

## **Fundamentals of Actuarial Mathematics – Long Term Topics**

### **Solutions to Sample Multiple Choice Questions**

**July 22, 2024**

Versions:

July 19, 2024	Questions deleted for material removed from the syllabus (questions 3.10, 3.17, 18.1-18.11)
June 7, 2023	Fixed solution to 5.12.
March 6, 2023	Fixed answer key typo in sample solution to 7.34 to match the exam answer key.
December 16, 2022	Added solutions for Fall 2022 exam questions
August 8, 2022	Revised solution to questions 18.1 and 18.4
July 29, 2022	Updated term “expense reserve” to “expense policy value”.
July 1, 2022	Original version for FAM-L.

These solutions are written to provide evidence that the stated answer is correct and to offer at least one approach to solving the problem. There may be additional approaches that can be employed.

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### **Question 1.1**

#### **Answer C**

Answer C is false. If the purchaser of a single premium immediate annuity has higher mortality than expected, this reduces the number of payments that will be paid. Therefore, the Actuarial Present Value will be less and the insurance company will benefit. Therefore, single premium life annuities do not need to be underwritten.

The other items are true.

A: Life insurance is typically underwritten to prevent adverse selection as higher mortality than expected will result in the Actuarial Present Value of the benefits being higher than expected.

B: In some cases, such as direct marketed products for low face amounts, there may be very limited underwriting. The actuary would assume that mortality will be higher than normal, but the expenses related to selling the business will be low and partially offset the extra mortality.

D: If the insured's occupation or hobby is hazardous, then the insured life may be rated.

E: If the purchaser of the pure endowment has higher mortality than expected, this reduces the number of endowments that will be paid. Therefore, the Actuarial Present Value will be less and the insurance company will benefit. Therefore, pure endowments do not need to be underwritten.

### **Question 1.2**

#### **Answer E**

Insurers have an increased interest in combining savings and insurance products so Item E is false.

The other items are all true.

### **Question 1.3**

#### **Answer C**

Key is that the benefit is linked to the performance of an investment fund. That is true only for C.

### **Question 1.4**

#### **Answer B**

Key is that benefit is contingent upon being sick, but not related to long-term care. Replacing salary and benefits tied to being unable to work also fit best with B.

### Question 2.1

**Answer: B**

Since  $S_0(t) = 1 - F_0(t) = \left(1 - \frac{t}{\omega}\right)^{\frac{1}{4}}$ , we have  $\ln[S_0(t)] = \frac{1}{4} \ln\left[\frac{\omega-t}{\omega}\right]$ .

Then  $\mu_t = -\frac{d}{dt} \log S_0(t) = \frac{1}{4} \frac{1}{\omega-t}$ , and  $\mu_{65} = \frac{1}{180} = \frac{1}{4} \frac{1}{\omega-65} \Rightarrow \omega = 110$ .

$$e_{106} = \sum_{t=1}^3 {}_tP_{106}, \text{ since } {}_4P_{106} = 0$$

$${}_tP_{106} = \frac{S_0(106+t)}{S_0(106)} = \frac{\left(1 - \frac{106+t}{110}\right)^{1/4}}{\left(1 - \frac{106}{110}\right)^{1/4}} = \left(\frac{4-t}{4}\right)^{1/4}$$

$$e_{106} = \sum_{i=1}^{i=4} {}_iP_{106} = \frac{1}{4^{0.25}} (1^{0.25} + 2^{0.25} + 3^{0.25}) = 2.4786$$

**Question 2.2**

**Answer: D**

This is a mixed distribution for the population, since the vaccine will apply to all once available.

Available?			S = # of survivors		
(A)	Pr(A)	${}_2p   A$	$E(S A)$	$Var(S A)$	$E(S^2 A)$
Yes	0.2	0.9702	97,020	2,891	9,412,883,291
No	0.8	0.9604	96,040	3,803	9,223,685,403
			$E(S)$		$E(S^2)$
			96,236		9,261,524,981
			$Var(S)$	157,285	
			$SD(S)$	397	

As an example, the formulas for the “No” row are

$$\Pr(\text{No}) = 1 - 0.2 = 0.8$$

$${}_2p \text{ given No} = (0.98 \text{ during year 1})(0.98 \text{ during year 2}) = 0.9604.$$

$E(S | \text{No}), Var(S | \text{No})$  and  $E(S^2 | \text{No})$  are just binomial,  $n = 100,000$ ;  $p(\text{success}) = 0.9604$

$E(S), E(S^2)$  are weighted averages,

$$Var(S) = E(S^2) - E(S)^2$$

Or, by the conditional variance formula:

$$\begin{aligned} Var(S) &= Var[E(S | A)] + E[Var(S | A)] \\ &= 0.2(0.8)(97,020 - 96,040)^2 + 0.2(2,891) + 0.8(3,803) \\ &= 153,664 + 3,621 = 157,285 \end{aligned}$$

$$StdDev(S) = 397$$

### Question 2.3

Answer: A

$$\begin{aligned} f_x(t) &= -\frac{d}{dt} S_x(t) = -\frac{d}{dt} \left( e^{-\frac{B}{\ln c}(c^x)(c^t-1)} \right) \\ &= -e^{-\frac{B}{\ln c}(c^x)(c^t-1)} \cdot \left( -\frac{B}{\ln c} \cdot c^x \right) \cdot c^t \cdot \ln c \\ &= e^{-\frac{B}{\ln c}(c^x)(c^t-1)} \cdot B c^{x+t} \\ &= 0.00027 \times 1.1^{x+t} \cdot e^{-\frac{0.00027}{\ln(1.1)}(1.1^x)(1.1^t-1)} \end{aligned}$$

$$f_{50}(10) = 0.00027 \times 1.1^{50+10} \cdot e^{-\frac{0.00027}{\ln(1.1)}(1.1^{50})(1.1^{10}-1)} = 0.04839$$

Alternative Solution:

$$f_x(t) = {}_t p_x \cdot \mu_{x+t}$$

Then we can use the formulas given for Makeham with  $A = 0$ ,  $B = 0.00027$  and  $c = 1.1$

$$f_x(t) = \left( e^{-\frac{0.00027}{\ln(1.1)}(1.1^{50})(1.1^{10}-1)} \right) (0.00027 \times 1.1^{50+10}) = 0.04839$$

### Question 2.4

Answer: E

$$e_{75:\overline{10}|}^{\circ} = \int_{t=0}^{t=10} {}_t p_{75} dt \quad \text{where } {}_t p_x = \frac{{}_{t+x} P_0}{{}_x P_0} = \frac{1 - \frac{(t+x)^2}{10000}}{1 - \frac{x^2}{10000}} = \frac{10000 - (t+x)^2}{10000 - x^2} \quad \text{for } 0 < t < 100 - x$$

$$\begin{aligned} &= \int_0^{10} \frac{10000 - 75^2 - 150t - t^2}{10000 - 75^2} dt \\ &= \frac{1}{4375} \cdot \left[ 4375t - 75t^2 - \frac{t^3}{3} \right]_{t=0}^{t=10} = 8.21 \end{aligned}$$

### Question 2.5

Answer: B

$$\begin{aligned} e_{40} &= e_{40:\overline{20}|} + {}_{20} p_{40} \cdot e_{60} \\ &= 18 + (1 - 0.2)(25) \\ &= 38 \end{aligned}$$

$$\begin{aligned} e_{40} &= e_{40:\overline{1}|} + P_{40} \cdot e_{41} \\ \Rightarrow e_{41} &= \frac{e_{40} - e_{40:\overline{1}|}}{P_{40}} = \frac{e_{40} - P_{40}}{P_{40}} = \frac{38 - 0.997}{0.997} = 37.11434 \end{aligned}$$

### Question 2.6

Answer: C

$$\begin{aligned} \mu_x &= -\frac{d}{d_x} \ln S_0(x) = -\frac{1}{3} \frac{d}{d_x} \ln \left( 1 - \frac{x}{60} \right) \\ &= \frac{1}{180} \left( 1 - \frac{x}{60} \right)^{-1} = \frac{1}{3(60-x)} \end{aligned}$$

$$\text{Therefore, } 1000\mu_{35} = (1000) \frac{1}{3(25)} = \frac{1000}{75} = 13.3.$$

**Question 2.7****Answer: B**

$$\begin{aligned}
{}_{20}q_{30} &= \frac{S_0(30) - S_0(50)}{S_0(30)} = \frac{\left(1 - \frac{30}{250}\right) - \left(1 - \left[\frac{50}{100}\right]^2\right)}{1 - \frac{30}{250}} = \frac{\frac{220}{250} - \frac{3}{4}}{\frac{220}{250}} \\
&= \frac{440 - 375}{440} = \frac{65}{440} = \frac{13}{88} = 0.1477
\end{aligned}$$

**Question 2.8****Answer: C**

The 20-year female survival probability =  $e^{-20\mu}$

The 20-year male survival probability =  $e^{-30\mu}$

We want 1-year female survival =  $e^{-\mu}$

Suppose that there were  $M$  males and  $3M$  females initially. After 20 years, there are expected to be  $Me^{-30\mu}$  and  $3Me^{-20\mu}$  survivors, respectively. At that time we have:

$$\frac{3Me^{-20\mu}}{Me^{-30\mu}} = \frac{85}{15} \Rightarrow e^{10\mu} = \frac{85}{45} = \frac{17}{9} \Rightarrow e^{-\mu} = \left(\frac{9}{17}\right)^{1/10} = 0.938$$

**Question 2.9****Answer: E**

The distribution is binomial with 10,000 trials.

$$\text{Var}[L_{15}] = npq = 10,000({}_{15}p_{50})({}_{15}q_{50})$$

$${}_{15}p_{50} = e^{\left[-A(15) - \frac{B}{\ln C} * c^{50} (c^{15} - 1)\right]} = 0.837445$$

$${}_{15}q_{50} = 1 - {}_{15}p_{50} = 0.162555$$

$$\text{Var}[L_{15}] = 10,000(0.837445)(0.162555) = 1361.3$$

### Question 2.10

Answer: A

$${}_{10}P_x = \frac{l_{x+10}}{l_x} = e^{-\int_0^{10} \mu_{x+t} \cdot dt} \implies \frac{400}{1000} = e^{-\int_0^{10} \beta t^2 \cdot dt} \implies 0.4 = e^{-\beta t^3 / 3} \Big|_0^{10}$$

$$\implies 0.4 = e^{-\beta \cdot 1000 / 3} \implies \ln(0.4) = -\beta \left( \frac{1000}{3} \right) \implies \beta = -\ln(0.4) \cdot (0.003) = 0.0027489$$

### Question 2.11

Answer: C

$$\text{Let } l_{80} = 1000 \implies l_{81} = 960 \implies l_{82} = 902.4$$

$$\text{Answer} = \frac{l_{81.1} - l_{81.6}}{l_{80.6}} = \frac{(960)^{0.9} (902.4)^{0.1} - (960)^{0.4} (902.4)^{0.6}}{(0.4)(1000) + (0.6)(960)} = 0.02978$$

The value for  $l_{80}$  is arbitrary. Any other starting value gives the same result.

Another form for the survivors between 81 and 82 would be

$$\mu = -\ln(902 / 960) = 0.0095833;$$

$$l_{81.1} = 960 \times e^{-0.1 \times 0.0095833} = 959.08;$$

$$l_{81.6} = 960 \times e^{-0.6 \times 0.0095833} = 954.496.$$

### Question 2.12

Answer: A

$$E[T_0] = \int_0^8 {}_t p_0 dt = \int_0^8 \left(1 - \frac{t^2 + t}{72}\right) dt \rightarrow \frac{1}{72} \left[72t - \frac{t^3}{3} - \frac{t^2}{2}\right]_0^8 = 5.1852$$

$$E[T_0^2] = 2 \int_0^8 ({}_t p_0 \times t) dt = \frac{2}{72} \int_0^8 (72t - t^3 - t^2) dt = \frac{2}{72} \left[36t^2 - \frac{t^4}{4} - \frac{t^3}{3}\right]_0^8 = 30.815$$

$$\text{Var}[T_0] = E[T_0^2] - (E[T_0])^2 = 3.9287$$



### Question 3.1

**Answer: B**

Under constant force over each year of age,  $l_{x+k} = (l_x)^{1-k} (l_{x+1})^k$  for  $x$  an integer and  $0 \leq k \leq 1$ .

$${}_{2|3}q_{[60]+0.75} = \frac{l_{[60]+2.75} - l_{[60]+5.75}}{l_{[60]+0.75}}$$

$$l_{[60]+0.75} = (80,000)^{0.25} (79,000)^{0.75} = 79,249$$

$$l_{[60]+2.75} = (77,000)^{0.25} (74,000)^{0.75} = 74,739$$

$$l_{[60]+5.75} = (67,000)^{0.25} (65,000)^{0.75} = 65,494$$

$${}_{2|3}q_{[60]+0.75} = \frac{l_{[60]+2.75} - l_{[60]+5.75}}{l_{[60]+0.75}} = \frac{74,739 - 65,494}{79,249} = 0.11679$$

$$1000 {}_{2|3}q_{[60]+0.75} = 116.8$$

### Question 3.2

**Answer: D**

$$l_{65+1} = 1000 - 40 = 960$$

$$l_{66+1} = 955 - 45 = 910$$

$$\dot{e}_{[65]} = \int_0^1 {}_tP_{[65]} dt + P_{[65]} \int_0^1 {}_tP_{66} dt + P_{[65]} P_{66} \dot{e}_{67}$$

$$15.0 = \left[ 1 - \left( \frac{1}{2} \right) \left( \frac{40}{1000} \right) \right] + \frac{960}{1000} \left[ 1 - \left( \frac{1}{2} \right) \left( \frac{50}{960} \right) \right] + \left( \frac{960}{1000} \right) \left( \frac{910}{960} \right) \dot{e}_{67}$$

$$\dot{e}_{67} = \frac{15(1000) - (980 + 935)}{910} = 14.37912$$

$$\dot{e}_{[66]} = \int_0^1 {}_tP_{[66]} dt + P_{[66]} \dot{e}_{67} = \left[ 1 - \left( \frac{1}{2} \right) \left( \frac{45}{955} \right) \right] + \left( \frac{910}{955} \right) \dot{e}_{67}$$

$$\dot{e}_{[66]} = \left[ 1 - \left( \frac{1}{2} \right) \left( \frac{45}{955} \right) \right] + \left( \frac{910}{955} \right) (14.37912) = 14.678$$

Note that because deaths are uniformly distributed over each year of age,  $\int_0^1 {}_tP_x dt = 1 - 0.5q_x$ .

### Question 3.3

**Answer: E**

$${}_{2.2}q_{[51]+0.5} = \frac{l_{[51]+0.5} - l_{53.7}}{l_{[51]+0.5}}$$

$$l_{[51]+0.5} = 0.5l_{[51]} + 0.5l_{[51]+1} = 0.5(97,000) + 0.5(93,000) = 95,000$$

$$l_{53.7} = 0.3l_{53} + 0.7l_{54} = 0.3(89,000) + 0.7(83,000) = 84,800$$

$${}_{2.2}q_{[51]+0.5} = \frac{95,000 - 84,800}{95,000} = 0.1074$$

$$10,000 {}_{2.2}q_{[51]+0.5} = 1,074$$

### Question 3.4

**Answer: B**

Let  $S$  denote the number of survivors.

This is a binomial random variable with  $n = 4000$  and success probability  $\frac{21,178.3}{99,871.1} = 0.21206$

$$E(S) = 4,000(0.21206) = 848.24$$

The variance is  $Var(S) = (0.21206)(1 - 0.21206)(4,000) = 668.36$

$$StdDev(S) = \sqrt{668.36} = 25.853$$

The 90% percentile of the standard normal is 1.282

Let  $S^*$  denote the normal distribution with mean 848.24 and standard deviation 25.853. Since  $S$  is discrete and integer-valued, for any integer  $s$ ,

$$\begin{aligned} \Pr(S \geq s) &= \Pr(S > s - 0.5) \approx \Pr(S^* > s - 0.5) \\ &= \Pr\left(\frac{S^* - 848.24}{25.853} > \frac{s - 0.5 - 848.24}{25.853}\right) \\ &= \Pr\left(Z > \frac{s - 0.5 - 848.24}{25.853}\right) \end{aligned}$$

For this probability to be at least 90%, we must have  $\frac{s - 0.5 - 848.24}{25.853} < -1.282$

$$\Rightarrow s < 815.6$$

So  $s = 815$  is the largest integer that works.

### Question 3.5

**Answer: E**

Using UDD

$$l_{63.4} = (0.6)66,666 + (0.4)(55,555) = 62,221.6$$

$$l_{65.9} = (0.1)(44,444) + (0.9)(33,333) = 34,444.1$$

$${}_{3.4|2.5}q_{60} = \frac{l_{63.4} - l_{65.9}}{l_{60}} = \frac{62,221.6 - 34,444.1}{99,999} = 0.277778 \quad (\text{a})$$

Using constant force

$$l_{63.4} = l_{63} \left( \frac{l_{64}}{l_{63}} \right)^{0.4} = l_{63}^{0.6} l_{64}^{0.4}$$
$$= (66,666^{0.6})(55,555^{0.4})$$

$$= 61,977.2$$

$$l_{65.9} = l_{65}^{0.1} l_{66}^{0.9} = (44,444^{0.1})(33,333^{0.9})$$
$$= 34,305.9$$

$${}_{3.4|2.5}q_{60} = \frac{61,977.2 - 34,305.9}{99,999}$$
$$= 0.276716 \quad (\text{b})$$

$$100,000(a - b) = 100,000(0.277778 - 0.276716) = 106$$

### Question 3.6

**Answer: D**

$$e_{[61]} = e_{[61]:\overline{3}} + {}_3p_{[61]}(e_{64})$$

$$p_{[61]} = 0.90,$$

$${}_2p_{[61]} = 0.9(0.88) = 0.792,$$

$${}_3p_{[61]} = 0.792(0.86) = 0.68112$$

$$e_{[61]:\overline{3}} = \sum_{k=1}^3 {}_k p_{[61]} = 0.9 + 0.792 + 0.68112 = 2.37312$$

$$e_{[61]} = 2.37312 + 0.68112e_{64} = 2.37312 + 0.68112(5.10) = 5.847$$

**Question 3.7****Answer: B**

$$\begin{aligned}
{}_{2.5}q_{[50]+0.4} &= 1 - {}_{2.5}p_{[50]+0.4} = 1 - {}_{2.9}p_{[50]} / (p_{[50]})^{0.4} \\
&= 1 - \left\{ p_{[50]} p_{[50]+1} (p_{52})^{0.9} \right\} / (1 - q_{[50]})^{0.4} \\
&= 1 - \left\{ (1 - q_{[50]})(1 - q_{[50]+1})(1 - q_{52})^{0.9} \right\} / (1 - q_{[50]})^{0.4} \\
&= 1 - \left\{ (1 - 0.0050)(1 - 0.0063)(1 - 0.0080)^{0.9} \right\} / (1 - 0.0050)^{0.4} \\
&= 0.01642
\end{aligned}$$

$$1000 {}_{2.5}q_{[50]+0.4} = 16.42$$

**Question 3.8****Answer: B**

$$E(N) = 1000({}_{40}p_{35} + {}_{40}p_{45}) = 1000 \left( \frac{85,203.5}{99,556.7} + \frac{61,184.9}{99,033.9} \right) = 1473.65$$

$$Var(N) = 1000 {}_{40}p_{35} (1 - {}_{40}p_{35}) + 1000 {}_{40}p_{45} (1 - {}_{40}p_{45}) = 359.50$$

$$\text{Since } 1473.65 + 1.645\sqrt{359.50} = 1504.84$$

$$N = 1505$$

**Question 3.9****Answer: E**

From the SULT, we have:

$${}_{25}p_{20} = \frac{l_{45}}{l_{20}} = \frac{99,033.9}{100,000.0} = 0.99034$$

$${}_{25}p_{45} = \frac{l_{70}}{l_{45}} = \frac{91,082.4}{99,033.9} = 0.91971$$

The expected number of survivors from the sons is 1980.68 with variance 19.133.

The expected number of survivors from fathers is 1839.42 with variance 147.687.

The total expected number of survivors is therefore 3820.10.

The standard deviation of the total expected number of survivors is therefore

$$\sqrt{19.133 + 147.687} = \sqrt{166.82} = 12.916$$

The 99<sup>th</sup> percentile equals  $3820.10 + (2.326)(12.916) = 3850$

**Question 3.10 DELETED**

**Question 3.11****Answer: B**

$${}_{2.5}q_{50} = {}_2q_{50} + {}_2P_{50} \cdot {}_{0.5}q_{52} = 0.02 + (0.98) \left( \frac{0.5}{2} \right) (0.04) = 0.0298$$

**Question 3.12****Answer: C**

$$\begin{aligned} 3.5P_{[61]} - 3.5P_{[60]+1} &= {}_{0.5}P_{64} ({}_3P_{[61]} - {}_3P_{[60]+1}) \\ &= \left( \frac{l_{65}}{l_{64}} \right)^{0.5} \left( \frac{l_{64}}{l_{[61]}} - \frac{l_{64}}{l_{[60]+1}} \right) \\ &= \left( \frac{4016}{5737} \right)^{0.5} \left( \frac{5737}{8654} - \frac{5737}{9600} \right) \\ &= 0.05466 \end{aligned}$$

**Question 3.13****Answer: B**

$$\begin{aligned} \overset{\circ}{e}_{[58]+2} &= e_{[58]+2} + 0.5 \\ e_{[58]+2} &= P_{[58]+2} (1 + e_{61}) = P_{[58]+2} \left[ 1 + \frac{e_{60}}{P_{60}} - 1 \right] \\ &= \frac{l_{61}}{l_{[58]+2}} \times \frac{e_{60}}{P_{60}} = \frac{2210}{3548} \times \frac{1}{(2210/3904)} = \frac{3904}{3549} = 1.100338 \\ \overset{\circ}{e}_{[58]+2} &= 1.100338 + 0.5 = 1.6 \end{aligned}$$

**Question 3.14****Answer: C**We need to determine  ${}_{3|2.5}q_{90}$ .

$${}_{3|2.5}q_{90} = \frac{l_{90+3} - l_{90+3+2.5}}{l_{90}} = \frac{l_{93} - l_{95.5}}{l_{90}} = \frac{l_{93} - (l_{95} - 0.5d_{95})}{l_{90}} = \frac{825 - [600 - 0.5(240)]}{1,000} = 0.3450$$

where  $l_{90} = 1,000$ ,  $l_{93} = 825$ ,  $l_{97} = \frac{d_{97}}{q_{97}} = \frac{72}{1} = 72$ ,  $l_{96} = \frac{l_{97}}{p_{96}} = \frac{72}{0.2} = 360$ ,

$$l_{95} = \frac{l_{96}}{p_{95}} = \frac{360}{1 - 0.4} = 600, \text{ and } d_{95} = l_{95} - l_{96} = 600 - 360 = 240.$$

**Question 3.15****Answer: A**

$$\mu_{25} = \frac{-s'(25)}{s(25)} = \frac{0.5(0.01)(1 - 0.01 \times 25)^{-0.5}}{(1 - 0.01 \times 25)^{0.5}} = 0.00667$$

**Question 3.16****Answer: C**

$[x]$	$q_{[x]}$	$q_{[x]+1}$	$q_{[x]+2}$	$q_{x+3}$	$x+3$
52	0.030	0.043	0.057	0.072	55

$$1000({}_{0.6|1.5}q_{[52]+1.7}) = 1000 \frac{l_{[52]+2.3} - l_{[52]+3.8}}{l_{[52]+1.7}}$$

$l_{[52]} = 1000$  (arbitrary, for convenience; any other value would work and give the same answer)

$$l_{[52]+1} = 1000(1 - 0.03) = 970$$

$$l_{[52]+2} = 970(1 - 0.043) = 928.29$$

$$l_{55} = 928.29(1 - 0.057) = 875.3775$$

$$l_{56} = 875.3775(1 - 0.072) = 812.3503$$

$$l_{[52]+2.3} = (0.7)l_{[52]+2} + (0.3)l_{55} = 912.42$$

$$l_{[52]+3.8} = l_{55.8} = (0.2)l_{55} + (0.8)l_{56} = 824.96$$

$$l_{[52]+1.7} = (0.3)l_{[52]+1} + (0.7)l_{[52]+2} = 940.80$$

$$1000({}_{0.6|1.5}q_{[52]+1.7}) = 1000 \frac{912.42 - 824.96}{940.80} = 92.96$$

**Question 3.17 DELETED**



### Question 4.1

**Answer: A**

$$E[Z] = 2 \cdot A_{40} - {}_{20}E_{40} A_{60} = (2)(0.36987) - (0.51276)(0.62567) = 0.41892$$

$$E[Z^2] = 0.24954 \text{ which is given in the problem.}$$

$$\text{Var}(Z) = E[Z^2] - (E[Z])^2 = 0.24954 - 0.41892^2 = 0.07405$$

$$SD(Z) = \sqrt{0.07405} = 0.27212$$

An alternative way to obtain the mean is  $E[Z] = 2A_{40:\overline{20}|}^1 + {}_{20}A_{40}$ . Had the problem asked for the evaluation of the second moment, a formula is

$$E[Z^2] = (2^2) \left( {}^2A_{40:\overline{20}|}^1 \right) + (v^2)^{20} ({}_{20}P_{40}) \left( {}^2A_{60} \right)$$

### Question 4.2

**Answer: D**

Half-year	PV of Benefit	
1	$300,000v^{0.5} = (300,000)(1.09)^{-1} = 275,229$	<i>PV</i> > 277,000 if and only if ( <i>x</i> ) dies in the 2 <sup>nd</sup> or 3 <sup>rd</sup> half years.
2	$330,000v^1 = (330,000)(1.09)^{-2} = 277,754$	
3	$360,000v^{1.5} = (360,000)(1.09)^{-3} = 277,986$	
4	$390,000v^2 = (390,000)(1.09)^{-4} = 276,286$	

Under CF assumption,  ${}_{0.5}p_x = {}_{0.5}p_{x+0.5} = (0.84)^{0.5} = 0.9165$  and

${}_{0.5}p_{x+1} = {}_{0.5}p_{x+1.5} = (0.77)^{0.5} = 0.8775$  Then the probability of dying in the 2<sup>nd</sup> or 3<sup>rd</sup> half-years is

$$\left( {}_{0.5}p_x \right) \left( 1 - {}_{0.5}p_{x+0.5} \right) + \left( p_x \right) \left( 1 - {}_{0.5}p_{x+1} \right) = (0.9165)(0.0835) + (0.84)(0.1225) = 0.1794$$

### Question 4.3

Answer: D

$$A_{60:\overline{3}|} = q_{60}v + (1 - q_{60})q_{60+1}v^2 + (1 - q_{60})(1 - q_{60+1})v^3 = 0.86545$$

$$q_{60+1} = \frac{A_{60:\overline{3}|} - q_{60}v - (1 - q_{60})v^3}{(1 - q_{60})v^2 - (1 - q_{60})v^3} = \frac{0.86545 - \frac{0.01}{1.05} - \frac{0.99}{1.05^3}}{\frac{0.99}{1.05^2} - \frac{0.99}{1.05^3}} = 0.017 \text{ when } v = 1/1.05.$$

The primes indicate calculations at 4.5% interest.

$$\begin{aligned} A'_{60:\overline{3}|} &= q_{60}v' + (1 - q_{60})q_{60+1}v'^2 + (1 - q_{60})(1 - q_{60+1})v'^3 \\ &= \frac{0.01}{1.045} + \frac{0.99(0.017)}{1.045^2} + \frac{0.99(0.983)}{1.045^3} \\ &= 0.87777 \end{aligned}$$

### Question 4.4

Answer: A

$$\text{Var}(Z) = E(Z^2) - E(Z)^2$$

$$\begin{aligned} E(Z) &= E\left[(1 + 0.2T)(1 + 0.2T)^{-2}\right] = E\left[(1 + 0.2T)^{-1}\right] \\ &= \int_0^{40} \frac{1}{(1 + 0.2t)} f_T(t) dt = \frac{1}{40} \int_0^{40} \frac{1}{1 + 0.2t} dt \\ &= \frac{1}{40} \frac{1}{0.2} \ln(1 + 0.2t) \Big|_0^{40} = \frac{1}{8} \ln(9) = 0.27465 \end{aligned}$$

$$\begin{aligned} E(Z^2) &= E\{(1 + 0.2T)^2 [(1 + 0.2T)^{-2}]^2\} = E[(1 + 0.2T)^{-2}] \\ &= \int_0^{40} \frac{1}{(1 + 0.2t)^2} f_T(t) dt = \frac{1}{40} \frac{1}{0.2} \left[ \frac{-1}{(1 + 0.2t)} \right]_0^{40} \\ &= \frac{1}{8} \left( 1 - \frac{1}{9} \right) = \frac{1}{9} = 0.11111 \end{aligned}$$

$$\text{Var}(Z) = 0.11111 - (0.27465)^2 = 0.03568$$

### Question 4.5

Answer: C

The earlier the death (before year 30), the larger the loss. Since we are looking for the 95<sup>th</sup> percentile of the present value of benefits random variable, we must find the time at which 5% of the insureds have died. The present value of the death benefit for that insured is what is being asked for.

$$l_{45} = 99,033.9 \Rightarrow 0.95l_{45} = 94,082.2$$

$$l_{65} = 94,579.7$$

$$l_{66} = 94,020.3$$

So, the time is between ages 65 and 66, i.e., time 20 and time 21.

$$l_{65} - l_{66} = 94,579.7 - 94,020.3 = 559.4$$

$$l_{65+t} - l_{66} = 94,579.7 - 94,082.2 = 497.5$$

$$497.5 / 559.4 = 0.8893$$

The time just before the last 5% of deaths is expected to occur is:  $20 + 0.8893 = 20.8893$

The present value of death benefits at this time is:

$$100,000e^{-20.8893(0.05)} = 35,188$$

### Question 4.6

Answer: B

Time	Age	$q_x^{SULT}$	Improvement factor	$q_x$
0	70	0.010413	100.00%	0.010413
1	71	0.011670	95.00%	0.011087
2	72	0.013081	90.25%	0.011806

$$v = 1/1.05 = 0.952381$$

$$EPV = 1,000[0.010413v + 0.989587(0.011087)v^2 + 0.989587(0.988913)(0.011806)v^3] \\ = 29.85$$

**Question 4.7****Answer: B**

$$\text{Var}(Z) = 0.10E[Z] \Rightarrow v^{50} {}_{25}p_x(1 - {}_{25}p_x) = 0.10 \cdot v^{25} {}_{25}p_x$$

$$\Rightarrow \frac{(1-0.57)}{(1+i)^{50}} = 0.10 \times \frac{1}{(1+i)^{25}}$$

$$\Rightarrow (1+i)^{25} = \frac{0.43}{0.10} = 4.3 \Rightarrow i = 0.06$$

**Question 4.8****Answer: C**

Let  $A_{51}^{SULT}$  designate  $A_{51}$  using the Standard Ultimate Life Table at 5%.

$$\begin{aligned} \text{APV (insurance)} &= 1000 \left( \frac{1}{1.04} \right) (q_{50} + p_{50} A_{51}^{SULT}) \\ &= 1000 \left( \frac{1}{1.04} \right) [0.001209 + (1 - 0.001209)(0.19780)] \\ &= 191.12 \end{aligned}$$

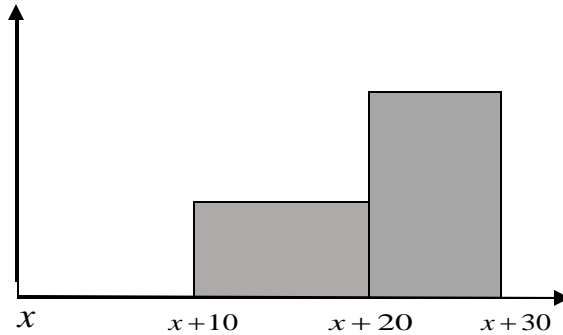
**Question 4.9****Answer: D**

$$\begin{aligned} A_{35} &= A_{35:\overline{15}|}^1 + A_{35:\overline{15}|}^{\frac{1}{2}} A_{50} \\ 0.32 &= 0.25 + 0.14 A_{50} \\ A_{50} &= \frac{0.07}{0.14} = 0.50 \end{aligned}$$

**Question 4.10**

**Answer: D**

Drawing the benefit payment pattern:



$$E[Z] = {}_{10}E_x \times \bar{A}_{x+10} + {}_{20}E_x \times \bar{A}_{x+20} - 2 {}_{30}E_x \times \bar{A}_{x+30}$$

**Question 4.11**

**Answer: A**

$$\begin{aligned} \text{Var}(Z_2) &= (1000)^2 \left[ {}^2A_{x:\overline{n}|} - \left( A_{x:\overline{n}|} \right)^2 \right] = 15,000 \\ &= (1000)^2 \left( {}^2A_{x:\overline{n}|}^1 + {}^2A_{x:\overline{n}|}^{\frac{1}{n}} \right) - (1000)^2 \left[ A_{x:\overline{n}|}^1 + A_{x:\overline{n}|}^{\frac{1}{n}} \right]^2 \\ &= (1000)^2 {}^2A_{x:\overline{n}|}^1 + (1000)^2 {}^2A_{x:\overline{n}|}^{\frac{1}{n}} - (1000)^2 \left( A_{x:\overline{n}|}^1 \right)^2 - (1000)^2 \left( A_{x:\overline{n}|}^{\frac{1}{n}} \right)^2 \\ &\quad - 2(1000)^2 \left( A_{x:\overline{n}|}^1 \right) \left( A_{x:\overline{n}|}^{\frac{1}{n}} \right) \\ &= (1000)^2 \left[ {}^2A_{x:\overline{n}|}^1 - \left( A_{x:\overline{n}|}^1 \right)^2 \right] + \left( 1000^2 {}^2A_{x:\overline{n}|}^{\frac{1}{n}} \right) - \left( 1000 A_{x:\overline{n}|}^{\frac{1}{n}} \right)^2 \\ &\quad - \left( 1000 A_{x:\overline{n}|}^1 \right)^2 - (2) \left( 1000 A_{x:\overline{n}|}^1 \right) \left( 1000 A_{x:\overline{n}|}^{\frac{1}{n}} \right) \\ &= V(Z_1) + (1000) \left( 1000 {}^2A_{x:\overline{n}|}^{\frac{1}{n}} \right) - \left( 1000 A_{x:\overline{n}|}^{\frac{1}{n}} \right)^2 - \left( 1000 A_{x:\overline{n}|}^1 \right)^2 \\ &\quad - (2) \left( 1000 A_{x:\overline{n}|}^1 \right) \left( 1000 A_{x:\overline{n}|}^{\frac{1}{n}} \right) \end{aligned}$$

$$15,000 = \text{Var}(Z_1) + (1000)(136) - (209)^2 - 2(528)(209)$$

$$\text{Therefore, } \text{Var}(Z_1) = 15,000 - 136,000 + 43,681 + 220,704 = 143,385.$$

**Question 4.12****Answer: C**

$$Z_3 = 2Z_1 + Z_2 \text{ so that } \text{Var}(Z_3) = 4\text{Var}(Z_1) + \text{Var}(Z_2) + 4\text{Cov}(Z_1, Z_2)$$

$$\text{where } \text{Cov}(Z_1, Z_2) = \underbrace{E[Z_1 Z_2]}_{=0} - E[Z_1]E[Z_2] = -(1.65)(10.75)$$

$$\begin{aligned} \text{Var}(Z_3) &= 4(46.75) + 50.78 - 4(1.65)(10.75) \\ &= 166.83 \end{aligned}$$

**Question 4.13****Answer: C**

$$\begin{aligned} {}_2|2A_{65} &= \underbrace{v^3}_{\text{payment year 3}} \underbrace{{}_2P_{[65]}}_{\text{Lives 2 years}} \times \underbrace{q_{[65]+2}}_{\text{Die year 3}} \\ &\quad + \underbrace{v^4}_{\text{payment year 4}} \underbrace{{}_3P_{[65]}}_{\text{Lives 3 years}} \times \underbrace{q_{65+3}}_{\text{Die year 4}} \\ &= \left(\frac{1}{1.04}\right)^3 (0.92)(0.9)(0.12) \\ &\quad + \left(\frac{1}{1.04}\right)^4 (0.92)(0.9)(0.88)(0.14) \\ &= 0.088 + 0.087 = 0.176 \end{aligned}$$

The actuarial present value of this insurance is therefore  $2000 \times 0.176 = 352$ .

**Question 4.14****Answer: E**

Out of 400 lives initially, we expect  $400 {}_{25}p_{60} = 400 \frac{l_{85}}{l_{60}} = 400 \left( \frac{61,184.9}{96,634.1} \right) = 253.26$  survivors

The standard deviation of the number of survivors is  $\sqrt{400 {}_{25}p_{60} (1 - {}_{25}p_{60})} = 9.639$

To ensure 86% funding, using the normal distribution table, we plan for  $253.26 + 1.08(9.639) = 263.67$

The initial fund must therefore be  $F = (264)(5000) \left( \frac{1}{1.05} \right)^{25} = 389,800$ .

**Question 4.15****Answer: E**

$$E[Z] = \int_0^{\infty} b_t \cdot v^t \cdot {}_t p_x \cdot \mu_{x+t} dt = \int_0^{\infty} e^{0.02t} \cdot e^{-0.06t} \cdot e^{-0.04t} \cdot 0.04 dt$$

$$= 0.04 \int_0^{\infty} e^{-0.08t} dt = \frac{0.04}{0.08} = \frac{1}{2}$$

$$E[Z^2] = \int_0^{\infty} (b_t \cdot v^t)^2 \cdot {}_t p_x \cdot \mu_{x+t} dt = \int_0^{\infty} (e^{0.04t}) (e^{-0.12t}) (0.04 e^{-0.04t}) dt = \frac{0.04}{0.12} = \frac{1}{3}$$

$$Var[Z] = \frac{1}{3} - \left( \frac{1}{2} \right)^2 = \frac{1}{12} = 0.0833$$

**Question 4.16****Answer: D**

$$A_{[50]:\overline{3}|}^1 = vq_{[50]} + v^2 p_{[50]} q_{[50]+1} + v^3 p_{[50]} p_{[50]+1} q_{52}$$

$$\text{where: } v = \frac{1}{1.04}$$

$$q_{[50]} = 0.7(0.045) = 0.0315$$

$$p_{[50]} = 1 - q_{[50]} = 0.9685$$

$$q_{[50]+1} = 0.8(0.050) = 0.040$$

$$p_{[50]+1} = 1 - q_{[50]+1} = 0.960$$

$$q_{52} = 0.055$$

$$\text{So: } A_{[50]:\overline{3}|}^1 = 0.1116$$

**Question 4.17****Answer: A**

The median of  $K_{48}$  is the integer  $m$  for which

$$P(K_{48} < m) \leq 0.5 \text{ and } P(K_{48} > m) \leq 0.5.$$

This is equivalent to finding  $m$  for which

$$\frac{l_{48+m}}{l_{48}} \geq 0.5 \text{ and } \frac{l_{48+m+1}}{l_{48}} \leq 0.5.$$

Based on the SULT and  $l_{48}(0.5) = (98,783.9)(0.5) = 49,391.95$ , we have  $m = 40$  since

$$l_{88} \geq 49,391.95 \text{ and } l_{89} \leq 49,391.95.$$

$$\begin{aligned} \text{So: } APV &= 5000A_{48} + 5000 {}_{40}E_{48} A_{88} = 5000A_{48} + 5000 {}_{20}E_{48} {}_{20}E_{68} \cdot A_{88} \\ &= 5000(0.17330) + 5000(0.35370)(0.20343)(0.72349) = 1126.79 \end{aligned}$$



**Question 4.18****Answer: A**

The present value random variable  $PV = 1,000,000e^{-0.05T}$ ,  $2 \leq T \leq 10$  is a decreasing function of  $T$  so that its 90<sup>th</sup> percentile is

$$1,000,000e^{-0.05p} \text{ where } p \text{ is the solution to } \int_2^p 0.4t^{-2} dt = 0.10.$$

$$\int_2^p 0.4t^{-2} dt = -0.4 \left( \frac{t^{-1}}{-1} \right) \Big|_2^p = 0.4 \left( \frac{1}{2} - \frac{1}{p} \right) = 0.10$$

$$p = 4$$

$$1,000,000e^{-0.05 \times 4} = 81,873.08$$

**Question 4.19****Answer: B**

Superscript SULT refers to values from the SULT. Values without superscripts refer to this select life.

$$q_{80} = 0.8q_{80}^{SULT} = 0.0261264 \Rightarrow p_{80} = 0.9738736$$

$$A_{80} = vq_{80} + vp_{80}A_{81}^{SULT}$$

$$= (1.05)^{-1}(0.0261264) + (1.05)^{-1}(0.9738736)(0.60984) = 0.59051$$

$$100,000A_{80} = 59,051$$

**Question 4.20****Answer: D**

$$\begin{aligned} E(Z) &= \int_0^{\infty} {}_t p_x \times \mu \times e^{0.05 \times t} e^{-\delta \times t} dt \\ &= \int_0^{\infty} e^{-0.03 \times t} \times 0.03 \times e^{0.05 \times t} e^{-0.06 \times t} dt \\ &= \frac{0.03}{0.04} \times e^{-0.04t} \Big|_0^{\infty} = 0.75 \end{aligned}$$

$$\begin{aligned} E(Z^2) &= \int_0^{\infty} {}_t p_x \times \mu \times e^{0.05 \times 2 \times t} e^{-\delta \times 2 \times t} dt \\ &= \int_0^{\infty} e^{-0.03 \times t} \times 0.03 \times e^{0.05 \times 2 \times t} e^{-0.06 \times 2 \times t} dt \\ &= \frac{0.03}{0.04} \times e^{-0.05t} \Big|_0^{\infty} = 0.6 \end{aligned}$$

$$\text{Var}(Z) = E(Z^2) - [E(Z)]^2 = 0.6 - 0.75^2 = 0.375$$

**Question 4.21****Answer: E**

$$E(Z) = \frac{i}{\delta} [vq_x + v^2 p_x q_{x+1}] = \frac{0.04}{\ln(1.04)} \left[ \frac{1}{1.04} (0.04) + \frac{1}{1.04^2} (0.96)(0.06) \right] = 0.09353831$$

$$E(Z^2) = \frac{(1+i)^2 - 1}{2\delta} [v^2 q_x + v^4 p_x q_{x+1}] = \frac{(1.04^2 - 1)}{2\ln(1.04)} \left[ \frac{1}{1.04^2} (0.04) + \frac{1}{1.04^4} (0.96)(0.06) \right] = 0.08969072$$

$$\text{Var}(Z) = E(Z^2) - [E(Z)]^2 = 0.08969072 - (0.09353831)^2 = 0.0809413$$

**Question 4.22****Answer: A**

Probability (50) survives one year under Standard Ultimate Life Table =  $1 - 0.001209 = 0.998791$

Probability (50) survives one year following surgery =  $0.55 \times 0.998791 = 0.5493 = p_{50}$

$$q_{50} = 1 - p_{50} = 0.4507$$

$$\begin{aligned} A_{50:\overline{3}|} &= q_{50} \left( \frac{1}{1.05} \right) + p_{50} q_{51} \left( \frac{1}{1.05^2} \right) + p_{50} p_{51} q_{52} \left( \frac{1}{1.05^3} \right) \\ &= (0.4507) \left( \frac{1}{1.05} \right) + (0.5493)(0.001331) \left( \frac{1}{1.05^2} \right) + (0.5493)(0.99867)(0.001469) \left( \frac{1}{1.05^3} \right) \\ &= 0.4306 \end{aligned}$$

Therefore, answer =  $100,000 \times 0.4306 \approx 43,000$  (A)

**Question 5.1****Answer: A**

$$E(Y) = \bar{a}_{\overline{10}|} + e^{-\delta(10)} e^{-\mu(10)} \bar{a}_{x+10}$$

$$= \frac{(1 - e^{-0.6})}{0.06} + e^{-0.7} \frac{1}{0.07}$$

$$= 14.6139$$

$$Y > E(Y) \Rightarrow \left( \frac{1 - e^{-0.06T}}{0.06} \right) > 14.6139$$

$$\Rightarrow T > 34.90$$

$$\Pr[Y > E(Y)] = \Pr(T > 34.90) = e^{-34.90(0.01)} = 0.705$$

### Question 5.2

Answer: B

$$A_{x:\overline{n}|}^1 = {}_n E_x$$

$$A_x = A_{x:\overline{n}|}^1 + {}_n E_x A_{x+n}$$

$$0.3 = A_{x:\overline{n}|}^1 + (0.35)(0.4) \Rightarrow A_{x:\overline{n}|}^1 = 0.16$$

$$A_{x:\overline{n}|} = A_{x:\overline{n}|}^1 + {}_n E_x = 0.16 + 0.35 = 0.51$$

$$\ddot{a}_{x:\overline{n}|} = \frac{1 - A_{x:\overline{n}|}}{d} = \frac{1 - 0.51}{(0.05/1.05)} = 10.29$$

$$a_{x:\overline{n}|} = \ddot{a}_{x:\overline{n}|} - 1 + {}_n E_x = 10.29 - 0.65 = 9.64$$

### Question 5.3

Answer: C

$$(\overline{I\bar{a}})_{40:\overline{t}|} = \int_0^t s {}_s p_{40} v^s ds \Rightarrow \frac{d(\overline{I\bar{a}})_{40:\overline{t}|}}{dt} = t {}_t p_{40} v^t$$

At  $t = 10.5$ ,

$$\begin{aligned} 10.5 {}_{10.5} E_{40} &= 10.5 {}_{10} P_{40} {}_{0.5} P_{50} v^{10.5} \\ &= 10.5 {}_{10} E_{40} {}_{0.5} P_{50} v^{0.5} \\ &= 10.5 \times 0.60920 \times (1 - 0.5 \times 0.001209)(0.975900073) \\ &= 6.239 \end{aligned}$$

### Question 5.4

Answer: A

$\overset{\circ}{e}_{40} = \frac{1}{\mu} = 50$  So, receive  $K$  for 50 years guaranteed and for life thereafter.

$$10,000 = K \left[ \bar{a}_{50|} + {}_{50|}\bar{a}_{40} \right]$$

$$\bar{a}_{50|} = \int_0^{50} e^{-\delta t} dt = \frac{1 - e^{-50\delta}}{\delta} = \frac{1 - e^{-50(0.01)}}{0.01} = 39.35$$

$${}_{50|}\bar{a}_{40} = {}_{50}E_{40} \bar{a}_{40+50} = e^{-(\delta+\mu)50} \frac{1}{\mu + \delta} = e^{-1.5} \frac{1}{0.03} = 7.44$$

$$K = \frac{10,000}{39.35 + 7.44} = 213.7$$

### Question 5.5

Answer: A

$$\ddot{a}_{45}^S = 1 + \nu p_{45}^S \ddot{a}_{46}^{SULT}$$

$$p_{45}^S = e^{-\int_0^1 \mu_{45+t}^S dt} = e^{-\int_0^1 (\mu_{45+t}^{SULT} + 0.05) dt} = e^{-\int_0^1 \mu_{45+t}^{SULT} dt} e^{-\int_0^1 (0.05) dt} = p_{45}^{SULT} \cdot e^{-0.05} = \left( \frac{98,957.6}{99,033.9} \right) e^{-0.05} = 0.9504966$$

$$\ddot{a}_{45}^S = 1 + \nu p_{45}^S \ddot{a}_{46}^{SULT} = 1 + (1.05)^{-1} (0.9504966)(17.6706) = 17.00$$

$$100 \ddot{a}_{45}^S = 1700$$

**Question 5.6****Answer: D**

Let  $Y_i$  be the present value random variable of the payment to life  $i$ .

$$E[Y_i] = \ddot{a}_x = \frac{1 - A_x}{d} = 11.55 \quad \text{Var}[Y_i] = \frac{{}^2A_x - (A_x)^2}{d^2} = \frac{0.22 - 0.45^2}{(0.05/1.05)^2} = 7.7175$$

Then  $Y = \sum_{i=1}^{100} Y_i$  is the present value of the aggregate payments.

$$E[Y] = 100E[Y_i] = 1155 \quad \text{and} \quad \text{Var}[Y] = 100\text{Var}[Y_i] = 771.75$$

$$\Pr[Y \leq F] = \Pr\left[Z \leq \frac{F - 1155}{\sqrt{771.75}}\right] = 0.95 \Rightarrow \frac{F - 1155}{\sqrt{771.75}} = 1.645$$

$$\Rightarrow F = 1155 + 1.645\sqrt{771.75} = 1200.699$$

**Question 5.7****Answer: C**

$$\ddot{a}_{35:\overline{30}|}^{(2)} \approx \ddot{a}_{35:\overline{30}|} - \frac{(m-1)}{2m} (1 - v^{30} {}_{30}P_{35})$$

$$\begin{aligned} \ddot{a}_{35:\overline{30}|} &= \frac{1 - A_{35:\overline{30}|}}{d} = \frac{1 - A_{35:\overline{30}|}^1 - {}_{30}E_{35}}{d} \\ &= \frac{1 - (A_{35} - {}_{30}E_{35} \times A_{65}) - {}_{30}E_{35}}{d} \end{aligned}$$

Since  ${}_{30}E_{35} = v^{30} {}_{30}P_{35} = 0.2722$ , then

$$\begin{aligned} \ddot{a}_{35:\overline{30}|} &= \frac{1 - (A_{35} - v^{30} {}_{30}P_{35} \times A_{65}) - v^{30} {}_{30}P_{35}}{d} \\ &= \frac{1 - (0.188 - (0.2722)(0.498)) - 0.2722}{(0.04/1.04)} \\ &= 17.5592 \end{aligned}$$

$$\ddot{a}_{35:\overline{30}|}^{(2)} \approx 17.5592 - \frac{1}{4} (1 - 0.2722) = 17.38$$

$$1000\ddot{a}_{35:\overline{30}|}^{(2)} \approx 1000 \times 17.38 = 17,380$$

**Question 5.8****Answer: C**

The expected present value is:

$$\ddot{a}_{\overline{5}|} + {}_5E_{55}\ddot{a}_{60} = 4.54595 + 0.77382 \times 14.9041 = 16.07904$$

The probability that the sum of the undiscounted payments will exceed the expected present value is the probability that at least 17 payments will be made. This will occur if (55) survives to age 71. The probability is therefore:

$${}_{16}P_{55} = \frac{l_{71}}{l_{55}} = \frac{90,134.0}{97,846.2} = 0.92118$$

**Question 5.9****Answer: C**

$$\ddot{a}_{[x]:\overline{n}|} = 1 + vp_{[x]}\ddot{a}_{x+1:\overline{n-1}|} = 1 + (1+k)\left(vp_x\ddot{a}_{x+1:\overline{n-1}|}\right) = 1 + (1+k)\left(\ddot{a}_{x:\overline{n}|} - 1\right)$$

Therefore, we have

$$k = \frac{\ddot{a}_{[x]:\overline{n}|} - 1}{\ddot{a}_{x:\overline{n}|} - 1} - 1 = \frac{21.167}{20.854} - 1 = 0.015$$

**Question 5.10****Answer: C**

$$\ddot{a}_{\overline{65:10}|} = \ddot{a}_{\overline{10}|} + {}_{10}E_{65} \times \ddot{a}_{75} = 8.10782 + 0.55305 \times 10.3178 = 13.8141$$

Assuming payments of 1 (any other payment amount would just cancel, giving the same number of years), 14 payments are needed for the sum of payments to exceed 13.8141. That requires surviving to the start of year 14, thus surviving 13 years.

$${}_{13}P_{65} = \frac{l_{78}}{l_{65}} = \frac{80,006.2}{94,579.7} = 0.846$$

**Question 5.11****Answer: B**

Discounted Payment	Probability
10	$(1-0.95) = 0.05$
$10+10v = 19.3$	$(0.95)(1-0.9) = 0.095$
$10+10v+10v^2=27.949$	$(0.95)(0.9) = 0.855$

$$\text{Var}(X) = E(X^2) - [E(X)]^2$$

$$E(X) = 10(1-0.95) + [10+10v](0.95)(1-0.9) + [10+10v+10v^2](0.95)(0.9) = 26.23$$

$$E(X^2) = 10^2(1-0.95) + [10+10v]^2(0.95)(1-0.9) + [10+10v+10v^2]^2(0.95)(0.9) = 708.27$$

$$\text{Var}(X) = 708.27 - (26.23)^2 = 20.26$$

$$\text{Standard Deviation} = (20.26)^{1/2} = 4.5$$

It's not significantly less work, but since everyone receives the first 10, you could use the formula  $\text{Var}(X-\text{constant}) = \text{Var}(X)$ , and work with the three payment amounts as 0, 10v, 10v+10v<sup>2</sup>, and you would get the same answer.

**Question 5.12****Answer: E**

Any of these ways of viewing the benefit structure would be fine; all give the same answer:

- (i) A temporary life annuity-due of  $X$  plus a deferred life annuity-due of  $0.75X$ .
- (ii) A whole life annuity-due of  $0.75X$  plus a temporary deferred life annuity-due of  $0.25X$
- (iii) A whole life annuity-due of  $X$  minus a deferred life annuity-due of  $0.25X$

This solution views it the first way.

$$X\ddot{a}_{55:\overline{10}|} + 0.75X\ddot{a}_{65} \cdot {}_{10}E_{55} = 8.0192X + (0.75X)(0.59342)(13.5498) = 14.05X = 250,000$$

$$\Rightarrow X = 17,794$$

### Question 6.1

**Answer: D**

The equation of value is given by

Actuarial Present Value of Premiums = Actuarial Present Value of Death Benefits.

The death benefit in the first year is  $1000 + P$ . The death benefit in the second year is  $1000 + 2P$ .

The formula is  $P\ddot{a}_{80:\overline{2}|} = 1000A_{80:\overline{2}|}^1 + P(IA)_{80:\overline{2}|}^1$ .

Solving for P we obtain  $P = \frac{1000A_{80:\overline{2}|}^1}{\ddot{a}_{80:\overline{2}|} - (IA)_{80:\overline{2}|}^1}$ .

$$\ddot{a}_{80:\overline{2}|} = 1 + p_{80}v = 1 + \frac{0.967342}{1.03} = 1.93917$$

$$1000A_{80:\overline{2}|}^1 = 1000(vq_{80} + v^2p_{80}q_{81}) = 1000\left(\frac{0.032658}{1.03} + \frac{(0.967342)(0.036607)}{1.03^2}\right) = 65.08552$$

$$(IA)_{80:\overline{2}|}^1 = vq_{80} + 2v^2p_{80}q_{81} = \frac{0.032658}{1.03} + (2)\frac{(0.967342)(0.036607)}{1.03^2} = 0.09846$$

$$P = \frac{65.08552}{1.93917 - 0.09846} = 35.36 \rightarrow D$$

### Question 6.2

**Answer: E**

$$G\ddot{a}_{x:\overline{10}|} = 100,000A_{x:\overline{10}|}^1 + G(IA)_{x:\overline{10}|}^1 + 0.45G + 0.05G\ddot{a}_{x:\overline{10}|} + 200\ddot{a}_{x:\overline{10}|}$$

$$G = \frac{(100,000)(0.17094) + 200(6.8865)}{(1 - 0.05)(6.8865) - 0.96728 - 0.45} = 3604.23$$



### Question 6.3

Answer: C

Let  $C$  be the annual contribution, then  $C = \frac{{}_{20}E_{45} \ddot{a}_{65}}{\ddot{a}_{45:\overline{20}|}}$

Let  $K_{65}$  be the curtate future lifetime of (65). The required probability is

$$\Pr\left(\frac{C \ddot{a}_{45:\overline{20}|}}{{}_{20}E_{45}} > \ddot{a}_{\overline{K_{65}+1}|}\right) = \Pr\left(\frac{{}_{20}E_{45} \ddot{a}_{65} \ddot{a}_{45:\overline{20}|}}{\ddot{a}_{45:\overline{20}|} {}_{20}E_{45}} > \ddot{a}_{\overline{K_{65}+1}|}\right) = \Pr\left(\ddot{a}_{65} > \ddot{a}_{\overline{K_{65}+1}|}\right) = \Pr\left(13.5498 > \ddot{a}_{\overline{K_{65}+1}|}\right)$$

Thus, since  $\ddot{a}_{\overline{21}|} = 13.4622$  and  $\ddot{a}_{\overline{22}|} = 13.8212$  we have

$$\Pr\left(\ddot{a}_{\overline{K_{65}+1}|} < 13.5498\right) = \Pr\left(K_{65} + 1 \leq 21\right) = 1 - {}_{21}p_{65} = 1 - \frac{l_{86}}{l_{65}} = 1 - \frac{57,656.7}{94,579.7} = 0.390$$

### Question 6.4

Answer: E

Let  $X_i$  be the present value of a life annuity of 1/12 per month on life  $i$  for  $i = 1, 2, \dots, 200$ .

Let  $S = \sum_{i=1}^{200} X_i$  be the present value of all the annuity payments.

$$E[X_i] = \ddot{a}_{62}^{(12)} = \frac{1 - A_{62}^{(12)}}{d^{(12)}} = \frac{1 - 0.4075}{0.05813} = 10.19267$$

$$\text{Var}(X_i) = \frac{{}^2A_{62}^{(12)} - \left(A_{62}^{(12)}\right)^2}{\left(d^{(12)}\right)^2} = \frac{0.2105 - (0.4075)^2}{(0.05813)^2} = 13.15255$$

$$E[S] = (200)(180)(10.19267) = 366,936.12$$

$$\text{Var}(S) = (200)(180)^2(13.15255) = 85,228,524$$

With the normal approximation, for  $\Pr(S \leq M) = 0.90$

$$M = E[S] + 1.282\sqrt{\text{Var}(S)} = 366,936.12 + 1.282\sqrt{85,228,524} = 378,771.45$$

$$\text{So } \pi = \frac{378,771.45}{200} = 1893.86$$

### Question 6.5

**Answer: D**

Let  $k$  be the policy year, so that the mortality rate during that year is  $q_{30+k-1}$ . The objective is to determine the smallest value of  $k$  such that

$$v^{k-1}({}_{k-1}P_{30})(1000P_{30}) < v^k({}_{k-1}P_{30})q_{30+k-1}(1000)$$

$$P_{30} < vq_{30+k-1}$$

$$\frac{0.07698}{19.3834} < \frac{q_{29+k}}{1.05}$$

$$q_{29+k} > 0.00417$$

$$29 + k > 61 \Rightarrow k > 32$$

Therefore, the smallest value that meets the condition is 33.

### Question 6.6

**Answer: B**

$$\text{Net Premium} = 10,000A_{62} / \ddot{a}_{62} = 10,000(0.31495) / 14.3861 = 218.93$$

$$G = 218.93(1.03) = 225.50$$

Let  ${}_0L^*$  be the present value of future loss at issue for one policy.

$$\begin{aligned} {}_0L^* &= 10,000v^{K+1} - (G - 5)\ddot{a}_{\overline{K+1}|} + 0.05G \\ &= 10,000v^{K+1} - (225.50 - 5)\frac{1 - v^{K+1}}{d} + 0.05(225.50) \\ &= (10,000 + 4630.50)v^{K+1} - 4630.50 + 11.28 \\ &= 14,630.50v^{K+1} - 4619.22 \end{aligned}$$

$$E({}_0L^*) = 14,630.50A_{62} - 4619.22 = 14,630.50(0.31495) - 4619.22 = -11.34$$

$$\text{Var}({}_0L^*) = (14,630.50)^2 ({}^2A_{62} - A_{62}^2) = (14,630.50)^2 (0.12506 - 0.31495^2) = 5,536,763$$

Let  ${}_0L$  be the aggregate loss for 600 such policies.

$$E({}_0L) = 600E({}_0L^*) = 600(-11.34) = -6804$$

$$\text{Var}({}_0L) = 600\text{Var}({}_0L^*) = 600(5,536,763) = 3,322,057,800$$

$$\text{StdDev}({}_0L) = 3,322,057,800^{0.5} = 57,637$$

$$\Pr({}_0L < 40,000) = \Phi\left(\frac{40,000 + 6804}{57,637}\right) = \Phi(0.81) = 0.7910$$

### Question 6.7

**Answer: C**

There are four ways to approach this problem. In all cases, let  $\pi$  denote the net premium.

The first approach is an intuitive result. The key is that in addition to the pure endowment, there is a benefit equal in value to a temporary interest only annuity due with annual payment  $\pi$ . However, if the insured survives the 20 years, the value of the annuity is not received.

$$\pi \ddot{a}_{40:\overline{20}|} = 100,000 {}_{20}E_{40} + \pi \ddot{a}_{40:\overline{20}|} - {}_{20}P_{40} \ddot{a}_{\overline{20}|5\%} \pi$$

Based on this equation,

$$\pi = \frac{100,000 {}_{20}E_{40}}{{}_{20}P_{40} \ddot{a}_{\overline{20}|}} = \frac{100,000 v^{20}}{\ddot{a}_{\overline{20}|}} = \frac{100,000}{{\ddot{s}}_{\overline{20}|}} = \frac{100,000}{34.71925} = 2880$$

The second approach is also intuitive. If you set an equation of value at the end of 20 years, the present value of benefits is 100,000 for all the people who are alive at that time. The people who have died have had their premiums returned with interest. Therefore, the premiums plus interest that the company has are only the premiums for those alive at the end of 20 years. The people who are alive have paid 20 premiums. Therefore  $\pi {\ddot{s}}_{\overline{20}|} = 100,000$ .

The third approach uses random variables to derive the expected present value of the return of premium benefit. Let  $K$  be the curtate future lifetime of (40). The present value random variable is then

$$\begin{aligned} Y &= \begin{cases} \pi {\ddot{s}}_{\overline{K+1}|} v^{K+1}, & K < 20 \\ 0, & K \geq 20 \end{cases} \\ &= \begin{cases} \pi \ddot{a}_{\overline{K+1}|}, & K < 20 \\ 0, & K \geq 20 \end{cases} \\ &= \begin{cases} \pi \ddot{a}_{\overline{K+1}|} - 0, & K < 20 \\ \pi \ddot{a}_{\overline{20}|} - \pi \ddot{a}_{\overline{20}|}, & K \geq 20 \end{cases} \end{aligned}$$

The first term is the random variable that corresponds to a 20-year temporary annuity. The second term is the random variable that corresponds to a payment with a present value of  $\pi \ddot{a}_{\overline{20}|}$  contingent on surviving 20 years. The expected present value is then  $\pi \ddot{a}_{40:\overline{20}|} - {}_{20}P_{40} \ddot{a}_{\overline{20}|} \pi$ .

The fourth approach takes the most steps.

$$\begin{aligned}
 \pi \ddot{a}_{40:\overline{20}|} &= 100,000 {}_{20}E_{40} + \pi \sum_{k=0}^{19} v^{k+1} \ddot{s}_{\overline{k+1}|} q_{40} = 100,000 {}_{20}E_{40} + \pi \sum_{k=0}^{19} v^{k+1} \frac{(1+i)^{k+1} - 1}{d} q_{40} \\
 &= 100,000 {}_{20}E_{40} + \frac{\pi}{d} \left( \sum_{k=0}^{19} q_{40} - v^{k+1} q_{40} \right) = 100,000 {}_{20}E_{40} + \frac{\pi}{d} \left( {}_{20}q_{40} - A_{40:\overline{20}|}^1 \right) \\
 &= 100,000 {}_{20}E_{40} + \frac{\pi}{d} \left( {}_{20}q_{40} - 1 + d \ddot{a}_{40:\overline{20}|} + v^{20} {}_{20}p_{40} \right) \\
 &= 100,000 {}_{20}E_{40} + \pi \ddot{a}_{40:\overline{20}|} - \pi {}_{20}p_{40} \frac{1-v^{20}}{d} \\
 &= 100,000 {}_{20}E_{40} + \pi \ddot{a}_{40:\overline{20}|} - {}_{20}p_{40} \ddot{a}_{\overline{20}|6\%} \pi.
 \end{aligned}$$

### Question 6.8

**Answer: B**

$$\ddot{a}_{60:\overline{10}|} = 7.9555$$

$$\ddot{a}_{60:\overline{20}|} = 12.3816$$

$$\text{Annual level amount} = \frac{40 + 5\ddot{a}_{60:\overline{10}|} + 5\ddot{a}_{60:\overline{20}|}}{\ddot{a}_{60}} = \frac{141.686}{14.9041} = 9.51$$

### Question 6.9

**Answer: D**

$$\ddot{a}_{50:\overline{10}|} = 8.0550$$

$$A_{50:\overline{20}|}^1 = A_{50:\overline{20}|} - {}_{20}E_{50} = 0.38844 - 0.34824 = 0.04020$$

$$\ddot{a}_{50:\overline{20}|} = 12.8428$$

APV of Premiums = APV Death Benefit + APV Commission and Taxes + APV Maintenance

$$G \ddot{a}_{50:\overline{10}|} = 100,000 A_{50:\overline{20}|}^1 + 0.12G \ddot{a}_{50:\overline{10}|} + 0.3G + 25\ddot{a}_{50:\overline{20}|} + 50$$

$$8.0550G = 4020 + 1.2666G + 371.07$$

$$6.7883G = 4391.07$$

$$\Rightarrow G = 646.86$$

**Question 6.10****Answer: D**

$$\ddot{a}_{x:\overline{3}|} = \frac{\text{Actuarial PV of the benefit}}{\text{Level Annual Premium}} = \frac{152.85}{56.05} = 2.727$$

$$\ddot{a}_{x:\overline{3}|} = 1 + \frac{0.975}{1.06} + \frac{0.975(p_{x+1})}{(1.06)^2} = 2.727$$

$$\Rightarrow p_{x+1} = 0.93$$

Actuarial PV of the benefit =

$$152.85 = 1,000 \left[ \frac{0.025}{1.06} + \frac{0.975(1-0.93)}{(1.06)^2} + \frac{0.975(0.93)(q_{x+2})}{(1.06)^3} \right]$$

$$\Rightarrow q_{x+2} = 0.09 \Rightarrow p_{x+2} = 0.91$$

**Question 6.11****Answer: C**For calculating  $P$ 

$$A_{50} = vq_{50} + vp_{50}A_{51} = v(0.0048) + v(1-0.0048)(0.39788) = 0.38536$$

$$\ddot{a}_{50} = (1 - A_{50}) / d = 15.981$$

$$P = A_{50} / \ddot{a}_{50} = 0.02411$$

For this particular life,

$$A'_{50} = vq'_{50} + vp'_{50}A_{51} = v(0.048) + (1-0.048)(0.39788) = 0.41037$$

$$\ddot{a}'_{50} = (1 - A'_{50}) / d = 15.330$$

$$\text{Expected PV of loss} = A'_{50} - P\ddot{a}'_{50} = 0.41037 - 0.02411(15.330) = 0.0408$$

**Question 6.12****Answer: E**

1,020 in the solution is the 1,000 death benefit plus the 20 death benefit claim expense.

$$A_x = 1 - d\ddot{a}_x = 1 - d(12.0) = 0.320755$$

$$G\ddot{a}_x = 1,020A_x + 0.65G + 0.10G\ddot{a}_x + 8 + 2\ddot{a}_x$$

$$G = \frac{1,020A_x + 8 + 2\ddot{a}_x}{\ddot{a}_x - 0.65 - 0.10\ddot{a}_x} = \frac{1,020(0.320755) + 8 + 2(12.0)}{12.0 - 0.65 - 0.10(12.0)} = 35.38622$$

Let  $Z = v^{K_x+1}$  denote the present value random variable for a whole life insurance of 1 on  $(x)$ .

Let  $Y = \ddot{a}_{\overline{K_x+1}|}$  denote the present value random variable for a life annuity-due of 1 on  $(x)$ .

$$\begin{aligned} L &= 1,020Z + 0.65G + 0.10GY + 8 + 2Y - GY \\ &= 1,020Z + (2 - 0.9G)Y + 0.65G + 8 \\ &= 1,020v^{K_x+1} + (2 - 0.9G)\frac{1 - v^{K_x+1}}{d} + 0.65G + 8 \\ &= \left(1,020 + \frac{0.9G - 2}{d}\right)v^{K_x+1} + \frac{2 - 0.9G}{d} + 0.65G + 8 \end{aligned}$$

$$\begin{aligned} \text{Var}(L) &= \left[ {}^2A_x - (A_x)^2 \right] \left( 1,020 + \frac{0.9G - 2}{d} \right)^2 \\ &= (0.14 - 0.320755^2) \left( 1,020 + \frac{0.9(35.38622) - 2}{d} \right)^2 \\ &= 0.037116(2,394,161) \\ &= 88,861 \end{aligned}$$

**Question 6.13****Answer: D**

If  $T_{45} = 10.5$ , then  $K_{45} = 10$  and  $K_{45} + 1 = 11$ .

$${}_0L = 10,000v^{K_{45}+1} - G(1-0.10)\ddot{a}_{\overline{K_{45}+1}|} + G(0.80-0.10) = 10,000v^{11} - 0.9G\ddot{a}_{\overline{11}|} + 0.7G$$

$$4953 = 10,000(0.58468) - 0.9G(8.72173) + 0.7G$$

$$G = (5846.8 - 4953) / (7.14956) = 125.01$$

$$\begin{aligned} E({}_0L) &= 10,000A_{45} - (1-0.1)G\ddot{a}_{45} + (0.8-0.1)G \\ &= (10,000)(0.15161) - (0.9)(125.01)(17.8162) + (0.7)(125.01) \end{aligned}$$

$$E({}_0L) = -400.87$$

**Question 6.14****Answer: D**

$$100,000A_{40} = P[\ddot{a}_{40:\overline{10}|} + 0.5 {}_{10|}\ddot{a}_{40:\overline{10}|}]$$

$$P = \frac{100,000A_{40}}{\ddot{a}_{40:\overline{10}|} + 0.5 {}_{10|}\ddot{a}_{40:\overline{10}|}} = \frac{100,000(0.12106)}{8.0863 + 0.5(4.9071)} = \frac{12,106}{10.53985} = 1148.59$$

where

$${}_{10|}\ddot{a}_{40:\overline{10}|} = {}_{10}E_{40} \left[ \ddot{a}_{50:\overline{10}|} \right] = 0.60920[8.0550] = 4.9071$$

There are several other ways to write the right-hand side of the first equation.

### Question 6.15

Answer: B

$$\text{Woolhouse: } {}^W\ddot{a}_x^{(4)} = 3.4611 - \frac{3}{8} = 3.0861$$

$${}^{UDD}\ddot{a}_x^{(4)} = \alpha(4)\ddot{a}_x - \beta(4)$$

$$\begin{aligned}\text{UDD: } &= 1.00019(3.4611) - 0.38272 \\ &= 3.0790\end{aligned}$$

$$\text{and } A_x = 1 - d\ddot{a}_x = 1 - (0.04762)(3.4611) = 0.83518$$

$$P^{(W)} = \frac{1000(0.83518)}{3.0861} = 270.63$$

$$P^{(UDD)} = \frac{1000(0.83518)}{3.0790} = 271.25$$

$$\frac{P^{(UDD)}}{P^{(W)}} = \frac{271.25}{270.63} = 1.0023$$

### Question 6.16

Answer: A

$$P_{\overline{30:20}|} = \frac{1}{\ddot{a}_{\overline{30:20}|}} - d \Rightarrow \frac{2,143}{100,000} + 0.05 = \frac{1}{\ddot{a}_{\overline{30:20}|}} \Rightarrow \ddot{a}_{\overline{30:20}|} = 14$$

$$A_{\overline{30:20}|} = 1 - d\ddot{a}_{\overline{30:20}|} = 1 - 0.05(14) = 0.3$$

$$G\ddot{a}_{\overline{30:20}|} = 100,000A_{\overline{30:20}|} + (200 + 50\ddot{a}_{\overline{30:20}|}) + (0.33G + 0.06G\ddot{a}_{\overline{30:20}|})$$

$$14G = 100,000(0.3) + [200 + 50(14)] + (0.33G + 0.84G)$$

$$12.83G = 30,900$$

$$G = 2408$$



**Question 6.17****Answer: A**

$$q_x^{\text{NS}} = q_{x+1}^{\text{NS}} = 1 - e^{-0.1} = 0.095$$

Then the annual premium for the non-smoker policies is  $P^{\text{NS}}$ , where

$$P^{\text{NS}}(1 + vp_x^{\text{NS}}) = 100,000vq_x^{\text{NS}} + 100,000v^2p_x^{\text{NS}}q_{x+1}^{\text{NS}} + 30,000v^2p_x^{\text{NS}}p_{x+1}^{\text{NS}}$$

$$P^{\text{NS}} = \frac{100,000(0.926)(0.095) + 100,000(0.857)(0.905)(0.095) + 30,000(0.857)(0.905)^2}{1 + (0.926)(0.905)}$$

$$P^{\text{NS}} = 20,251$$

And then  $P^{\text{S}} = 40,502$ .

$$q_x^{\text{S}} = q_{x+1}^{\text{S}} = 1.5(1 - e^{-0.1}) = 0.143$$

$$EPV(L^{\text{S}}) = 100,000vq_x^{\text{S}} + 100,000v^2p_x^{\text{S}}q_{x+1}^{\text{S}} + 30,000v^2p_x^{\text{S}}p_{x+1}^{\text{S}} - P^{\text{S}} - P^{\text{S}}vp_x^{\text{S}}$$

$$= 100,000(0.926)(0.143) + 100,000(0.857)(0.857)(0.143)$$

$$+ 30,000(0.857)(0.857)^2 - 40,502 - 40,502(0.926)(0.857)$$

$$= -30,017$$

**Question 6.18****Answer: D**

$$P = 30,000 {}_{20|}\ddot{a}_{40} + PA_{40:\overline{20}|}^1$$

$$\Rightarrow P = 30,000 {}_{20|}\ddot{a}_{40} / (1 - A_{40:\overline{20}|}^1)$$

$$= 30,000(5.46429) / (1 - 0.0146346) = 166,363$$

**Question 6.19****Answer: C**

Let  $\pi$  be the annual premium, so that  $\pi \ddot{a}_{50} = A_{50} + 0.01 \ddot{a}_{50} + 0.19$

$$\Rightarrow \pi = \frac{A_{50} + 0.19}{\ddot{a}_{50}} + 0.01 = \frac{0.18931 + 0.19}{17.0245} + 0.01 = 0.03228$$

Loss at issue:  $L_0 = v^{k+1} - (\pi - 0.01) \ddot{a}_{\overline{k+1}|} (1 - v^{k+1}) / d + 0.19$

$$\begin{aligned} \Rightarrow \text{Var}[L_0] &= \left(1 + \frac{(\pi - 0.01)}{d}\right)^2 ({}^2A_{50} - A_{50}^2) \\ &= (2.15467)(0.05108 - 0.18931^2) \\ &= (2.15467)(0.015242) \\ &= 0.033 \end{aligned}$$

**Question 6.20****Answer: B**

EPV(premiums) = EPV(benefits)

$$P(1 + v p_x + v^2 {}_2 p_x) = P(v q_x + 2v^2 p_x q_{x+1}) + 10000(v^3 {}_2 p_x q_{x+2})$$

$$P\left(1 + \frac{0.9}{1.04} + \frac{0.9 \times 0.88}{1.04^2}\right) = P\left(\frac{0.1}{1.04} + \frac{2 \times 0.9 \times 0.12}{1.04^2}\right) + 10000\left(\frac{0.9 \times 0.88 \times 0.15}{1.04^3}\right)$$

$$2.5976P = 0.29588P + 1056.13$$

$$P = 459$$

**Question 6.21****Answer: C**

$$P \times \ddot{a}_{75:\overline{15}|} = 1000\left(A_{75:\overline{15}|}^1 + 15 \times P \times A_{75:\overline{15}|}^{\frac{1}{2}}\right) \rightarrow P = \frac{1000A_{75:\overline{15}|}^1}{\ddot{a}_{75:\overline{15}|} - 15 \times A_{75:\overline{15}|}^{\frac{1}{2}}}$$

$$A_{75:\overline{15}|}^1 = A_{75:\overline{15}|} - A_{75:\overline{15}|}^{\frac{1}{2}} = 0.7 - 0.11 = 0.59$$

$$\ddot{a}_{75:\overline{15}|} = \frac{1 - A_{75:\overline{15}|}}{d} = (1 - 0.7) / 0.04 = 7.5$$

$$\text{So } P = \frac{590}{7.5 - 15(0.11)} = 100.85$$

**Question 6.22****Answer: C**Let the monthly net premium =  $\pi$ 

$$12\pi = \frac{100,000\bar{A}_{45}}{\ddot{a}_{45:\overline{20}|}^{(12)}} \quad \begin{array}{l} \alpha(12) = 1.00020 \\ \beta(12) = 0.46651 \\ \frac{i}{\delta} = 1.02480 \end{array}$$

$$100,000\bar{A}_{45} = 100,000 \frac{i}{\delta} A_{45} = (1.02480)(15,161) = 15,536.99$$

$$\begin{aligned} \ddot{a}_{45:\overline{20}|}^{(12)} &= \alpha(12)\ddot{a}_{45:\overline{20}|} - \beta(12)(1 - {}_{20}E_{45}) \\ &= 1.00020[12.9391] - 0.46651(1 - 0.35994) \\ &= 12.6431 \end{aligned}$$

$$12\pi = \frac{15,536.99}{12.6431}$$

$$12\pi = 1228.891$$

$$\pi = 102.41$$

**Question 6.23****Answer: D**

$$\begin{aligned} G\ddot{a}_{x:\overline{30}|} &= \text{APV}[\text{gross premium}] = \text{APV}[\text{Benefits} + \text{expenses}] \\ &= FA_x + (30 + 30\ddot{a}_x) + G(0.6 + 0.10\ddot{a}_{x:\overline{30}|} + 0.10\ddot{a}_{x:\overline{15}|}) \end{aligned}$$

$$\begin{aligned} G &= \frac{FA_x + 30 + 30\ddot{a}_x}{\ddot{a}_{x:\overline{30}|} - 0.6 - 0.1\ddot{a}_{x:\overline{30}|} - 0.1\ddot{a}_{x:\overline{15}|}} \\ &= \frac{FA_x + 30 + 30(15.3926)}{14.0145 - 0.6 - 0.1(14.0145) - 0.1(10.1329)} \\ &= \frac{FA_x + 491.78}{10.9998} \\ &= \frac{FA_x}{10.9998} + \frac{491.78}{10.9998} = \frac{FA_x}{10.9998} + 44.71 \end{aligned}$$

$$\Rightarrow h = 44.71$$

**Question 6.24****Answer: E**

In general, the loss at issue random variable can be expressed as:

$$L = \bar{Z}_x - P \times \bar{Y}_x = \bar{Z}_x - P \times \left( \frac{1 - \bar{Z}_x}{\delta} \right) = \bar{Z}_x \times \left( 1 + \frac{P}{\delta} \right) - \frac{P}{\delta}$$

Using actuarial equivalence to determine the premium rate:

$$P = \frac{\bar{A}_x}{\bar{a}_x} = \frac{0.3}{(1-0.3)/0.07} = 0.03$$

$$\text{Var}(L) = \left( 1 + \frac{P}{\delta} \right)^2 \times \text{Var}(\bar{Z}_x) = \left( 1 + \frac{0.03}{0.07} \right)^2 \times \text{Var}(\bar{Z}_x) = 0.18$$

$$\text{Var}(\bar{Z}_x) = \frac{0.18}{\left( 1 + \frac{0.03}{0.07} \right)^2} = 0.088$$

$$\text{Var}(L^*) = \left( 1 + \frac{P^*}{\delta} \right)^2 \times \text{Var}(\bar{Z}_x) = \left( 1 + \frac{0.06}{0.07} \right)^2 (0.088) = 0.304$$

**Question 6.25****Answer: C**

Need  $\text{EPV}(\text{Ben} + \text{Exp}) - \text{EPV}(\text{Prem}) = -800$

$$\text{EPV}(\text{Prem}) = G \ddot{a}_{55:\overline{10}|} = 8.0192G$$

$$\begin{aligned} \text{EPV}(\text{Ben} + \text{Exp}) &= 12,000 {}_{10|}\ddot{a}_{55}^{(12)} + 300\ddot{a}_{55} \\ &= 12,000 {}_{10}E_{55} \ddot{a}_{65}^{(12)} + 300\ddot{a}_{55} \\ &= 12,000 {}_{10}E_{55} \left( \ddot{a}_{65} - \frac{m-1}{2m} \right) + 300\ddot{a}_{55} \\ &= 12,000(0.59342) \left( 13.5498 - \frac{11}{24} \right) + 300(16.0599) \\ &= 98,042.83 \end{aligned}$$

Therefore,  $98,042.83 - 8.0192G = -800$

$$G = 12,326$$

**Question 6.26****Answer: D**

$$\text{EPV (Premiums)} = Pa_{90} = P(\ddot{a}_{90} - 1) = (4.1835)P$$

$$\text{EPV(Benefits)} = 1000A_{90} = 1000(0.75317) = 753.17$$

Therefore,

$$P = \frac{753.17}{4.1835} = 180.03$$

**Question 6.27****Answer: D**

$$\text{EPV(Premiums)} = \text{EPV(Benefits)}$$

$$\begin{aligned} \text{EPV(Premiums)} &= 3P\bar{a}_x - 2P {}_{20}E_x \bar{a}_{x+20} \\ &= 3P\left(\frac{1}{\mu + \delta}\right) - 2P\left(e^{-20(\mu + \delta)}\right)\left(\frac{1}{\mu + \delta}\right) \\ &= 3P\left(\frac{1}{0.09}\right) - 2Pe^{-1.8} - \frac{1}{0.09} \\ &= 29.66P \end{aligned}$$

$$\begin{aligned} \text{EPV(Benefits)} &= 1,000,000\bar{A}_x - 500,000 {}_{20}E_x \bar{A}_{x+20} \\ &= 1,000,000\left(\frac{\mu}{\mu + \delta}\right) - 500,000e^{-20(\mu + \delta)} \frac{\mu}{\mu + \delta} \\ &= 1,000,000\left(\frac{0.03}{0.09}\right) - 500,000e^{-1.8} \frac{0.03}{0.09} \\ &= 305,783.5 \end{aligned}$$

$$29.66P = 305,783.5$$

$$P = \frac{305,783.5}{29.66}$$

$$P = 10,309.62$$

**Question 6.28****Answer: B**

$$G\ddot{a}_{40:\overline{5}|} = 1000A_{40} + 0.15G + 0.05G\ddot{a}_{40:\overline{5}|} + 5 + 5\ddot{a}_{40:\overline{5}|}$$

$$\ddot{a}_{40:\overline{5}|} = \ddot{a}_{40} - {}_5E_{40} \cdot \ddot{a}_{45} = 18.4578 - (0.78113)(17.8162) = 4.5410$$

$$G = \frac{121.06 + 5 + 5(4.5410)}{-0.15 + 0.95(4.5410)} = 35.73$$



**Question 6.29****Answer: B**

Per equivalence Principle:

$$G\ddot{a}_{35} = 100,000A_{35} + 0.4G + 150 + 0.1G\ddot{a}_{35} + 50\ddot{a}_{35}$$

$$1770\ddot{a}_{35} = 100,000(1 - d\ddot{a}_{35}) + 0.4(1770) + 150 + 0.1(1770)\ddot{a}_{35} + 50\ddot{a}_{35}$$

$$1770\ddot{a}_{35} = 100,000 + 708 + 150 + \ddot{a}_{35} \left( 177 + 50 - 100,000 \left( \frac{0.035}{1.035} \right) \right)$$

Solving for  $\ddot{a}_{35}$ , we have

$$\ddot{a} = \frac{100,858}{1770 + 3154.64} = \frac{100,858}{4924.64} = 20.48$$

**Question 6.30****Answer: A**

The loss at issue is given by:

$$L_0 = 100v^{K+1} + 0.05G + 0.05G\ddot{a}_{\overline{K+1}|} - G\ddot{a}_{\overline{K+1}|}$$

$$= 100v^{K+1} + 0.05G - 0.95G \left( \frac{1 - v^{K+1}}{d} \right)$$

$$= \left( 100 + \frac{0.95G}{d} \right) v^{K+1} + 0.05G - 0.95 \frac{G}{d}$$

Thus, the variance is

$$\text{Var}(L_0) = \left[ 100 + \frac{0.95(2.338)}{0.04/1.04} \right]^2 \left( {}^2A_x - (A_x)^2 \right)$$

$$= \left[ 100 + \frac{0.95(2.338)}{0.04/1.04} \right]^2 \left( 0.17 - \left( 1 - \frac{0.04}{1.04}(16.50) \right)^2 \right)$$

$$= 908.1414$$

**Question 6.31****Answer: D**

$$\bar{A}_{35} = \left(1 - e^{-35(\mu+\delta)}\right) \times \left(\frac{\mu}{\mu+\delta}\right) + e^{-35(\mu+\delta)} \bar{A}_{70} = 0.063421 + 0.146257 = 0.209679$$

$$\bar{a}_{35} = \frac{1 - \bar{A}_{35}}{\delta} = \frac{1 - 0.209679}{0.05} = 15.80642$$

$$\bar{P}_{35} = \frac{\bar{A}_{35}}{\bar{a}_{35}} = \frac{0.209679}{15.80642} = 0.0132654$$

The annual net premium for this policy is therefore  $100,000 \times 0.0132654 = 1,326.54$

**Question 6.32****Answer: C**

Assuming UDD

Let  $P$  = monthly net premium

$$\text{EPV}(\text{premiums}) = 12P \ddot{a}_x^{(12)} \cong 12P [\alpha(12) \dot{a}_x - \beta(12)]$$

$$= 12P [1.00020(9.19) - 0.46651]$$

$$= 104.7039P$$

$$\text{EPV}(\text{benefits}) = 100,000 \bar{A}_x$$

$$= 100,000 \frac{i}{\delta} A_x = 100,000 \frac{i}{\delta} (1 - d \ddot{a}_x)$$

$$= 100,000 \frac{0.05}{\ln(1.05)} \left(1 - \frac{0.05}{1.05} (9.19)\right)$$

$$= 57,632.62$$

$$P = \frac{57,632.62}{104.7039} = 550.43$$



**Question 6.33****Answer: B**

The probability that the endowment payment will be made for a given contract is:

$$\begin{aligned}
 {}_{15}p_x &= \exp\left(-\int_0^{15} 0.02t \, dt\right) \\
 &= \exp\left(-0.01t^2 \Big|_0^{15}\right) \\
 &= \exp\left(-0.01(15)^2\right) \\
 &= 0.1054
 \end{aligned}$$

Because the premium is set by the equivalence principle, we have  $E[{}_0L] = 0$ . Further,

$$\begin{aligned}
 \text{Var}({}_0L) &= 500 \left[ (10,000v^{15})^2 ({}_{15}p_x)(1 - {}_{15}p_x) \right] \\
 &= 1,942,329,000
 \end{aligned}$$

Then, using the normal approximation, the approximate probability that the aggregate losses exceed 50,000 is

$$P({}_0L > 50,000) = P\left(Z > \frac{50,000 - 0}{\sqrt{1,942,329,000}}\right) = P(Z > 1.13) = 0.13$$

**Question 6.34****Answer: A**

Let B be the amount of death benefit.

$$\text{EPV}(\text{Premiums}) = 500\ddot{a}_{61} = 500(14.6491) = 7324.55$$

$$\text{EPV}(\text{Benefits}) = B \cdot A_{61} = (0.30243)B$$

$$\text{EPV}(\text{Expenses}) = (0.12)(500) + (0.03)(500)\ddot{a}_{61} = (0.12)(500) + (0.03)(7324.55) = 279.74$$

$$\text{EPV}(\text{Premiums}) = \text{EPV}(\text{Benefits}) + \text{EPV}(\text{Expenses})$$

$$7324.55 = (0.30243)B + 279.74$$

$$7044.81 = (0.30243)B$$

$$B = 23,294$$

**Question 6.35****Answer: D**

Let  $G$  be the annual gross premium. By the equivalence principle, we have

$$G\ddot{a}_{35} = 100,000A_{35} + 0.15G + 0.04G\ddot{a}_{35}$$

so that

$$G = \frac{100,000A_{35}}{0.96\ddot{a}_{35} - 0.15} = \frac{100,000(0.09653)}{0.96(18.9728) - 0.15} = 534.38$$

**Question 6.36****Answer: B**

By the equivalence principle,

$$4500\bar{a}_{x:\overline{20}|} = 100,000\bar{A}_{x:\overline{20}|}^1 + R\bar{a}_{x:\overline{20}|}$$

where

$$\bar{A}_{x:\overline{20}|}^1 = \frac{\mu}{\mu + \delta} (1 - e^{-20(\mu + \delta)}) = \frac{0.04}{0.12} (1 - e^{-20(0.12)}) = 0.3031$$

$$\bar{a}_{x:\overline{20}|} = \frac{1 - e^{-20(\mu + \delta)}}{\mu + \delta} = \frac{1 - e^{-20(0.12)}}{0.12} = 7.5774$$

Solving for  $R$ , we have

$$R = 4500 - 100,000 \left( \frac{0.3031}{7.5774} \right) = 500$$

**Question 6.37****Answer: D**

By the equivalence principle, we have

$$G\ddot{a}_{35:\overline{10}|} = 50,000A_{35} + 100a_{35} + 100A_{35}$$

so

$$G = \frac{50,100A_{35} + 100(\ddot{a}_{35} - 1)}{\ddot{a}_{35:\overline{10}|}} = \frac{50,100(0.09653) + 100(17.9728)}{8.0926} = 819.69$$

**Question 6.38****Answer: B**Let  $P$  be the annual net premium

$$P = \frac{1000\bar{A}_{x:\overline{n}|}}{\ddot{a}_{x:\overline{n}|}} = \frac{1000(0.192)}{\ddot{a}_{x:\overline{n}|}}$$

where

$$\ddot{a}_{x:\overline{n}|} = \frac{1 - A_{x:\overline{n}|}}{d} = \frac{(1.05)}{(0.05)} \left( 1 - A_{x:\overline{n}|}^1 - A_{x:\overline{n}|}^{\frac{1}{2}} \right)$$

$$A_{x:\overline{n}|} = \frac{i}{\delta} \left( A_{x:\overline{n}|}^1 \right) + {}_nE_x$$

$$\Rightarrow 0.192 = \frac{0.05}{0.04879} \left( A_{x:\overline{n}|}^1 \right) + 0.172$$

$$\Rightarrow A_{x:\overline{n}|}^1 = 0.019516$$

$$\Rightarrow \ddot{a}_{x:\overline{n}|} = \frac{1.05}{0.05} (1 - 0.019516 - 0.172) = 16.978$$

Therefore, we have

$$P = \frac{1000(0.192)}{16.978} = 11.31$$

**Question 6.39****Answer: A**

$$\text{Premium at issue for (40): } \frac{1000A_{40}}{\ddot{a}_{40}} = \frac{121.06}{18.4578} = 6.5587$$

$$\text{Premium at issue for (80): } \frac{1000A_{80}}{\ddot{a}_{80}} = \frac{592.93}{8.5484} = 69.3615$$

Lives in force after ten years:

$$\text{Issued at age 40: } 10,000 {}_{10}p_{40} = 10,000 \times \frac{98,576.4}{99,338.3} = 9923.30$$

$$\text{Issued at age 80: } 10,000 {}_{10}p_{80} = 10,000 \times \frac{41,841.1}{75,657.2} = 5530.35$$

The total number of lives after ten years is therefore:  $9923.30 + 5530.35 = 15,453.65$ 

The average premium after ten years is therefore:

$$\frac{(6.5587 \times 9923.30) + (69.3615 \times 5530.35)}{15,453.65} = 29.03$$

**Question 6.40****Answer: C**

Let  $P$  be the annual net premium at  $x+1$ . Also, let  $A_y^*$  be the expected present value for the special insurance described in the problem issued to  $(y)$ .

$$P\ddot{a}_{x+1} = 1000 \sum_{k=0}^{\infty} (1.03)^{k+1} v^{k+1} {}_k|q_{x+1} = 1000A_{x+1}^*$$

We are given

$$110\ddot{a}_x = 1000 \sum_{k=0}^{\infty} (1.03)^{k+1} v^{k+1} {}_k|q_x = 1000A_x^*$$

Which implies that

$$110(1 + vp_x \ddot{a}_{x+1}) = 1000(1.03vq_x + 1.03vp_x A_{x+1}^*)$$

Solving for  $A_{x+1}^*$ , we get

$$A_{x+1}^* = \frac{\frac{110}{1000} [1 + v(0.95)(7)] - 1.03v(0.05)}{1.03v(0.95)} = 0.8141032$$

Thus, we have

$$P = \frac{1000(0.8141032)}{7} = 116.3005$$

**Question 6.41****Answer: B**

Let  $P$  be the net premium for year 1.

Then:

$$P + 1.01Pvp_x = 100,000vq_x + (1.01)(100,000)v^2 p_x q_{x+1}$$

$$P \left[ 1 + \frac{1.01}{1.05} 0.99 \right] = 100,000 \left( \frac{0.01}{1.05} + \frac{(1.01)(0.99)(0.02)}{(1.05)^2} \right) \Rightarrow P = 1416.93$$

**Question 6.42****Answer: D**

The policy is fully discrete, so all cash flows occur at the start or end of a year.

$$\text{Die Year 1} \implies L_0 = 1000v - 315.80 = 625.96$$

$$\text{Die Year 2} \implies L_0 = 1000v^2 - 315.80(1+v) = 273.71$$

$$\text{Survive Year 2} \implies L_0 = 1000v^3 - 315.80(1+v+v^2) = -58.03$$

There is a loss if death occurs in year 1 or year 2, otherwise the policy was profitable.

$$\Pr(\text{death in year 1 or 2}) = 1 - e^{-2\mu} = 0.113$$

**Question 6.43****Answer: C**

$$APV(\text{expenses}) = 0.35G + 8 + 0.15Ga_{\overline{30}|4} + 4a_{\overline{30}|9}$$

$$= 0.20G + 4 + 0.15G\ddot{a}_{\overline{30}|5} + 4\ddot{a}_{\overline{30}|10}$$

$$G\ddot{a}_{\overline{30}|5} = 0.20G + 4 + 0.15G\ddot{a}_{\overline{30}|5} + 4\ddot{a}_{\overline{30}|10} + 200,000A_{\overline{30}|10}^1$$

$$G = \frac{200,000A_{\overline{30}|10}^1 + 4 + 4\ddot{a}_{\overline{30}|10}}{0.85\ddot{a}_{\overline{30}|5} - 0.20}$$

$$200,000A_{\overline{30}|10}^1 = 200,000 \left[ A_{\overline{30}|10} - {}_{10}E_{30} \right]$$

$$= 200,000(0.61447 - 0.61152) = 590$$

$$G = \frac{590 + 4 + 4(8.0961)}{0.85(4.5431) - 0.20} = 171.07$$

**Question# 6.44****Answer: D**

Let  $P$  be the premium per 1 of insurance.

$$P\ddot{a}_{50:\overline{10}|} = P(IA)_{50:\overline{10}|}^1 + {}_{10}E_{50}A_{60}$$

$$\ddot{a}_{50:\overline{10}|} = \ddot{a}_{50} - {}_{10}E_{50}\ddot{a}_{60} = 17.0 - 0.60 \times 15.0 = 8$$

$$A_{60} = 1 - d\ddot{a}_{60} = 1 - \left(\frac{0.05}{1.05}\right)15 = 0.285714$$

$$P\left(\ddot{a}_{50:\overline{10}|} - (IA)_{50:\overline{10}|}^1\right) = {}_{10}E_{50}A_{60}$$

$$P = \frac{{}_{10}E_{50}A_{60}}{\ddot{a}_{50:\overline{10}|} - (IA)_{50:\overline{10}|}^1} = \frac{0.6 \times 0.285714}{8 - 0.15} = 0.021838$$

$$100P = 2.18$$

**Question 6.45****Answer: E**

$$L_0 = 100,000v^T - 560\bar{a}_{\overline{T}|} = \left(100,000 + \frac{560}{\delta}\right)e^{-\delta T} - \frac{560}{\delta}$$

Since  $L_0$  is a decreasing function of  $T$ , the 75<sup>th</sup> percentile of  $L_0$  is  $L_0(t)$  where  $t$  is such that  $\Pr[T_{35} > t] = 0.75$ .

$$\frac{l_{35+t}}{l_{35}} = 0.75$$

$$l_{35+t} = 0.75l_{35} = 0.75 \times 99,556.7 = 74,667.5$$

$$l_{81} < 74,667.5 < l_{80}$$

$$t = (80 - 35) + s$$

$$l_{80+s} = sl_{81} + (1-s)l_{80}$$

$$74,667.5 = 73,186.3s + 75,657.2(1-s)$$

$$s = 0.40054$$

$$t = 45.40054$$

$$L_0(45.40054) = \left(100,000 + \frac{560}{\ln(1.05)}\right)e^{-45.40054 \ln(1.05)} - \frac{560}{\ln(1.05)} = 689.25$$

**Question 6.46****Answer: E**

Let  $P$  be the premium per 1 of insurance.

$$P\ddot{a}_{55:\overline{10}|} = 0.51213P + v^{10} {}_{10}p_{55} \ddot{a}_{65}$$

$$\ddot{a}_{55} = \ddot{a}_{55:\overline{10}|} + v^{10} {}_{10}p_{55} \ddot{a}_{65} \Rightarrow v^{10} {}_{10}p_{55} \ddot{a}_{65} = 12.2758 - 7.4575 = 4.8183$$

$$7.4575P = 0.51213P + 4.8183 \Rightarrow P = 0.693742738$$

$$300P = 208.12$$



**Question 6.47****Answer: D**

$$G\ddot{a}_{\overline{70:10}|} = 100,000 \cdot {}_{10}E_{70}\ddot{a}_{80} + 0.05G\ddot{a}_{\overline{70:10}|} + 0.7G$$

$$7.6491G = (100,000)(0.50994)(8.5484) + 0.05G(7.6491) + 0.7G$$

$$\Rightarrow G = 66,383.54$$

**Question 6.48****Answer: A**

Actuarial present value of insured benefits:

$$100,000 \left[ \frac{0.95 \times 0.02}{1.06^6} + \frac{0.95 \times 0.98 \times 0.03}{1.06^7} + \frac{0.95 \times 0.98 \times 0.97 \times 0.04}{1.06^8} \right] = 5,463.32$$

$$\Rightarrow P \left( 1 + \frac{0.95}{1.06^5} \right) = 5,463.32 \Rightarrow P = 3,195.12$$

**Question 6.49****Answer: C**

$$G\ddot{a}_{40:\overline{20}|}^{(12)} = 100,000 \left( \frac{i}{\delta} \right) A_{40} + 200 + 0.04G\ddot{a}_{40:\overline{20}|}^{(12)}$$

$$\begin{aligned} \ddot{a}_{40:\overline{20}|}^{(12)} &= \alpha(12)\ddot{a}_{40:\overline{20}|} - (1 - {}_{20}E_{40})\beta(12) \\ &= 1.00020 \cdot 12.9935 - (1 - 0.36663) \cdot 0.46651 = 12.700625 \end{aligned}$$

$$G = \frac{(100,000)(1.02480)(0.12106) + 200}{0.96 \times 12.700625} = 1033.92$$

$$\Rightarrow G/12 = 86.16$$

**Question 6.50****Answer: A**

$$1,000P = 1,000 \frac{A_{35}}{\ddot{a}_{35}} = \frac{96.53}{18.9728} = 5.0878$$

Benefits paid during July 2018:

$$10,000 \times 1,000 \times q_{35} = 10,000 \times 0.391 = 3910$$

Premiums payable during July 2018:

$$10,000 \times (1 - q_{35}) \times 5.0878 = 9,996.09 \times 5.0878 = 50,858.10$$

Cash flow during July 2018:

$$3910 - 50,858 = -46,948$$

**Question 6.51****Answer: D**

Under the Equivalence Principle

$$P \ddot{a}_{62:\overline{10}|} = 50,000 \left( \ddot{a}_{62} - \ddot{a}_{62:\overline{10}|} \right) + P \left( (IA)_{62:\overline{10}|}^1 \right)$$

$$\text{where } (IA)_{62:\overline{10}|}^1 = 11A_{62:\overline{10}|}^1 - \sum_{k=1}^{10} A_{62:k|}^1 = 11(0.091) - 0.4891 = 0.5119$$

$$\text{So } P = \frac{50,000 \left( \ddot{a}_{62} - \ddot{a}_{62:\overline{10}|} \right)}{\ddot{a}_{62:\overline{10}|} - (IA)_{62:\overline{10}|}^1} = \frac{50,000(12.2758 - 7.4574)}{7.4574 - 0.5119} = 34,687$$

**Question 6.52****Answer: E**

$$G\ddot{a}_{45:\overline{10}|} = HA_{45} + G + 0.05G\ddot{a}_{45:\overline{10}|} + 80 + 10\ddot{a}_{45} + 10\ddot{a}_{45:\overline{10}|}$$

$$G = \frac{HA_{45} + 80 + 10(\ddot{a}_{45} + \ddot{a}_{45:\overline{10}|})}{0.95\ddot{a}_{45:\overline{10}|} - 1}$$

$$G = \frac{HA_{45} + 80 + 10(17.8162 + 8.0751)}{(0.95 \times 8.0751) - 1}$$

$$G = \frac{A_{45}}{(0.95 \times 8.0751) - 1} H + \frac{80 + 10(17.8162 + 8.0751)}{(0.95 \times 8.0751) - 1}$$

$$g = \frac{A_{45}}{(0.95 \times 8.0751) - 1}$$

$$f = \frac{80 + 10(17.8162 + 8.0751)}{(0.95 \times 8.0751) - 1} = 50.80$$

**Question 6.53****Answer: D**

$$G = 0.35G + 2000 \left( \frac{0.1}{1.08} + \frac{0.9 \times 0.1}{1.08^2} + \frac{0.9 \times 0.9 \times 0.1}{1.08^3} \right)$$

$$0.65G = 468.107$$

$$G = 720.16$$

**Question 6.54****Answer: A**

$$\begin{aligned}
\text{On a unit basis, } \text{Var}(L_0) &= \left(1 + \frac{P}{d}\right)^2 \left[ {}^2A_{45} - (A_{45})^2 \right] = \left(1 + \frac{A_{45}}{d\ddot{a}_{45}}\right)^2 \left[ {}^2A_{45} - (A_{45})^2 \right] \\
&= \left(\frac{d\ddot{a}_{45} + 1 - d\ddot{a}_{45}}{d\ddot{a}_{45}}\right)^2 \left[ {}^2A_{45} - (A_{45})^2 \right] = \frac{{}^2A_{45} - (A_{45})^2}{(d\ddot{a})^2} \\
&= \frac{0.03463 - 0.15161^2}{\left(\frac{0.05}{1.05} \times 17.8162\right)^2} = 0.016178038
\end{aligned}$$

The standard deviation of  $L_0 = 0.127193$

$(200,000)(\text{The standard deviation of } L_0) = 25,439$

**Question 6.55****Answer: D**

$$\begin{aligned}
\text{EPV}(\text{benefits}) &= 1000 \times A_{45:\overline{20}|}^1 + 2500 \times {}_{20}\ddot{a}_{45} \\
&= 1000(0.15161 - 0.35994 \times 0.35477) + 2500 \times 0.35994 \times 13.5498 = 12,216.7
\end{aligned}$$

$$\text{EPV}(\text{premiums}) = P \times (17.8161 - 0.35994 \times 13.5498) = P \times 12.9391$$

$$P = \frac{12216.7}{12.9391} = 944.17$$

**Question 6.56****Answer: D**

$$A_{\overline{35:20}|} = 0.37981$$

$$\ddot{a}_{\overline{35:20}|} = 13.0240$$

APV of Premium = APV of Benefits + APV of Expenses

$$\text{APV of Benefits} + \text{APV of \$ Expenses} = 1,000,000 \times A_{\overline{35:20}|} + 50 \times \ddot{a}_{\overline{35:20}|} + 100 = 380,561.20$$

$$\text{APV of Premium} - \text{APV of \% Expenses} = (0.95) \times P \times \ddot{a}_{\overline{35:20}|} - (0.5) \times P = 11.8728 \times P$$

$$P = \frac{380,561.20}{11.8728} = 32,053.20$$

### Question 7.1

Answer: C

$$\begin{aligned} {}_{10}V &= 50,000(A_{50} + {}_{10}E_{50}A_{60}) - (875)\left[\ddot{a}_{50:\overline{10}}\right] \\ &= 50,000[0.18931 + (0.60182)(0.29028)] - 875[8.0550] \\ &= 11,152 \end{aligned}$$

### Question 7.2

Answer: C

$${}_0V = 0$$

$${}_2V = 2000$$

$$\text{Year 1: } ({}_0V + P)(1+i) = q_x(2000 + {}_1V) + p_x {}_1V$$

$$P(1.1) = 0.15(2000 + {}_1V) + 0.85({}_1V)$$

$$1.1P - 300 = {}_1V$$

$$\text{Year 2: } ({}_1V + P)(1+i) = q_{x+1}(2000 + {}_2V) + p_{x+1}(2000)$$

$$(1.1P - 300 + P)(1.1) = 0.165(2000 + 2000) + 0.835(2000)$$

$$2.31P - 330 = 2330$$

$$P = \frac{2330 + 330}{2.31} = 1152$$

### Question 7.3

**Answer: E**

$i^{(4)} = 0.08$  means an interest rate of  $j = 0.02$  per quarter. This problem can be done with two quarterly recursions or a single calculation.

Using two recursions:

$${}_{10.75}V = \frac{[{}_{10.5}V + 60(1-0.1)](1.02) - \frac{800-706}{800}(1000)}{\frac{706}{800}}$$

$$753.72 = \frac{[{}_{10.5}V + 54](1.02) - 117.50}{0.8825} \Rightarrow {}_{10.5}V = 713.31$$

$${}_{10.5}V = \frac{[{}_{10.25}V](1.02) - \frac{898-800}{898}(1000)}{\frac{800}{898}} \Rightarrow 713.31 = \frac{[{}_{10.25}V](1.02) - 109.13}{0.8909}$$

$${}_{10.25}V = 730.02$$

Using a single step,  ${}_{10.25}V$  is the *EPV* of cash flows through time 10.75 plus  ${}_{0.5}E_{80.25}$  times the *EPV* of cash flows thereafter (that is,  ${}_{10.75}V$ ).

$${}_{10.25}V = (1000) \left[ \frac{898-800}{898(1.02)} + \frac{800-706}{898(1.02)^2} \right] - (60)(1-0.1) \left[ \frac{800}{898(1.02)} \right] + \left[ \frac{706}{898(1.02)^2} \right] (753.72) = 730$$

#### Question 7.4

Answer: B

$$\text{EPV of benefits at issue} = 1000A_{40} + 4 {}_{11}E_{40}(1000A_{51})$$

$$= 121.06 + (4)(0.57949)(197.80) = 579.55$$

$$\text{EPV of expenses at issue} = 100 + 10(\ddot{a}_{40} - 1) = 100 + 10(17.4578) = 274.58$$

$$\pi = (579.55 + 274.58) / \ddot{a}_{40} = 854.13 / 18.4578 = 46.27$$

$$G = 1.02\pi = 47.20$$

$$\text{EPV of benefits at time 1} = 1000A_{41} + 4 {}_{10}E_{41} \times 1000A_{51}$$

$$= 126.65 + (4)(0.60879)(197.80) = 608.32$$

$$\text{EPV of expenses at time 1} = 10(\ddot{a}_{41}) = 10(18.3403) = 183.40$$

$$\text{Gross Prem Policy Value} = 608.32 + 183.40 - G\ddot{a}_{41} = 791.72 - 47.20(18.3403) = -73.94$$

#### Question 7.5

Answer: E

$${}_{4.5}V = v^{0.5} {}_{0.5}p_{x+4.5} {}_5V + v^{0.5} {}_{0.5}q_{x+4.5} b, \text{ where } b = 10,000 \text{ is the death benefit during year 5}$$

$${}_{0.5}q_{x+4.5} = \frac{{}_{0.5}q_{x+4}}{1 - {}_{0.5}q_{x+4}} = \frac{0.5(0.04561)}{1 - 0.5(0.04561)} = 0.02334$$

$${}_{0.5}p_{x+4.5} = 0.97666$$

$${}_5V = \frac{({}_4V + P)(1.03) - q_{x+4}b}{p_{x+4}}$$

$${}_5V = \frac{(1,405.08 + 647.46)(1.03) - 0.04561(10,000)}{0.95439} = 1,737.25$$

$$\begin{aligned} {}_{4.5}V &= (1.03)^{-0.5}(0.97666)(1,737.25) + (1.03)^{-0.5}(0.02334)(10,000) \\ &= 1,671.81 + 229.98 = 1,902 \end{aligned}$$

${}_{4.5}V$  can also be calculated recursively:

$${}_{0.5}q_{x+4} = 0.5(0.04561) = 0.02281$$

$${}_{4.5}V = \frac{(1,405.08 + 647.16)(1.03)^{0.5} - 0.02281(10,000) / (1.03)^{0.5}}{1 - 0.02281} = 1,902$$

The interest adjustment on the death benefit term is needed because the death benefit will not be paid for another one-half year.



### Question 7.6

**Answer: E**

Gross premium =  $G$

$$G\ddot{a}_{45} = 2000A_{45} + \underbrace{\left(1\left(\frac{2000}{1000}\right) + 20\right)}_{22} + \underbrace{\left(0.5\left(\frac{2000}{1000}\right) + 10\right)}_{11} \ddot{a}_{45} + 0.20G + 0.05G\ddot{a}_{45}$$

$$(0.95\ddot{a}_{45} - 0.20)G = 2000A_{45} + 22 + 11\ddot{a}_{45}$$

$$G = \frac{2000A_{45} + 22 + 11\ddot{a}_{45}}{0.95\ddot{a}_{45} - 0.20} = \frac{2000(0.15161) + 22 + 11(17.8162)}{0.95(17.8162) - 0.20} = 31.16$$

There are two ways to proceed. The first is to calculate the gross premium policy value (with equivalence principle gross premium and original assumptions, both of which do apply here) and the net premium policy value and take the difference.

$$\text{The net premium is } \frac{2000A_{45}}{\ddot{a}_{45}} = \frac{2000(0.15161)}{17.8162} = 17.02$$

$$\text{The net premium policy value is } 2000A_{55} - 17.02\ddot{a}_{55} = 2000(0.23524) - 17.02(16.0599) = 197.14$$

The gross premium policy value is

$$2000A_{55} + [0.05(31.16) + 0.5(2000/1000) + 10]\ddot{a}_{55} - 31.16\ddot{a}_{55} \\ = 2000(0.23524) + (12.56 - 31.16)(16.0599) = 171.77$$

$$\text{Expense policy value is } 171.77 - 197.14 = -25$$

The second is to calculate the expense policy value directly based on the pattern of expenses. The first step is to determine the expense premium.

The present value of expenses is

$$[0.05G + 0.5(2000/1000) + 10]\ddot{a}_{45} + 0.20G + 1.0(2000/1000) + 20 \\ = 12.558(17.8162) + 28.232 = 251.97$$

$$\text{The expense premium is } 251.97/17.8162 = 14.14$$

The expense policy value is the expected present value of future expenses less future expense premiums, that is,

$$[0.05G + 0.5(2000/1000) + 10]\ddot{a}_{55} - 14.14\ddot{a}_{55} = -1.582(16.0599) = -25$$

There is a shortcut with the second approach based on recognizing that expenses that are level throughout create no expense policy value (the level expense premium equals the actual expenses). Therefore, the expense policy value in this case is created entirely from the extra first year expenses. They occur only at issue, so the expected present value is  $0.20(31.16)+1.0(2000/1000)+20 = 28.232$ . The expense premium for those expenses is then  $28.232/17.8162 = 1.585$  and the expense policy value is the present value of future non-level expenses (0) less the present value of those future expense premiums, which is  $1.585(16.0599) = 25$  for a reserve of  $-25$ .

### Question 7.7

**Answer: D**

$$\ddot{a}_{x+10} = (1 - A_{x+10}) / d = (1 - 0.4) / (0.05 / 1.05) = 12.6$$

$$\ddot{a}_{x+10}^{(12)} \approx 12.6 - 11 / 24 = 12.142$$

$${}_{10}V = 10,000A_{x+10} + 100\ddot{a}_{x+10} - 12\ddot{a}_{x+10}^{(12)}(30)(1 - 0.05)$$

$${}_{10}V = 10,000(0.4) + 100(12.6) - 12(12.142)(28.50)$$

$${}_{10}V = 1107$$

### Question 7.8

**Answer: C**

The simplest solution is recursive:

${}_0V = 0$  since the policy values are net premium policy values.

$$q_{[70]} = (0.7)(0.010413) = 0.007289$$

$${}_1V = \frac{(0 + 35.168)(1.05) - (1000)(0.007289)}{1 - 0.007289} = 29.86$$

Prospectively,  $q_{[70]+1} = (0.8)(0.011670) = 0.009336$ ;  $q_{[70]+2} = (0.9)(0.013081) = 0.011773$

$$A_{[70]+1} = (0.009336)v + (1 - 0.009336)(0.011773)v^2 \\ + (1 - 0.009336)(1 - 0.011773)(0.47580)v^2 = 0.44197$$

$$\ddot{a}_{[70]+1} = (1 - A_{[70]+1}) / d = (1 - 0.44197) / (0.05 / 1.05) = 11.7186$$

$${}_1V = (1000)(0.44197) - (11.7186)(35.168) = 29.85$$

### Question 7.9

**Answer: A**

Let  $P = 0.00253$  be the monthly net premium per 1 of insurance.

$$\begin{aligned} {}_{10}V &= 100,000 \left[ \frac{i}{\delta} A_{55:\overline{10}|}^1 + A_{55:\overline{10}|}^{\frac{1}{2}} - 12P\ddot{a}_{55:\overline{10}|}^{(12)} \right] \\ &= 100,000 [1.02480(0.02471) + 0.59342 - (12)(0.00253)(7.8311)] \\ &\approx 38,100 \end{aligned}$$

Where

$$A_{55:\overline{10}|}^1 = A_{55:\overline{10}|} - {}_{10}E_{55} = 0.61813 - 0.59342 = 0.02471$$

$$A_{55:\overline{10}|}^{\frac{1}{2}} = {}_{10}E_{55} = 0.59342$$

$$\ddot{a}_{55:\overline{10}|} = 8.0192$$

$$\begin{aligned} \ddot{a}_{55:\overline{10}|}^{(12)} &= \alpha(12)\ddot{a}_{55:\overline{10}|} - \beta(12)[1 - {}_{10}E_{55}] \\ &= 1.00020(8.0192) - 0.46651(1 - 0.59342) = 7.8311 \end{aligned}$$

**Question 7.10****Answer: C**Use superscript  $g$  for gross premiums and gross premium policy values.Use superscript  $n$  (representing “net”) for net premiums and net premium policy values.Use superscript  $e$  for expense premiums and expense policy values.

$$P^g = 977.60 \text{ (given)}$$

$$P^e = \frac{0.58P^g + 450 + (0.02P^g + 50)\ddot{a}_{45}}{\ddot{a}_{45}}$$

$$= \frac{0.58(977.60) + 450 + [0.02(977.60) + 50]17.8162}{17.8162} = 126.64$$

Alternatively,

$$P^n = \frac{100,000A_{45}}{\ddot{a}_{45}} = 850.97 \quad P^e = P^g - P^n = 126.63$$

$${}_5V^e = (0.02P^g + 50)\ddot{a}_{50} - P^e\ddot{a}_{50} = [0.02(977.60) + 50](17.0245) - 126.64(17.0245) = -972$$

Alternatively,

$${}_5V^n = 100,000A_{50} - P^n\ddot{a}_{50}$$

$$= 100,000(0.18931) - 850.97(17.0245) = 4443.66$$

$${}_5V^g = 100,000A_{50} + (50 + 0.02P^g - P^g)\ddot{a}_{50}$$

$$= 100,000(0.18931) + [50 + 0.02(977.60) - 977.60](17.0245) = 3471.93$$

$${}_5V^e = {}_5V^g - {}_5V^n = -972$$

**Question 7.11****Answer: B**

$$L = 10,000v^{K_{45}+1} - P\ddot{a}_{\overline{K_{45}+1}|} = 10,000v^{11} - P\ddot{a}_{\overline{11}|}$$

$$4450 = 10,000(0.58468) - 8.7217P$$

$$P = (5,846.8 - 4,450) / 8.7217 = 160.15$$

$$A_{55} = 1 - d\ddot{a}_{55} = 1 - (0.05 / 1.05)(13.4205) = 0.36093$$

$${}_{10}V = 10,000A_{55} - P\ddot{a}_{55} = (10,000)(0.36093) - (160.15)(13.4205) = 1,460$$

**Question 7.12****Answer: E**

$$\text{In the final year: } ({}_{24}V + P)(1+i) = b_{25}(q_{68}) + 1(p_{68})$$

$$\text{Since } b_{25} = 1, \text{ this reduces to } ({}_{24}V + P)(1+i) = 1 \Rightarrow (0.6 + P)(1.04) = 1 \Rightarrow P = 0.36154$$

$$\text{Looking back to the 12}^{\text{th}} \text{ year: } ({}_{11}V + P)(1+i) = b_{12}(q_{55}) + {}_{12}V(p_{55})$$

$$\Rightarrow (5.36154)(1.04) = 14(0.15) + {}_{12}V(0.85) \Rightarrow {}_{12}V = 4.089$$

**Question 7.13****Answer: A**

This first solution recognizes that the full preliminary term reserve at the end of year 10 for a 30 year endowment insurance on (40) is the same as the net premium policy value at the end of year 9 for a 29 year endowment insurance on (41). Then, using superscripts of FPT for full preliminary term reserve and NLP for net premium policy value to distinguish the symbols, we have

$$\begin{aligned} 1000 {}_{10}V^{FPT} &= 1000 {}_9V^{NLP} = 1000(A_{50:\overline{20}|} - P_{41:\overline{29}|} \ddot{a}_{50:\overline{20}|}) \\ &= 1000[0.38844 - 0.01622(12.8428)] = 180 \end{aligned}$$

$$\text{or } = 1000 \left( 1 - \frac{\ddot{a}_{50:\overline{20}|}}{\ddot{a}_{41:\overline{29}|}} \right) = 1000 \left( 1 - \frac{12.8428}{15.6640} \right) = 180$$

where

$$\begin{aligned} \ddot{a}_{41:\overline{29}|} &= \ddot{a}_{41} - {}_{29}E_{41} \ddot{a}_{70} \\ &= 18.3403 - (0.2228726)(12.0083) \\ &= 15.6640 \end{aligned}$$

$$A_{41:\overline{29}|} = 1 - d(15.6640) = 0.254095$$

$${}_{29}E_{41} = v^{29} \left( \frac{l_{70}}{l_{41}} \right) = (0.242946) \left( \frac{91,082.4}{99,285.9} \right) = 0.2228726$$

$$P_{41:\overline{29}|} = \frac{0.254095}{15.6640} = 0.01622$$

Alternatively, working from the definition of full preliminary term reserves as having  ${}_1V^{FPT} = 0$  and the discussion of modified net premium reserves in the Notation and Terminology Study Note, let  $\alpha$  be the valuation premium in year 1 and  $\beta$  be the valuation premium thereafter. Then (with some of the values taken from above),

$$\alpha = 1000vq_{40} = 0.5019$$

APV (valuation premiums) = APV (benefits)

$$\alpha + {}_1E_{40}(\ddot{a}_{41:\overline{29}})\beta = 1000A_{40:\overline{30}}$$

$$0.5019 + 0.95188(15.6640)\beta = 242.37$$

$$\beta = \frac{242.37 - 0.5019}{14.9102} = 16.22$$

Where

$${}_1E_{40} = (1 - 0.000527)v = 0.95188$$

$$A_{40:\overline{30}} = A_{40} + {}_{20}E_{40}({}_{10}E_{60})(1 - A_{70})$$

$$= 0.12106 + 0.36663(0.57864)(1 - 0.42818) = 0.24237$$

$${}_{10}V^{FPT} = 1000A_{50:\overline{20}} - \beta\ddot{a}_{50:\overline{20}} = 1000(0.38844) - 16.22(12.8427) = 180$$

#### Question 7.14

**Answer: A**

$$({}_5V + 0.96G - 50)(1.05) = q_{50}(100, 200) + p_{50}{}_6V$$

$$(5500 + 0.96G - 50)(1.05) = (0.009)(100, 200) + (1 - 0.009)(7100)$$

$$(1.05)(0.96)G + 5722.5 = 7937.9$$

$$(1.05)(0.96)G = 2215.4$$

$$G = 2197.8$$

#### Question 7.15

**Answer: E**

$${}_{15.6}V(1+i)^{0.4} = {}_{0.4}P_{x+15.6}{}_{16}V + {}_{0.4}q_{x+15.6}100$$

$${}_{15.6}V(1.05)^{0.4} = 0.957447(49.78) + 0.042553(100)$$

$${}_{15.6}V = 50.91$$

**Question 7.16****Answer: D**

$$\text{APV future benefits} = 1000 \left[ 0.04v + 0.05 \times 0.96v^2 + 0.96 \times 0.95 \times (0.06 + 0.94 \times 0.683)v^3 \right] = 630.25$$

$$\text{APV future premiums} = 130(1 + 0.96v) = 248.56$$

$$E[{}_3L] = 630.25 - 248.56 = 381.69$$

**Question 7.17****Answer: D**

$$\frac{V[{}_{10}L]}{V[{}_{11}L]} = \frac{\left(1 + \frac{p}{d}\right)^2 ({}^2A_{x+10} - A_{x+10}^2)}{\left(1 + \frac{p}{d}\right)^2 ({}^2A_{x+11} - A_{x+11}^2)}$$

$$A_{x+10} = vq_{x+10} + vp_{x+10} A_{x+11}$$

$$= (0.90703)^{\frac{1}{2}}(0.02067) + (0.90703)^{\frac{1}{2}}(1 - 0.02067)(0.52536) = 0.50969$$

$${}^2A_{x+10} = v^2q_{x+10} + v^2p_{x+10} {}^2A_{x+11}$$

$$= (0.90703)(0.02067) + (0.90703)(1 - 0.02067)(0.30783) = 0.29219$$

$$\Rightarrow \frac{\text{Var}({}_kL)}{\text{Var}({}_{k+1}L)} = \frac{(0.29219) - (0.50969)^2}{(0.30783) - (0.52536)^2} = \frac{0.03241}{0.03183} = 1.018$$

**Question 7.18****Answer: A**

$$\begin{aligned}
{}_1V_x &= A_{x+1} - P_x \ddot{a}_{x+1} = 1 - d\ddot{a}_{x+1} - P_x \ddot{a}_{x+1} \\
&= 1 - \underbrace{(P_x + d)} \ddot{a}_{x+1} = 1 - \ddot{a}_{x+1} / \ddot{a}_x
\end{aligned}$$

$$\Rightarrow \ddot{a}_x (1 - {}_1V_x) = \ddot{a}_{x+1}$$

Since  $\ddot{a}_x = 1 + vp_x \ddot{a}_{x+1}$  substituting we get

$$\ddot{a}_x (1 - {}_1V_x) = \frac{\ddot{a}_x - 1}{vp_x} \Rightarrow \ddot{a}_x (1 - {}_1V_x) vp_x = \ddot{a}_x - 1$$

$$\begin{aligned}
\text{Solving for } \ddot{a}_x, \text{ we get } \ddot{a}_x &= \frac{1}{1 - (1 - {}_1V_x) vp_x} = \frac{1}{1 - (1 - 0.012) \left( \frac{1}{1.04} \right) (1 - 0.009)} \\
&= 17.07942
\end{aligned}$$

**Question 7.19****Answer: D**

Let  $G$  be the annual gross premium.

$$\text{Using the equivalence principle, } 0.90G\ddot{a}_{40} - 0.40G = 100,000A_{40} + 300$$

$$\text{So } G = \frac{100,000(0.12106) + 300}{0.90(18.4578) - 0.40} = 765.2347$$

The gross premium policy value after the first year and immediately after the second premium and associated expenses are paid is

$$\begin{aligned}
&100,000A_{41} - 0.90G(\ddot{a}_{41} - 1) \\
&= 12,665 - 0.90(765.2347)(17.3403) \\
&= 723
\end{aligned}$$



**Question 7.20****Answer: A**If  $G$  denotes the gross premium, then

$$G = \frac{1000A_{35} + 30\ddot{a}_{35} + 270}{0.96\ddot{a}_{35} - 0.26} = \frac{1000(0.09653) + 30(18.9728) + 270}{0.96(18.9728) - 0.26} = 52.12$$

So that,

$$\begin{aligned} R &= 1000A_{36} + (30 - 0.96G)\ddot{a}_{36} \\ &= 1000(0.10101) + (30 - 0.96 \times 52.12)(18.8788) = -277.23 \end{aligned}$$

Note that  $S = 0$  as per definition of FPT reserve.**Question 7.21****Answer: D**

$$\pi = \frac{1000 \cdot {}_{10}\ddot{a}_{55}}{\ddot{a}_{55:\overline{10}|} - (IA)_{55:\overline{10}|}^1} = \frac{1000(0.59342)(13.5498)}{8.0192 - 0.14743} = 1021.46$$

$$\begin{aligned} {}_9V &= 1000 \cdot {}_1\ddot{a}_{64} + 10\pi A_{64:\overline{1}|}^1 - \pi\ddot{a}_{64:\overline{1}|} \\ &= 1000 \cdot \frac{1}{1.05} \left( \frac{94,579.7}{95,082.5} \right) 13.5498 + 10(1021.46) \cdot \frac{1}{1.05} (0.005288) - 1021.46 \\ &= 11,866 \end{aligned}$$

**Question 7.22****Answer: C**

$$V[L_0 \#1] = \left( B_1 + \frac{P_1}{d} \right)^2 ({}^2A_x - A_x^2) = 20.55 \quad \implies \left( 8 + \frac{1.25(1.06)}{0.06} \right)^2 ({}^2A_x - A_x^2) = 20.55$$

$${}^2A_x - A_x^2 = \frac{20.55}{\left( 8 + \frac{1.25(1.06)}{0.06} \right)^2} = 0.0227$$

$$V[L_0 \#2] = \left( 12 + \frac{1.875(1.06)}{0.06} \right)^2 ({}^2A_x - A_x^2) = \left( 12 + \frac{1.875(1.06)}{0.06} \right)^2 (0.0227) = 46.24$$

**Question 7.23****Answer: D**

We have Present Value of Modified Premiums = Present Value of level net premiums

$$vq_x + \beta(\ddot{a}_{25:\overline{20}|} - 1) + P \cdot {}_{20}E_{25} \cdot \ddot{a}_{45:\overline{20}|} = P\ddot{a}_{25:\overline{40}|}$$

$$\Rightarrow \beta = \frac{P(\ddot{a}_{25:\overline{40}|}) - P \cdot {}_{20}E_{25} \cdot \ddot{a}_{45:\overline{20}|} - vq_x}{\ddot{a}_{25:\overline{20}|} - 1} = \frac{P\ddot{a}_{25:\overline{20}|} - vq_x}{\ddot{a}_{25:\overline{20}|} - 1}$$

We are given that  $P = 0.0216$ 

$$\Rightarrow \beta = \frac{0.0216(11.087) - (1.04)^{-1}(0.005)}{11.087 - 1} = 0.023265$$

For insurance of 10,000,  $\beta = 233$ .**Question 7.24****Answer: C**

$$P^s = P^n + P^e \quad \text{where } P^e \text{ is the expense loading}$$

$$P^n = 1,000,000 \frac{A_{50}}{\ddot{a}_{50}} = 1,000,000 \left( \frac{0.18931}{17.0245} \right) = 11,119.86$$

$$P^e = P^s - P^n = 11,800 - 11,120 = 680$$

**Question 7.25****Answer: B**

$${}_3V^{FPT} = 100,000 A_{[55]+3} - 100,000 P_{[55]+1} \ddot{a}_{[55]+3}$$

$$= 100,000 A_{58} - 100,000 \frac{A_{[55]+1}}{\ddot{a}_{[55]+1}} \ddot{a}_{58}$$

$$= 100,000 \left( 0.27 - \frac{0.24}{1-0.24} \cdot \frac{1-0.27}{d} \right)$$

$$= 3947.37$$

**Question 7.26****Answer: D**

$${}_1V = ({}_0V + P)(1+i) - (25,000 + {}_1V - {}_0V)q_x = P(1+i) - (25,000)q_x$$

$${}_2V = ({}_1V + P)(1+i) - (50,000 + {}_2V - {}_1V)q_{x+1} = 50,000$$

$$\left( (P(1+i) - 25,000q_x) + P \right) (1+i) - 50,000q_{x+1} = 50,000$$

$$\left( (P(1.05) - 25,000(0.15)) + P \right) (1.05) - 50,000(0.15) = 50,000$$

Solving for  $P$ , we get

$$P = \frac{61,437.50}{2.1525} = 28,542.39$$

**Question 7.27****Answer: B**Since  $G$  is determined using the equivalence principle,  ${}_0V = 0$ 

$$\text{Then, } {}_1V^e = \frac{\left( 0 + \overbrace{G - 187}^{P^e} - 0.25G - 10 \right) (1.03)}{0.992} = -38.7$$

$$\Rightarrow 0.75G = \frac{-38.7(0.992)}{1.03} + 187 + 10 = 159.72$$

$$\Rightarrow G = 212.97$$

**Question 7.28****Answer: D**

$${}_{20}V = 0 \implies 1000A_{65} = (P + W) \times \ddot{a}_{65}$$

At issue, present value of benefits must equal present value of premium, so:

$$1000A_{45} = P\ddot{a}_{45} + W {}_{20}E_{45} \times \ddot{a}_{65}$$

$$354.77 = (P + W)(13.5498) \implies P + W = 26.182674 \implies P = 26.182674 - W$$

$$151.61 = 17.8162P + W(0.35994)(13.5498)$$

$$151.61 = 17.8162(26.182674 - W) + W(0.35994)(13.5498)$$

$$\implies W = 24.33447$$

**Question 7.29****Answer: E**

$$V_{10} = 2,290 = B \left( 1 - \frac{\ddot{a}_{x+10}}{\ddot{a}_x} \right) = B \left( 1 - \frac{11.4}{14.8} \right) \Rightarrow B = 9,968.24$$

$$G\ddot{a}_x = 25 + 5\ddot{a}_x + B \times A_x$$

$$A_x = 1 - d\ddot{a}_x = 1 - \left( \frac{0.04}{1.04} \times 14.8 \right) = 0.430769231$$

$$G \times 14.8 = 25 + 5 \times 14.8 + 9,968.24 \times 0.430769231$$

$$\Rightarrow G = 296.82$$

$${}_{10}V^s = 9,968.24A_{x+10} + 5\ddot{a}_{x+10} - 296.82\ddot{a}_{x+10}$$

$$A_{x+10} = 1 - d\ddot{a}_{x+10} = 1 - \left( \frac{0.04}{1.04} \times 11.4 \right) = 0.561538462$$

$${}_{10}V^s = 9,968.24 \times 0.561538462 + 5 \times 11.4 - 296.82 \times 11.4$$

$$\Rightarrow {}_{10}V^s = 2,270.80$$

Alternatively, the expense net premium is based on the extra expenses in year 1, so

$$P^e = (30 - 5) / 14.8 = 1.68919$$

$${}_{10}V^e = 0 - 1.68919(11.4) = -19.26$$

$${}_{10}V^s = {}_{10}V^n + {}_{10}V^e = 2290 - 19.26 = 2270.74$$

**Question 7.30****Answer: E**

$$L_{10} = 10,000A_{35} = 965.30$$

$$L_{10}^* = 10,000$$

$$L_{10}^* - L_{10} = 10,000 - 965.30 = 9034.70$$

**Question 7.31****Answer: E**Future expenses at  $x + 2 = 0.08G + 5$ Expense load at  $x + 2 = P^e$ 

$$-23.64 = (0.08G + 5) - P^e$$

$$\Rightarrow P^e = 58.08$$

$$1000P_{x:\overline{3}|} = 368.05 - 58.08 = 309.97$$

**Question 7.32****Answer: B**

$$L_A = v^T - 0.10\bar{a}_{\overline{7}|} = \left(1 + \frac{10}{6}\right)v^T - \frac{10}{6}$$

$$\text{Var}(L_A) = \left(1 + \frac{10}{6}\right)^2 \text{Var}(v^T) = 0.455 \Rightarrow \text{Var}(v^T) = 0.06398$$

$$L_B = 2v^T - 0.16\bar{a}_{\overline{7}|} = \left(2 + \frac{16}{6}\right)v^T - \frac{16}{6}$$

$$\text{Var}(L_B) = \left(2 + \frac{16}{6}\right)^2 \text{Var}(v^T) = \left(2 + \frac{16}{6}\right)^2 (0.06398) = 1.39$$

**Question 7.33****Answer: C**

$$A_{80:\overline{10}|}^1 = A_{80:\overline{10}|} - {}_{10}E_{80} = 0.67674 - 0.33952 = 0.33722$$

$$\bar{A}_{80:\overline{10}|}^1 = \frac{0.05}{\ln(1.05)} 0.33722 = 0.34559$$

$$\text{EPV future benefits} = 1000 (0.34559) + 500 (0.33952) = 515.35$$

$$\text{EPV future net premiums} = \ddot{a}_{80:\overline{10}|} P = 6.7885 (35.26) = 239.36$$

$$\text{Policy value} = 515.36 - 239.36 = 276$$

**Question 7.34****Answer: D**

$$\begin{aligned}
({}_{10.5}V + P)(1+i)^{0.25} &= 1,000,000 \times {}_{0.25}q_{50.5} + {}_{0.25}P_{50.5} \times {}_{50.75}V \\
(10,000 + 750)(1+0.05)^{0.25} &= 1,000,000 \times \left(1 - \frac{(1-0.75 \times 0.01)}{(1-0.5 \times 0.01)}\right) + \left(\frac{(1-0.75 \times 0.01)}{(1-0.5 \times 0.01)}\right) \times {}_{50.75}V \\
{}_{50.75}V &= 8,390
\end{aligned}$$

**Question 7.35****Answer: D**

The modified net premium reserve at duration 2 is:  ${}_2V^{\text{mod}} = 100000A_{62} - P^{\text{mod}}\ddot{a}_{62}$

$$P^{\text{mod}} = 100000 \frac{A_{62}}{\ddot{a}_{62}} = 100000 \left( \frac{0.31495}{14.3861} \right) = 2189$$

**Question 7.36****Answer: B**

There is no change to  ${}_9V$  because there were no errors after time 9.

An equation for  ${}_8V^{\text{orig}}$  is  ${}_8V^{\text{orig}} + P - e = 1000 (0.003736) / 1.05 + 100 (0.996264) / 1.05$

An equation for  ${}_8V^{\text{now}}$  is  ${}_8V^{\text{now}} + P - e = 1000 (0.002736) / 1.05 + 100 (0.997264) / 1.05$

By subtraction,  ${}_8V^{\text{now}} - {}_8V^{\text{orig}} = 1000 (-0.001) / 1.05 + 100 (0.001) / 1.05 = -0.86$

$${}_8V^{\text{now}} = 86.74 - 0.86 = 85.88 \text{ (B)}$$

In those equations,  $P - e$ , which cancels, represents the gross premium less all expenses. You could think of  $P$  as the gross premium less commissions, and  $e$  as the flat expenses. However you think of representing them, they cancel.

**Question 7.37****Answer: D**

$$\ddot{a}_{65:\overline{5}|} = \ddot{a}_{65} - {}_5E_{65} \times \ddot{a}_{70} = 13.5498 - 0.75455(12.0083) = 4.4889$$

$$\text{EPV future net premiums} = 4.4889 (5,808) = 26,072$$

$$130,580 = \text{EPV future benefits} - \text{EPV future net premiums} = \text{EPV future benefits} - 26,072$$

$$\text{EPV future benefits} = 156,652$$

**Question 7.38****Answer: A**

There is no change in the EPV of future benefits or of future gross premiums. Each year that a premium is due, the expense changes by  $(3\% - 2\%)$  of 16, or 0.16. Since at the end of year 10, the insured is 60 with 20 remaining premiums, the EPV of future expense rises by

$$0.16\ddot{a}_{\overline{60:20}|} = 0.16(12.3816) = 1.981$$

The new reserve is  $110 + 1.981 = 112$

**Question 7.39****Answer: C**

$$({}_{18}V + P)(1 + i) = p_{[45]+18} \times {}_{19}V$$

$${}_{18}V = 0.976956 \times 575,000 / 1.05 - 20,000 = 515,000$$

**Question 7.40****Answer: A**

The policy value at duration 2 is  ${}_2V^{\text{mod}} = 1000A_{62} - (P^{\text{mod}})(\ddot{a}_{62}) = 0$

$$P^{\text{mod}} = 1000A_{62} / \ddot{a}_{62} = 1000(0.31495) / 14.3861 = 21.89$$

$${}_5V^{\text{mod}} = 1000A_{65} - (P^{\text{mod}})(\ddot{a}_{65}) = 1000(0.35477) - 21.89(13.5498) = 58$$

**ALL QUESTIONS FROM CHAPTER 18 HAVE BEEN DELETED**