.42	102.10
Total		66.58	226.47

Macaulay duration =  $226.47/66.58 = 3.401472$  years

229. Solution: C

When a company's position is Redington immunized, its position is definitely protected from sufficiently small changes in yield rate, in either direction. However, its position may or may not be protected from large changes in yield rate.

230. Solution: D

Amount of loan =  $L$

Initial expected yield rate = 10.00%

Annual payment =  $L / a_{\overline{10}|10\%}$

Accumulated value at time 10 =  $(L / a_{\overline{10}|10\%})(s_{\overline{4}|10\%} 1.07^6 + s_{\overline{6}|7\%})$

$$\begin{aligned} \text{Yield rate} &= \left( \frac{\text{Accum Value}}{L} \right)^{1/10} - 1 \\ &= \left( \frac{s_{\overline{4}|10\%} 1.07^6 + s_{\overline{6}|7\%}}{a_{\overline{10}|10\%}} \right)^{1/10} - 1 \\ &= \left( \frac{4.6410(1.5007) + 7.1533}{6.1446} \right)^{1/10} - 1 \\ &= 8.67\% \end{aligned}$$

231. Solution: E

Let  $L$  = the loan amount. Note that  $1+i = (1+j)^5$ . The equation of value is

$$P \cdot a_{\overline{k}|i} = L = 120 \cdot a_{\overline{5k}|j}$$

so that

$$\begin{aligned} P &= \frac{120a_{\overline{5k}|j}}{a_{\overline{k}|i}} \\ &= 120 \frac{1-(1+j)^{-5k}}{j} \frac{i}{1-(1+i)^{-k}} \\ &= 120 \frac{1-(1+i)^{-k}}{j} \frac{i}{1-(1+i)^{-k}} \\ &= 120 \frac{i}{j} \\ &= 120 \frac{(1+j)^5 - 1}{j} \end{aligned}$$

Next, using the fact that  $0 < j < 0.04$ , we get

$5 < \frac{(1+j)^5 - 1}{j} < 5.41633$  by plugging in a small value like 0.000000001 and 0.04 resulting in  $P$  equaling more than 600 but less than 650.

232. Solution: B

Using the retrospective method:

$$4000(1.05)^6 - 250s_{\overline{6}|0.05}$$

$$5360.38 - 1700.48 = 3659.90$$

233. Solution: D

Let  $t$  represent the number of years since the beginning of year 1. Since the annual effective interest rate is 3% in each of years 1 through 10, and 2% each year thereafter, the present value of an amount is calculated by multiplying it by a discounting factor of  $\frac{1}{(1.03)^t}$  if  $0 \leq t \leq 10$ , and

$$\frac{1}{(1.03)^{10}(1.02)^{t-10}} \text{ if } t > 10.$$

The balance is initially 0 (the account is new before the first deposit). Deposits of  $X$  are made at times  $t = 0, 1, 2, 3, \dots, 25$ , or equivalently at time  $t = k - 1$  for each whole number  $k$  from 1 to 26 inclusive.

For the final balance to become 0, a withdrawal of 100,000 at time  $t = 25$  would be needed. Since the net present value of the cash flows (withdrawals minus deposits) must be zero, in a time period from a zero balance to another zero balance, we have

$$\frac{100,000}{(1.03)^{10}(1.02)^{25-10}} - X \sum_{k=1}^{11} \frac{1}{(1.03)^{k-1}} - X \sum_{k=12}^{26} \frac{1}{(1.03)^{10}(1.02)^{k-1-10}} = 0$$
$$\frac{100,000}{(1.03)^{10}(1.02)^{15}} = X \sum_{k=1}^{11} \frac{1}{(1.03)^{k-1}} + X \sum_{k=12}^{26} \frac{1}{(1.03)^{10}(1.02)^{k-11}}$$

234. Solution: A

For the first bond:  $P = (0.076)(6000)a_{\overline{20}|0.065} + 6000v^{20}$

$$P = 6727.22.$$

For the second bond:  $6727.22 = (r)(7500)a_{\overline{20}|0.065} + 7500v^{20}$

$$7500r = 417.37, \text{ so } r = 0.0556.$$

235. Solution: E

$$P_X = Fra_{\overline{40}|} + Cv^{40} = 75a_{\overline{40}|} + Cv^{40}$$

$$P_Y = P_X - 257.18 = 60a_{\overline{40}|} + (C + K)v^{40}$$

$$P_X - P_Y = 257.18 = 15a_{\overline{40}|} - Kv^{40}$$

$$Kv^{40} = 15a_{\overline{40}|} - 257.18$$

$$K(0.252572) = 320.33 - 257.18$$

$$K = 250.01$$

236. Solution: C

$$\text{The PV of the first twenty payments is: } 14,000 \left( \frac{1 - \left( \frac{1.04}{1.10} \right)^{20}}{0.10 - 0.04} \right) = 157,337.48$$

The PV of the remaining payments starting at time 21 is:

$$14,000(1.04)^{19}(1.01) \left( \frac{1}{0.10 - 0.01} \right) (1.10)^{-20} = 49,202.44$$

Total equals 206,539.92.

237. Solution: B

$$16,000 = 1,000 \left( \frac{1}{0.057 - (-r/100)} \right)$$

$$16 = \frac{1}{0.057 + r/100}$$

$$0.057 + r/100 = \frac{1}{16}$$

$$r = 0.55$$

238. Solution: E

Let  $j$  equal the five-year interest rate.

$$(1 + j) = (1.09)^5$$

$$j = 0.538624$$

$$PV = \frac{2}{0.538624} + \frac{10}{(0.538624)^2} = 38.18$$

239. Solution: A

$$\text{From the first bond: } P = 25a_{\overline{8}|0.03} + Cv^8$$

$$\text{From the second bond: } 0.93P = 25a_{\overline{4}|0.03} + Cv^4$$

Multiply the first equation by 0.93 and plug into the second equation:

$$0.93(25)a_{\overline{8}|0.03} + 0.93C = 25a_{\overline{4}|0.03} + Cv^4$$

$$163.2078 + 0.73415C = 92.9275 + 0.88849C$$

$$70.2804 = 0.15434C$$

$$C = 455.37$$

240. Solution: D

The PV of the liability is  $\frac{600,000}{1.046^2} = 548,387.92$  and its Macaulay duration is 2.

Then, equating present values:

$$\frac{x}{1.046} + \frac{y}{1.046^4} = 548,387.92$$

And equating durations:

$$\frac{(x/1.046)}{548,387.92}(1) + \frac{(y/1.046^4)}{548,387.92}(4) = 2$$

Solving the system of equations results in  $x = 382,409$

241. Solution: E

Solution:  $4000(1.04^3) = 1400[1 + (1+i) + (1+i)^2]$ , or  $i$  solves the quadratic equation:

$i^2 + 3i - 0.2139 = 0$ . Thus,  $i = 6.97\%$  because the other root is negative.

242. Solution: B

$$100,000 = Xa_{\overline{6}|0.01}v_{0.01}^4 + \frac{X}{0.05}v_{0.01}^{10}$$

$$100,000 = X(5.79548)(0.96098) + X(20)(0.90529)$$

$$100,000 = X(23.6751)$$

$$X = 4223.85$$



243. Solution: D

Using BA II Plus:

$$60,000 = Xa_{\overline{180}|0.075/12}$$

$$X = 556.21$$

$$49,893 = 556.21a_{\overline{m}|0.075/12}$$

$$m = 132$$

So,  $(180 - 132) = 48$  payments have been made so far:

$$49,893 = 556.21 a_{\overline{n}|0.6/12}$$

$$n = 119.3$$

Use 120 future payments including the smaller one.

$$48+120=168.$$

244. Solution: C

$$2000 = 110a_{\overline{20}|0.10} + X \left[ \frac{a_{\overline{20}|0.10} - 20v^{20}}{0.10} \right]$$

$$2000 = 110(8.51356) + X(55.40691)$$

$$1063.51 = X(55.40691)$$

$$X = 19.1945$$

245. Solution: E

$$PV = 100 \left[ \frac{1 - \left( \frac{1.15}{1.05} \right)^{10}}{0.05 - 0.15} \right] = 1483.62$$

246. Solution: D

$$OB_{n-1} = Ra_{\overline{1}} = 1144.5 = Rv$$

$$P_{n-4} = Rv^{n-(n-4)+1} = Rv^5 = 865$$

$$\frac{Rv^5}{Rv} = v^4 = \frac{865}{1144.5} = 0.75579$$

$$v^4 = 0.75579, \quad v = 0.93240, \quad i = 0.07251$$

$$I_1 = X(0.07251) = 797.50$$

$$X = 10,999.02$$

247. Solution: C

Given that the problem states that the inequality is true for all interest rates from 0% to 10% and all values of  $Y$ , it is sufficient to determine it for one set of values. Select  $i = 7\%$  and  $Y = 121$ .

Then,

$$Q = 121 / (1 + 3(0.07)) = 100$$

$$R = 121 / (1.07)^3 = 98 / 77$$

$$S = 121(1 - 0.07(3)) = 95.59$$

$$T = 121(0.93)^3 = 97.33$$

Hence,

$$S < T < R < Q$$

248. Solution: C

The yield rate on Kate's bond is

$$(1000 - 100) = 25a_{\overline{10}| \frac{i^{(2)}}{2}} + 1000v^{10}$$

$$\frac{i^{(2)}}{2} = 0.0371551$$

The discount on Wallace's bond is

$$(1000 - D) = 25a_{\overline{8}|0.05} + 1000v^8$$

$$1000 - D = 838.42, \quad D = 161.58$$

The book value of Kate's bond at time 1 is

$$B = 25a_{\overline{8}|0.0371551} + 1000v^8$$

$$B = 917.19$$

The difference is  $B - D = 917.19 - 161.58 = 755.61$

249. Solution: B

$$P_L = 1000(1+i)^{-2} + 300(1+i)^{-4}$$

$$P_L = 1153.84$$

$$P'_L = -2000(1+i)^{-3} - 1200(1+i)^{-5}$$

$$P'_L = -2667.91$$

$$P_A = X(1+i)^{-1} + Y(1+i)^{-3}$$

$$1153.84 = 0.95238X + 0.86384Y$$

$$P'_L = -X(1+i)^{-2} - 3Y(1+i)^{-4}$$

$$-2667.91 = -0.90703X - 2.46811Y$$

So, we have two equations and two unknowns. Solving simultaneously, we get:

$$Y = 953.57, \quad X = 346.61, \quad \frac{Y}{X} = 2.75.$$

250. Solution: C

$$1000 = Pa_{\overline{20}|0.03}$$

$$P = 67.22$$

$$67.22(20) = 1344.31$$

$$1344.31 = 1000 + i(1000 + 950 + 900 + \cdots + 50)$$

$$344.31 = i(10,500)$$

$$i = 0.03279$$

251. Solution: B

$$\frac{i^{(12)}}{12} = 1.08^{1/12} - 1 = 0.006434$$

$$600 = P(1/1.006434)^{120-6+1}$$

$$P = 1254.47$$

$$P_{24} = 1254.47v^{120-24+1} = 673.42$$

252. Solution: B

$$e^{\int_0^{20} \frac{2}{1+2t} dt} = e^{\ln(1+2t)|_0^{20}} = 41$$

$$41 = (1+i)^{20}$$

$$i = 0.204035$$

$$(1+0.204035)^5 = 2.53$$

253. Solution: B

$$1000 \cdot a_{\overline{20}|i} = \frac{600}{i} + \frac{600v^{10}}{i}$$

$$5 \left( \frac{1-v^{20}}{i} \right) = \frac{3}{i} (1+v^{10})$$

$$5 - 5v^{20} = 3 + 3v^{10}$$

$$0 = 5v^{20} + 3v^{10} - 2$$

$$\text{Let } x = v^{10} = \frac{-3 \pm \sqrt{9 + 4(5)(2)}}{2(5)} = \frac{-3 \pm 7}{10} = 0.4 \Rightarrow i = 9.59582\%$$

$$X = \frac{600}{0.0959582} (1+0.4) = 8753.8$$

254. Solution: B

$$5000 = \frac{150}{i} + \frac{10}{i^2}$$

$$5000i^2 - 150i - 10 = 0$$

$$i = \frac{150 \pm \sqrt{(-150)^2 - 4(5000)(-10)}}{10,000}$$

$$i = 0.06217$$

255. Solution: D

If paid in one lump sum, the total interest paid is  $X(1.05^{20} - 1) = 1.65330X$ .

With level payments for 10 years, the total interest paid is

$$10 \left( \frac{X}{a_{\overline{10}|0.05}} \right) - X = 0.29505X.$$

Then,

$$1.65330X = 1000 + 0.29505X$$

$$X = 736.24.$$

256. Solution: B

$$P_t = I_t$$

$$v^{20-t+1} = (1 - v^{20-t+1})$$

$$2v^{20-t+1} = 1$$

$$v^{21-t} = 0.5$$

$$1.05^{21-t} = 2$$

$$(21-t)\ln(1.05) = \ln 2$$

$$t = 7$$

257. Solution: B

$$10,000 + 10,815v^2 = 20,800v$$

$$v = \frac{20,800 \pm \sqrt{20,800^2 - 4(10,815)(10,000)}}{21,630}$$

$$v = 0.970873 \text{ or } 0.952381$$

$$i = 0.03 \text{ or } 0.05$$

$$|0.03 - 0.05| = 0.02$$

258. Solution: A

$$P_A = 45a_{\overline{30}|0.042} + 1200v^{30}$$

$$P_A = 1108.85$$

$$1108.85 = 20a_{\overline{4n}|0.021} + 1376.69v^{4n}$$

Using the BA II Plus:

$$PV = 1108.85$$

$$PMT = 20$$

$$FV = 1376.69$$

$$I/Y = 2.1$$

Solve for  $4n$  and get  $4n = 48$ ,  $n = 12$ .

Or solve the equation to get  $v^{4n} = 0.36876$  and then solve for  $n$ .

259. Solution: C

Statement I should have an  $\ddot{s}_{\overline{60}|}$  on the left.

Statement II has an annuity-due rather than an annuity-immediate on the right.

Statement III is correct.

260. Solution: D

$$\frac{15(961.54)(1) + 20(966.14)(2) + 30(878.41)(3)}{15(961.54) + 20(966.14) + 30(878.41)} = 2.198495$$

261. Solution: B

$$1,000,000(1.10)^{17} - Ps_{\overline{18}|0.10} = 1,000,000$$

$$5,054,470.285 - P(45.59917) = 1,000,000$$

$$4,054,470.285 = P(45.59917)$$

$$P = 88,915.43$$

262. Solution: D

Using time 5 as the first reference point, then bringing that value back to time 0:

$$v^5 \left[ 500\ddot{a}_{\overline{5}|i} + 500(I\ddot{a})_{\overline{5}|i} \right]$$

This combines a five-year level annuity-due of 500 plus an increasing annuity-due starting with 500 and increasing by 500.

263. Solution: B

Let face amount equal 1.

$$1.61 = 2.25i \left( \frac{1 - v^{18}}{i} \right) + v^{18}$$

$$1.61 = 2.25(1 - v^{18}) + v^{18}$$

$$1.25v^{18} = 0.64$$

$$v = 0.96342$$

$$1.45 = 2.25i \left( \frac{1 - v^n}{i} \right) + v^n$$

$$1.45 = 2.25(1 - v^n) + v^n$$

$$1.25v^n = 0.8$$

$$v^n = 0.64$$

$$n \ln 0.963492 = \ln 0.64$$

$$n = 12$$

264. Solution: C

$$450 = Xa_{\overline{10}|0.10} + 1000v^{10}$$

Using BA II Plus calculator:

$$X = 10.50$$

$$1.10 = (1 + j)^2, j = 0.048881$$

$$P = 5.25a_{\overline{20}|0.048881} + 1000v^{20}$$

$$P = 451.64$$

265. Solution: E

Let  $I$  be the amount of interest in the first month.

$$P_0 - m + I = P_1, I = P_1 - (P_0 - m)$$

In the first month, the interest  $P_1 - (P_0 - m)$  was charged on a principal of  $P_0$ , so the effective monthly interest rate (expressed as a decimal) of the first loan is  $\frac{P_1 - (P_0 - m)}{P_0} = \frac{P_1 - P_0 + m}{P_0}$ .

The nominal annual interest rate (expressed as a decimal) for both loans is therefore

$12\left(\frac{P_1 - P_0 + m}{P_0}\right)$ , so the effective daily rate (expressed as a decimal) for the second loan is

$$\frac{12}{365}\left(\frac{P_1 - P_0 + m}{P_0}\right).$$

Finally, the effective monthly rate (expressed as a decimal) for the second loan is

$$\left[1 + \frac{12}{365}\left(\frac{P_1 - P_0 + m}{P_0}\right)\right]^{365/12} - 1.$$

266. Solution: D.

$$P(0, m) = (1 + i)^{-m}$$

$$P(0, n) = (1 + i)^{-n}$$

$$X = \frac{(1 + i)^{-m}}{(1 + i)^{-n}} = (1 + i)^{-m+n}$$

$$X = \frac{P(0, m)}{P(0, n)}$$

267. Solution: A

$$OB_{180} = 2000a_{\overline{120}|0.005} = 180,146.91$$

$$180,146.91 = Pa_{\overline{180}|0.005}$$

$$P = 1520.18$$

$$L = 1520.18a_{\overline{360}|0.005} = 253,553.61$$

268. Solution: E

The conditions

- 1) assets and liabilities have equal present values and equal modified durations, and
  - 2) the convexity of its assets exceeds the convexity of its liabilities
- are precisely what is required for Redington immunization.



269. Solution: E

$$90.17 = 4a_{\overline{6}|j} + Xv^6$$

$$132.47 = 4a_{\overline{6}|j} + 1.6Xv^6$$

Multiply the first equation by 1.6:

$$144.272 = 6.4a_{\overline{6}|j} + 1.6Xv^6$$

Subtract the second equation:

$$11.802 = 2.4a_{\overline{6}|j}$$

Use annuity calculation on BA II Plus:

$$j = 6\% = \frac{i^{(2)}}{2}, i^{(2)} = 12\%$$

270. Solution: B

$$9297 = Pa_{\overline{20}|0.05}$$

$$P = 5000$$

$$\frac{5000}{1.05^2} = 4535.12$$

271. Solution: B

Using the BA II Plus calculator:

$$1.04 = \left(1 + \frac{i^{(2)}}{2}\right)^2$$

$$\frac{i^{(2)}}{2} = 0.019804$$

$$P_A = 25a_{\overline{10}|0.019804} + 1000v^{10}$$

$$P_A = 1046.72$$

$$1046.72 - 100 = 30a_{\overline{5}|j} + 1000v_j^5$$

$$j = 4.2036\%$$

272. Solution: E

Value at time 5 years:

$$100,000(1.02)^{20} - 2500s_{\overline{20}|0.02}$$

$$87,851.32 = 5000a_{\overline{m}|0.02}$$

$m = 21.86$ , using the BA II Plus. Since we want a balloon payment, use  $m = 21$ .

$$87,851.32 = 5000a_{\overline{21}|0.02} + Bv^{21}$$

$$B = 4236.70, \text{ so balloon equals } 5000 + 4236.70 = 9236.70$$

273. Solution: B

$$900 = 1000ra_{\overline{40}|0.05} + 900v^{40}$$

Using BA II Plus:

$$1000r = 45$$

$$P = 45a_{\overline{20}|0.04} + 900v^{20}$$

$$P = 1022.31$$

$$1022.31 = Fra_{\overline{20}|0.04} + 1100v^{20}$$

$$Fr = 38.28$$

274. Solution: C

Let  $Y$  indicate the nominal value of the two-year bond, then:

$$9465 = \frac{0.05Y}{1.08} + \frac{1.05Y}{1.08^2}, \text{ so } Y = 10,000.$$

Thus, the amount of liability at the end of the second year is 10,500.

Hence, the liability at the end of the first year is:

$$\frac{10,500}{2} = 5250.$$

So, the amount invested in the one-year bond is:

$$\frac{5250 - 10,000(0.05)}{1.06} = 4481.$$

275. Solution: C

Macaulay duration of the liability is 3. Asset duration must equal 3.

Let  $P_1$  and  $P_4$  be the present values of the two assets.

$$\frac{P_1 \cdot 1 + P_4 \cdot 4}{P_1 + P_4} = 3, \text{ then } P_4 = 2P_1$$

$$P_1 = \frac{20,000}{1+i}, P_4 = \frac{50,000}{(1+i)^4}, \frac{P_1}{P_4} = \frac{P_1}{2P_1} = \frac{1}{2} = \frac{\frac{20,000}{1+i}}{\frac{50,000}{(1+i)^4}} : 1 = 0.8(1+i)^3 :$$

$$(1+i)^3 = 1.25; (1+i) = 1.077217$$

PV of assets must equal PV of liabilities. So, PV of assets equals:

$$P_1 + 2P_1 = 3P_1 = 3 \frac{20,000}{1.077217} = 55,699.07.$$

Amount of liability equals:  $55,699.07(1.077217)^3 = 69,623.83$ .