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Assessment Ended

MA 373 - Practice Final - with solutions has ended.

Problem 1

You are given the following two bonds:

Term	Annual Coupon	Maturity Value	Price
1 Year Bond	60	1,000	1,000.00
2 Year Bond	80	1,000	1,053.20

Calculate the 2 year spot interest rate.

(Round your answer to 4 decimal places.)

IATEX 2

Incorrect

 ${\bf Correct\ Answer:}\ 0.0510$

We find the one year spot rate, r_1 , using the first bond.

$$1,\!000 = \frac{1,\!060}{1+r_1} \Longrightarrow r_1 = \frac{1,\!060}{1,\!000} - 1 = 0.06$$

We find the two year spot rate, r_2 , using the second bond and r_1 .

$$1,053.20 = \frac{80}{1+r_1} + \frac{1,080}{(1+r_2)^2}$$

$$\frac{1,080}{(1+r_2)^2} = 1,053.20 - \frac{80}{1.06} = 977.728$$

$$r_2 = \left(\frac{1{,}080}{977.728}\right)^{.5} - 1 = 0.051$$

Unlimited attempts

VARIABLE NAME	VALUE	RESPONSE	ANSWER	ANSWER RANGE
Price	1,053.20	1.	0.0510	[0.0509, 0.0511]
Rate	0.051			
X	977.728			

Problem 2

A 3 year bond has annual coupons.

- a. The coupon at the end of the first year is 100.
- b. The coupon at the end of the second year is 300.
- c. The coupon at the end of the third year is 500.

The bond matures for 700.

Calculate the modified convexity of this bond at an annual effective rate of interest of 7.25%.

(Round your answer to 2 decimal places.)

LATEX ?



Incorrect

Correct Answer: 8.80

$$v^2 \frac{\sum_{t} C_t(t)(t+1)v^t}{\sum_{t} C_t \cdot v^t} = (1+7.25\%)^{-2} \left(\frac{100(1)(2)(1+7.25\%)^{-1}+300(2)(3)(1+7.25\%)^{-2}+1,200(3)(4)(1+7.25\%)^{-3}}{100(1+7.25\%)^{-1}+300(1+7.25\%)^{-2}+1,200(1+7.25\%)^{-3}} \right)$$

$$=\frac{11,670.47}{1,326.77}=8.7962$$

Unlimited attempts

VARIABLE NAME	VALUE	RESPONSE ANSWER ANSWER RANGE
Den	1,326.77	1. 8.80 [8.70, 8.90]
ModCon	8.7962	
Num	11,670.47	
Rate	7.25%	

Problem 3

An n year bond has a par value of 8,000. The bond pays semi-annual coupons of 240 and matures for par.

Linfeng purchases the bond for 8,837.21 based on a yield of 5% convertible semi-annually.

Determine n.

(Round your answer to the nearest whole number)

LATEX ?

Round your answer to 2 decimal places.

Incorrect

 ${\bf Correct\ Answer:}\ 15.00$

I/Y = 5/2 = 2.5, PMT = 240, PV = -8,837.21, FV = 12,000, CPT N ==> 30 ==>15 years

$$8,837.21 = 240a_{\widetilde{n}} + 8,000 (1.025)^{-n} = 240 \left(\frac{1 - (1.025)^{-n}}{0.025}\right) + 8,000 (1.025)^{-n} =$$

$$\frac{240}{0.025} - \left(\frac{240}{0.025}\right) \! (1.025)^{-n} + 8,000 \, (1.025)^{-n} = 9,600 - (9,600 - 8,000) \! (1.025)^{-n} = 8,837.21$$

$$(1.025)^{-n} = \frac{9,600 - 8,837.21}{1,600} \Longrightarrow n = -\frac{\ln\left(\frac{9,600 - 8,837.21}{1,600}\right)}{\ln(1.025)} = 30 \Longrightarrow 15$$

Unlimited attempts

j 0.025 1. 15.00 - j1 1.025 n 30 n2 15	VARIABLE NAME	VALUE	RESPONSE ANSWER	ANSWER RANGE
n 30 n2 15	j	0.025	1. 15.00	-
n2 15	j1	1.025		
	n	30		
mice \$ 337.91	n2	15		
9,001.21	price	8,837.21		

Problem 4 :

A 40 year bond with a par value of 100,000 matures at par. The semi-annual coupons are paid at a rate of 9.8% convertible semi-annually. The bond is bought to yield 7% convertible semiannually. Calculate the amortization of premium in the coupon paid at the end of the 15^{th} year of this bond.

(Round your answer to the nearest two decimal places)

LATEX ?



Incorrect

 ${\sf Correct\ Answer:}\ 242.20$

Coupon = 100,000(9.8%/2) = 4,900.00

Book Value after the 29th Coupon is the present value of future cash flows:

N = 51, I/Y = 3.5, PMT = 4,900.00, FV = 100,000, CPT PV = 133,080.06

$$=4,\!900.00\left(\frac{1-(1.035)^{-51}}{0.035}\right)+100,\!000\left(1.035\right)^{-51}=133,\!080.06$$

133,080.06(0.035) = 4,657.80 is the interest in the 30^{th} coupon.

4,900.00-4,657.80=242.20 is the principal in the 30^{th} coupon

Unlimited attempts

ANSWER RANGE

[242.15, 242.25]

VARIABLE NAME	VALUE
ans	242.20
coup	4,900.00
coupon	4,900.000000000
int	4,657.80
p	9.8%
p1	0.049
pv	133,080.06

Problem 5

RESPONSE

1.

ANSWER

242.20

Sparks-Norris Asset Partners (SNAP) manages the following portfolio of bonds:

Bond	Price	Macaulay Duration
1	5,000	10
2	10,000	6
3	20,000	8
4	15,000	2

The duration is calculated at an annual effective interest rate of 6.00%.

Calculate the modified duration of SNAP's portfolio.

(Round your answer to 2 decimal places.)

Incorrect

Problem 6

Correct Answer: 5.66

$$\boldsymbol{D}^{Port} = \frac{\boldsymbol{\Sigma} \boldsymbol{D}^t \boldsymbol{P}^t}{\boldsymbol{\Sigma} \boldsymbol{P}^t}$$
 where \boldsymbol{D}^t is Modified Duration.

In order to $\,$ get Modified Duration from Macaulay Duration, we multiply Macaulay by v.

$$\frac{\Sigma D^t P^t}{\Sigma P^t} = \frac{10(5,000) + 6(10,000) + 8(20,000) + 2(15,000)}{5,000 + 10,000 + 20,000 + 15,000}v = 5.660$$

Unlimited attempts

LATEX ?

VARIABLE NAME	VALUE	RESPONSE	ANSWER	ANSWER RANGE
DPort	5.660	1.	5.66	[5.64, 5.68]
Rate	6.00%			

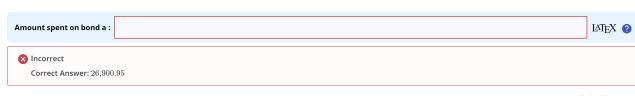
Sue wants to fully immunize a future payment of 100,000 at time 10 using the following two bonds:

a. A zero coupon bond maturing in 5 years; and

b. A zero coupon bond maturing in 20 years.

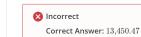
Determine the amount that Sue should spend on each bond at an annual effective interest rate of 9.50%.

(Round your answer to 2 decimal places.)



Unlimited attempts

LATEX ?



Amount spent on bond b :

Present Value Matching:

Let A = present value of 5 year bond

Let B = present value of 20 year bond

PV(assets)=PV(liabilities)

 $A+B=100,\!000\,(1+9.50\%)^{-10}$

Duration Matching:

Duration(assets)=Duration(liabilities)

 $5A + 20B = 10 (100,000 (1 + 9.50\%)^{-10})$

Now we have two equations with two unknowns. We can solve for A and B.

 $A = 100,000 (1 + 9.50\%)^{-10} - B$

A = 40,351.41867 - B

5(40,351.41867 - B) + 20B = 10(40,351.41867)

 $15B = 403,\!514.18670 - 201,\!757.09335$

B = 13,450.47

A = 40,351.41867 - 13,450.47

A = 26,900.95

Unlimited attempts

VARIABLE NAME	VALUE
\boldsymbol{A}	26,900.95
B	13,450.47
R	403,514.18670
Rate	9.50%
S	201,757.09335
V	40,351.41867

RESPONSE	ANSWER	ANSWER RANGE
a.	26,900.95	[26,900.85,26,901.05]
b.	13,450.47	$[13,\!450.37,13,\!450.57]$

Problem 7

Wang Life Insurance Company issues a three year annuity that pays 40,000 at the end of each year. Wang uses the following three bonds to absolutely match the cash flows under this annuity: a. A zero coupon bond which matures in one year for 1,000.

b. A two year bond which matures for 1,200 and pays an annual coupon of 100. This bond is priced using an annual yield of 7%.

c. A three year bond which matures for 2,000 and pays annual coupons of 75. This bond has a price of 1,750.

It cost Wang 104,000 to purchase all three bonds to absolutely match this annuity.

Calculate the one year spot interest rate.

(Round your answer to 4 decimal places.)

LATEX ?





Absolute matching means that the cash flow from the bonds at any given time should be exactly the same as the cash flow from the annuity. In this case, we need the bonds to produce a cash flow of \$40,000 at times 1,2, and 3. In order to do this we will need to buy a certain amount of each bond. These values will be "x, y, and z".

Sometimes it is easiest to set up a table that summarizes cash flows:

Bond	Amount	Price	CF at t=1	CF at t=2	CF at t=3
A	X	1000/(1+r ₁)	1000	0	0
В	Y	1228.928291	100	1300	0
С	Z	1750	75	75	2075

In the table above, the Price for bond B was found using the calculator.

$\boxed{N = 2 \, \left| PMT = 100 \right| \left| I \middle/ Y = 7 \right| FV = 1200 \, \left| CPT \right| \left| PV = 1228.928291 \right|}$

Now we can set up a few equations in order to solve for the unknown variables.

First, since there is only one bond with a cash flow at time 3 we can find z very easily:

2,075Z = 40,000

Z = 19.277

Then we can find Y:

 $1.300V \pm 75\,Z - 40.000$

$$Y = \frac{40,000 - 75(19.277)}{1,300} = 29.657$$

Finally we can find X.

1,000X + 100Y + 75Z = 40,000

$$X = \frac{40,000 - 75(19.277) - 100(29.657)}{1,000} = 35.5885$$

Now that we know the amount of each bond we are purchasing and we are given our total cost, we just need to find the price of the one year bond which will in turn give

$$104,\!000 = \frac{1,\!000}{1+r_1}X + 1,\!228.928291Y + 1,\!750Z$$

Note that the price of the first bond $=rac{1{,}000}{1+r_1}$

Now just plug in the values we found above to get $r_1=0.05233. \,$

Unlimited attempts

RESPONSE	ANSWER	ANSWER RANGE
1.	0.0523	[0.0522, 0.0524]

Problem 8

Trevor, Andrew, Emma, and Maddie enter a four-way loan agreement. As part of this agreement, Trevor pays Andrew 1,000 at time 0. At the end of 3 years, Andrew agrees to pay 1,200 to Emma. At the end of 5 years, Emma pays 1,355.09 to Maddie. Finally, at the end of N years, Maddie pays Trevor a total of 2,000.

Using the bottom line approach, all four participants have the same annual yield.

Determine N to three decimal places. (Please note that N is not an integer.)

LATEX ?



Round your answer to 3 decimal places.



 ${\bf Correct\ Answer:}\ 11.405$

Andrew's cashflows are 1000 at time 0, and -1200 at time 3; His equation of value is

$$1000 \, (1+i)^3 = 1200 \Rightarrow i = \left(\frac{1200}{1000}\right)^{\frac{1}{3}} - 1 = 0.062658569$$

To find N, Trevor's cashflows are -1000 at time 0, and 2000 at time N. His equation of value is

$$1000 (1.062658569)^N = 2000 \Rightarrow N = \frac{\ln(\frac{2000}{1000})}{\ln 1.062658569} = 11.405$$

Unlimited attempts

RESPONSE	ANSWER	ANSWER RANGE
1.	11.405	

Problem 9

An annuity immediate pays 100 at the end of each year for 5 years. Calculate the Macaulay convexity and the Modified convexity of this annuity at an annual effective rate of 6.75%. (Round your answers to 2 decimal places.)



Modified Convexity:





Incorrect

 ${\bf Correct\ Answer:}\ 11.49$

$$v^2 \frac{\sum_t C_t(t)(t+1)v^t}{\sum_t C_t \cdot v^t} =$$

$$(1.0675)^{-2} \left(\frac{100 \, (1)(2)(1.0675)^{-1} + 100 \, (2)(3)(1.0675)^{-2} + 100 \, (3)(4)(1.0675)^{-3} + 100 \, (4)(5)(1.0675)^{-4} + 100 \, (5)(6)(1.0675)^{-5}}{100 \, (1.0675)^{-1} + 100 \, (1.0675)^{-2} + 100 \, (1.0675)^{-3} + 100 \, (1.0675)^{-4} + 100 \, (1.0675)^{-5}}\right) \right) + 100 \, (1.0675)^{-1} + 100 \, (1.06$$

= 11.4897



Correct Answer: 10.22

Note: As you can see from Modified Convexity, the 100 in the numerator and denominator will cancel since all cash flows are 100. Therefore, the 100's have not been

$$\frac{\sum_{t} C_{t}(t)^{2} v^{t}}{\sum_{t} C_{t} \cdot v^{t}}$$

$$= \frac{1(1.0675)^{-1} + 2^2(1.0675)^{-2} + 3^2(1.0675)^{-3} + 4^2(1.0675)^{-4} + 5^2(1.0675)^{-5}}{(1.0675)^{-1} + (1.0675)^{-2} + (1.0675)^{-3} + (1.0675)^{-4} + (1.0675)^{-5}}$$

$$= \frac{1(1.0675)^{-1} + 4(1.0675)^{-2} + 9(1.0675)^{-3} + 16(1.0675)^{-4} + 25(1.0675)^{-5}}{(1.0675)^{-1} + (1.0675)^{-2} + (1.0675)^{-3} + (1.0675)^{-4} + (1.0675)^{-5}}$$

$$= 10.2236$$

Unlimited attempts

ANSWER RANGE

[11.48, 11.50] [10.21, 10.23]

VARIABLE NAME	VALUE	RESPONSE
MacCon	10.2236	a.
ModCon	11.4897	b.
r	0.0675	
r1	1.0675	
Rate	6.75%	

Problem 10

ANSWER

11.49

A 20 year bond has a par value of F. The bond has a maturity value of 1.25 F. The bond pays semi-annual coupons at a rate of 6% compounded semi-annually.

The bond is bought at a discount of 28.99 when purchased to yield 5% convertible semi-annually.

(Round your answer to the nearest two decimal places)

LATEX ?



Correct Answer: 923.88

$$r = \frac{0.06}{2} = 0.03; F = F; C = 1.25F; j = \frac{0.05}{2} = 0.025; n = (20)(2) = 40$$

$$P = C - 28.99 = 1.25F - 28.99$$
 Since the bond sells at a discount

$$P = Fra_{\overline{40}} + Cv^{40} = > 1.25F - 28.99 = F(0.03) \left(\frac{1 - (1.025)^{-40}}{0.025}\right) + 1.25F(1.025)^{-40}$$

Solve for F.

RESPONSE	ANSWER	ANSWER RANGE
1.	923.88	[923.78, 923.98]

Problem 11

A callable bond matures in 10 years for 1000. The bond pays semi-annual coupons of 42.

The bond may be called at the end of year 6 or year 8. The call value at the end of year 6 is 1085. The call value at the end of year 8 is 1043.

Andrew purchased this bond at issue to yield 6% convertible semi-annually. The price is P_1 .

Two years after issue, Andrew sells the bond for a price of P₂. to yield 5% convertible semi-annually.

Calculate P₂ - P₁.

(Round your answer to the nearest two decimal places)

IATEX ?



Correct Answer: 14.13

N	I/Y	PMT	FV	Price
12	3	-42	-1085	1179.07
16	3	-42	-1043	1177.53
20	3	-42	-1000	1178.53

 $P_1 = 1177.53$

At the end of two years, to find the price, you now have a bond that matures in 8 years and can be called at the end of 4 years or 6 years. You calculate the price at each of those points and then choose the lowest.

N	I/Y	PMT	FV	Price
8	2.5	-42	-1085	1191.66
12	2.5	-42	-1043	1206.35
16	2.5	-42	-1000	1221.94

 $P_2 = 1191.66$

 $P_2 - P_1 = 1191.66 - 1177.53 = 14.13$

Unlimited attempts

RESPONSE	ANSWER	Answer range
1.	14.13	[14.10, 14.16]

Problem 12

A 30 year special bond has annual coupons and a maturity value of 100,000. The annual coupons are 500 in the first year, 1,000 in the second year, 1,500 in the third year and continue to increase by 500 each year.

Calculate the amount of principal in the 29th coupon assuming that the bond was purchased to a yield rate of 4.4%.

(Round your answer to the nearest two decimal places)

LATEX ?





Correct Answer: 9.246.41

We need the book value right after the 28th coupon which is the present value of future cash flows = Present value of 29th coupon + the present value of the 30th coupon + the present value of the maturity value.

29th coupon = (29)(500) = 14,500 and 30th coupon = (30)(500) = 15,000

Then present value =

 $14{,}500v + 15{,}000v^2 + 100{,}000v^2 = 14{,}500 \, (1.044)^{-1} + 115{,}000 \, (1.044)^{-2} = 119{,}399.67$

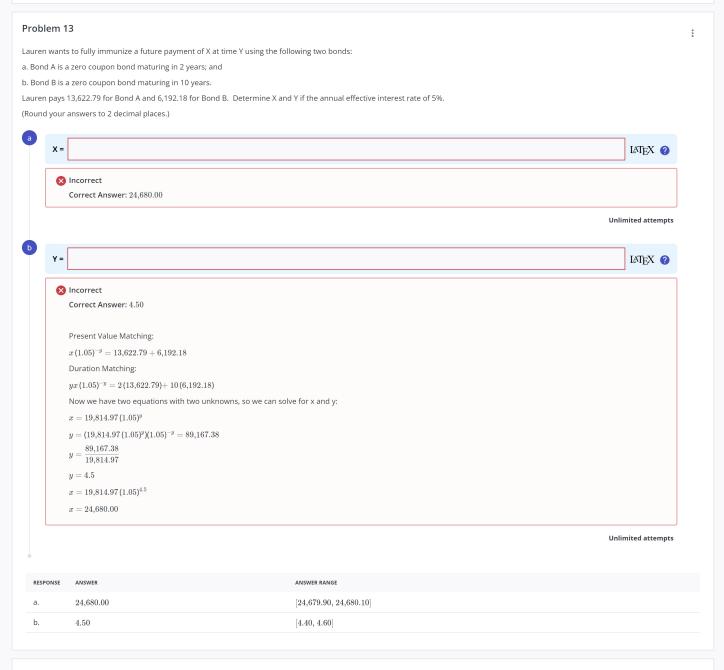
Coupon = 14,500

Interest = 119,399.67(0.044) = 5,253.59

Principal = 14,500 - 5,253.59 = 9,246.41

Unlimited attempts

VARIABLE NAME ANSWER RANGE [0 046 96 0 046 46]



Problem 14 A 10-year bond has a maturity value of 10,000 and semi-annual coupons of 500. Calculate the price of this bond immediately after the 13th coupon at a yield rate of 5.6% convertible semi-annually. (Remember that the price of a bond at any point during its lifetime is the present value of future cash flows). (Round your answer to the nearest two decimal places)

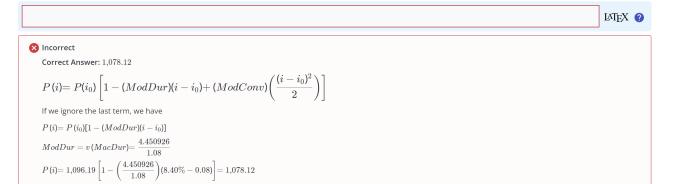


Unlimited attempts

VARIABLE NAME	VALUE	RESPONSE	ANSWER	ANSWER RANGE
ans	11,381.05	1.	11,381.05	$[11,\!380.95,11,\!381.15]$
i	5.6%			
i1	0.056			
i2	5.6			
i3	2.800			

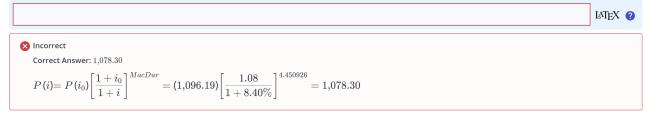
Problem 15

- 15.1 A bond has a Macaulay Duration of 4.450926 and a Macaulay Convexity of 21.19773 when calculated using an annual effective interest rate of 8%. The price of the bond is 1096.19. (Round your answers to 2 decimal places.)
 - $a.\ Estimate the price of the bond if the annual interest rate increases to 8.40\% using the first order modified approximation.$



Unlimited attempts

15.2 b. Estimate the price of the bond if the annual interest rate increases to 8.40% using the first order Macaulay approximation.



Unlimited attempts

15.3 c. The values in this problem are based on a 5 year bond with annual coupons of 70 and a maturity value of 1,200. What would the actual price be at 8.40%.



Unlimited attempts

VARIABLE NAME	VALUE	RESPONSE	ANSWER	ANSWER RANGE
\boldsymbol{A}	1,078.12	15.1.	1,078.12	$[1,\!078.02,1,\!078.22]$
B	1,078.30	15.2.	1,078.30	$[1,\!078.20,1,\!078.40]$
C	1 078 31	15.2	1 በ78 21	[1 078 21 -1 078 41]

Problem 16

Yegor borrows 25,000 from Kunyang. The loan will be repaid with three annual payments of 9619.95.

Kunyang reinvests the payments at an annual effective rate of $r. \ \ \,$

After reinvestment, Kunyang realizes an annual yield rate of 7% on the loan.

Determine r.

LATEX ?

Incorrect

Correct Answer: 0.06

We know the accumulated value for Kunyang at time 3 is the same:

$$25,000 (1.07)^3 = 9,619.95 (1+r)^2 + 9,619.95 (1+r) + 9,619.85 = 30,626.075$$

$$\Rightarrow 9{,}619.95\,(1+r)^2 + 9{,}619.95\,(1+r) - 21{,}006.125 = 0$$

$$\mathsf{Let}\,(1+r) = x$$

$$\Rightarrow 9,619.95x^2 + 9,619.95x - 21,006.125 = 0$$

$$=\frac{-9{,}619.95\pm\sqrt[2]{(9{,}619.95)^2-4\,(9{,}619.95)(-21{,}006.125)}}{2\,(9{,}619.95)}$$

$$x = -2.060000, 1.060000$$

So,
$$1+r=1.060000\Rightarrow r=0.060000$$

Unlimited attempts

:

RESPONSE	ANSWER	ANSWER RANGE	
1.	0.06	[0.05999, 0.06001]	

Problem 17 :

Haokun borrowed money to buy a new car. Payments are made monthly. The loan has a nominal interest rate of 6.0% compounded monthly. Immediately after the 18th payment, Haokun has an outstanding balance of 10,500.

Calculate the amount of interest in the 19th payment.

(Round your answer to the nearest two decimal places)

IATEX ? Incorrect

Correct Answer: 52.50

 $Interest = (OLB_{k-1})(i)$

 $10,\!500 \left(\frac{0.060}{12}\right) = 52.50$

Unlimited attempts

VARIABLE NAME	VALUE	RESPONSE ANSWER	ANSWER RANGE
ans	52.50	1. 52.50	[52.40, 52.60]
i	6.0%		
i1	0.060		
p	10,500		

Problem 18

The following is a list of prices for zero-coupon bonds with par value of 1,000 and maturities as follows: $\frac{1}{2} \left(\frac{1}{2} \right) = \frac{1}{2} \left(\frac{1}{2} \right) \left(\frac{1}{2} \right)$

Years to Maturity	Price of Bond
1	836.51
2	702.20

4	/03.30
3	719.12
4	905.26

Calculate $f_{[2,4]}$.

LATEX ?



 ${\bf Correct\ Answer:}\ -0.12$

First, we need to find spot rates using bootstrapping.

Using the second bond,

$$703.30 = \frac{1000}{(1+r_2)^2} \Longrightarrow r_2 = \sqrt{\frac{1000}{703.30}} - 1 = 0.192421207$$

We can also use the forth bond,

$$905.26 = \frac{1000}{(1+r_4)^4} \Longrightarrow r_4 = \left(\frac{1000}{905.26}\right)^{\frac{1}{4}} - 1 = 0.025195443$$
 $(1+r_2)^2 \left(1+f_{[2,4]}\right)^2 = (1+r_4)^4$

$$(1+r_2)^2 (1+f_{[2,4]})^2 = (1+r_4)^4$$

$$\implies f_{[2,4]} = \sqrt{\frac{(1+r_4)^4}{(1+r_2)^2}} - 1 = \sqrt{\frac{(1+0.025195443)^4}{(1+0.192421207)^2}} - 1 = -0.12$$

Unlimited attempts

VARIABLE NAME	VALUE
ans	-0.12
P1	836.51
P2	703.30
P3	719.12
P4	905.26
r2	0.192421207
r4	0.025195443

RESPONSE	ANSWER	ANSWER RANGE
1	_0.12	[_2 12 1 88]