Student Name:_____

Purdue ID:_____

MA 373 – Spring 2025
Midterm 1

PHYS 112 8:00 – 9:00 PM
Wednesday, February 19th, 2025

INSTRUCTIONS

- Do not open this exam until you are told to do so.
- There are 60 points possible on this test divided into two sections, each worth 30 points.
- You have 60 minutes to complete this exam.
- Be sure you have filled in your name and Purdue ID in the slots at the top of the page.
- Show all work to maximize partial credit.
- Be sure all cell phones are silenced and put away out of view. This policy applies to smart watches as well.
- Headphones are not permitted unless prior approval was granted by your instructor.
- Formula sheets are not permitted.
- You are only permitted to use calculator(s) from the following list:
    - BA II Plus
    - BA II Plus Professional
    - BA-35
    - TI–30Xa or TI–30XA (same model just different casing)
    - TI-30X II (IIS solar or IIB battery)
    - TI-30XS MultiView (or XB battery)
- When time expires, put your pencil down and close your exam. Failure to do so will result in automatic disqualification from obtaining University-Earned Credit.

PURDUE HONORS PLEDGE

"As a boilermaker pursuing academic excellence, I pledge to be honest and true in all that I do. Accountable together - we are Purdue."

STUDENT AGREEMENT

By signing below,

- I agree with the Purdue Honors Pledge stated above.
- I will not give or receive any assistance on this exam, and I will report any infractions of the honors pledge.
- I acknowledge that I only used calculator(s) from the above list.
- I am claiming all work in this exam as my own.

X_____

SECTION 1

30 Points Total

8 Questions (3-4 points each)

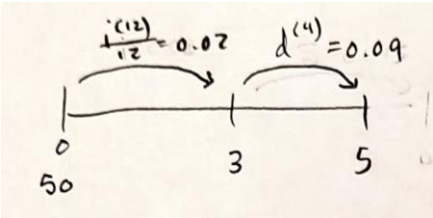
1. (4 points) You invest 50 into an account today.

   During the first 3 years, the account earns an effective monthly interest rate of 2%.

   During the next 2 years, the account earns a discount rate of 9% compounded quarterly.

   Determine the amount you have at the end of 5 years to two decimal places.

   Solution:



$$A(5) = 50(1+0.02)^{(12)(3)}\left(1-\frac{0.09}{4}\right)^{(-4)(2)}$$

$$\Rightarrow A(5) = 122.36$$

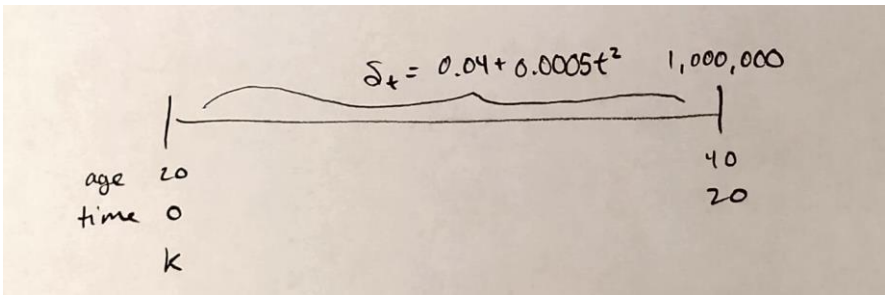
Points	
2	Correct equation that uses $d^{(4)} = 0.09$ • 1 point for recognizing correct symbol for what's given • 1 point for using properly in accumulation function
2	Correct equation that uses $\frac{i^{(12)}}{12}$ • 1 point for recognizing correct symbol for what's given • 1 point for using properly in accumulation function

2. (4 points) You want to have 1,000,000 on your 40th birthday. Your 20th birthday is today.

   You want to invest K in an account which has a force of interest of $\delta_t = 0.04 + 0.0005t^2$ so that you will exactly achieve your goal on your 40th birthday.

   Determine K to two decimal places.

   Solution:



$$Ka(20) = 1,000,000$$

$$Ke^{\int_0^{20} \delta_t \cdot dt} = 1,000,000 \Rightarrow Ke^{\int_0^{20} (0.04+0.0005t^2) \cdot dt} = 1,000,000$$

$$Ke^{0.04t + \frac{0.0005t^3}{3}\Big]_0^{20}} = 1,000,000$$

$$K = \frac{1,000,000}{e^{2.13333333}} = 118,441.83$$

Points	
1	Correct equation of value
2	Correct treatment of accumulation function with delta
1	Integration done correctly (this includes correctly defining limits of integration

3. (3 points) You are given that $a(t) = 1 + 0.02t + 0.0005t^2$.

   Calculate δ_{10} to three decimal places.

   Solution:

$$\delta_t = \frac{a'(t)}{a(t)} = \frac{0.02 + 0.0005(2)t}{1 + 0.02t + 0.0005t^2} = \frac{0.02 + 0.001t}{1 + 0.02t + 0.0005t^2}$$

$$\delta_{10} = \frac{0.02 + 0.001(10)}{1 + 0.02(10) + 0.0005(10)^2} = \frac{0.03}{1.25} = 0.024$$

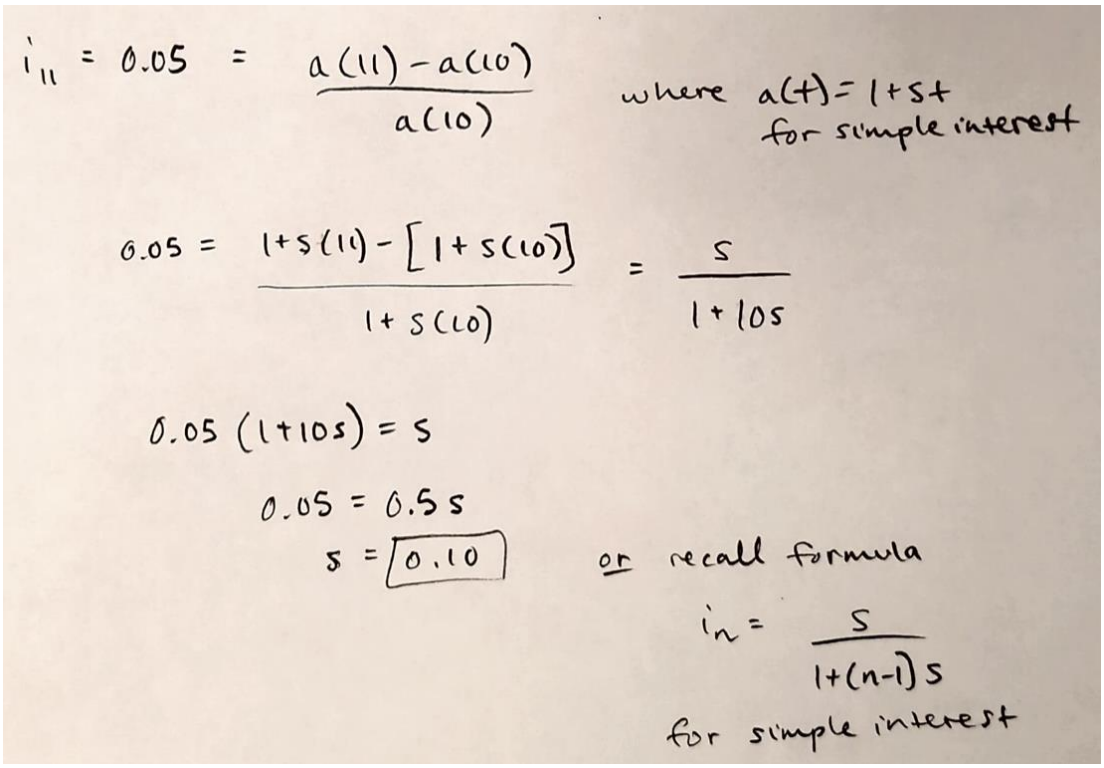
Points	
2	Correct relationship defined between equation that uses δ_r and the accumulation function
1	Correct method to find δ_{10}

4. (3 points) You invest money in an account earning simple interest s.

   You are also given that $i_{11} = 0.05$.

   Calculate s to four decimal places.

   Solution:

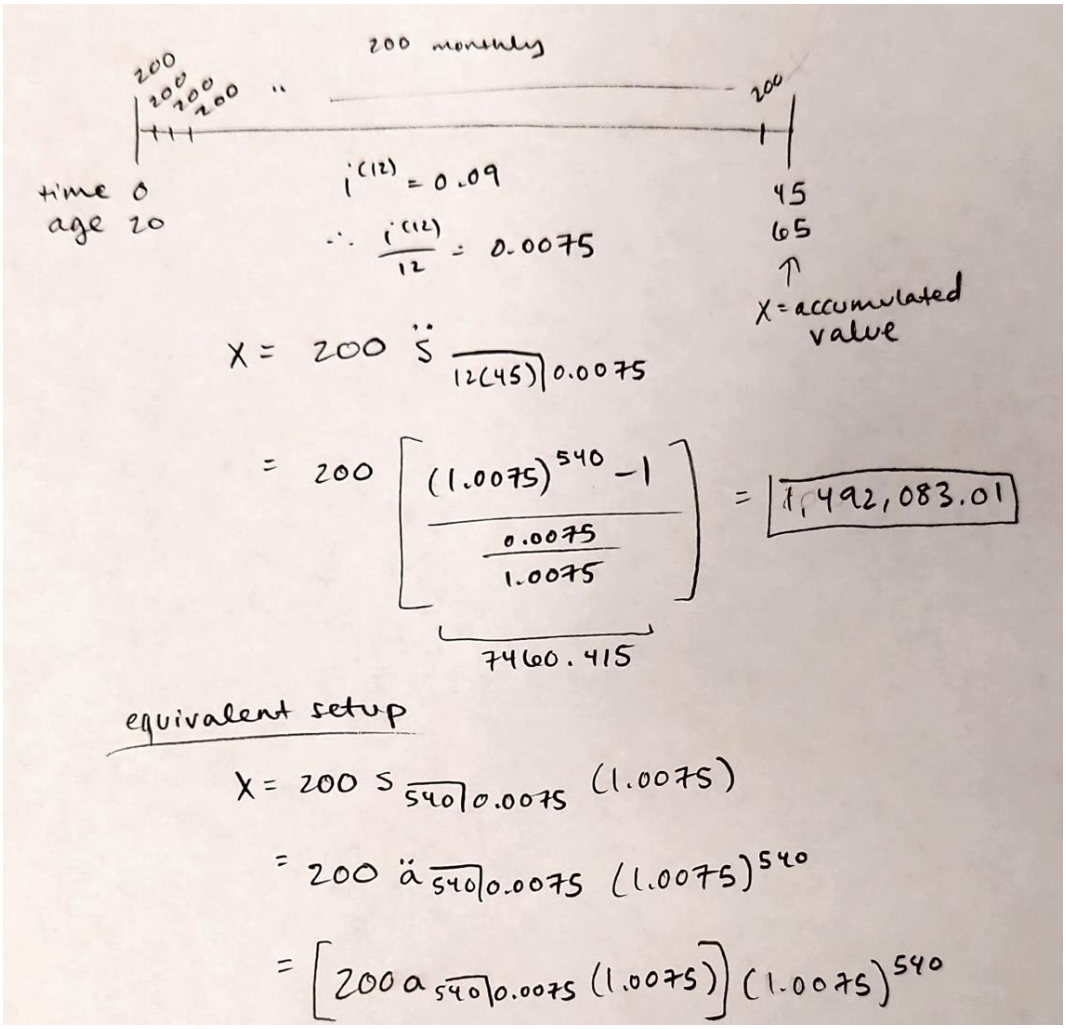


Points	
3	Correct setup for effective interest rate (whether from accumulation function or recalling the direct formula)

5. (4 points) You want to accumulate a sum of money at age 65 so you can retire. In order to accomplish this goal, you plan to deposit 200 per month at the beginning of each month. You are currently exactly 20 years old and your first deposit will be today. Your final deposit will be 1 month before you turn 65.

   Given that you earn an interest rate of 9% compounded monthly, calculate your accumulated value at age 65 to the nearest dollar.

   Solution:



Points	
1	Correct interest rate
3	Correct setup for finding accumulated value of stream of payments • 1 point for correct number of payments • 1 point for correct timing of payments (beginning) • 1 point for valid annuity formula setup

6. (4 points) You are the beneficiary of a deferred annuity. The deferred annuity will pay you 60 monthly payments of 1000 with the first payment at the end of 8 months.

   Given an interest rate of 6% compounded monthly, calculate the present value of your annuity to two decimal places.

   Solution:

$$n = 60,\ i^{(12)} = 0.06 \Rightarrow \frac{i^{(12)}}{12} = \frac{0.06}{12} = 0.005$$

$$PMT_{Monthly} = 1,000$$

$$PV\left(\text{at Month } 7\right) = P_7$$

Using the Annuity Formula:

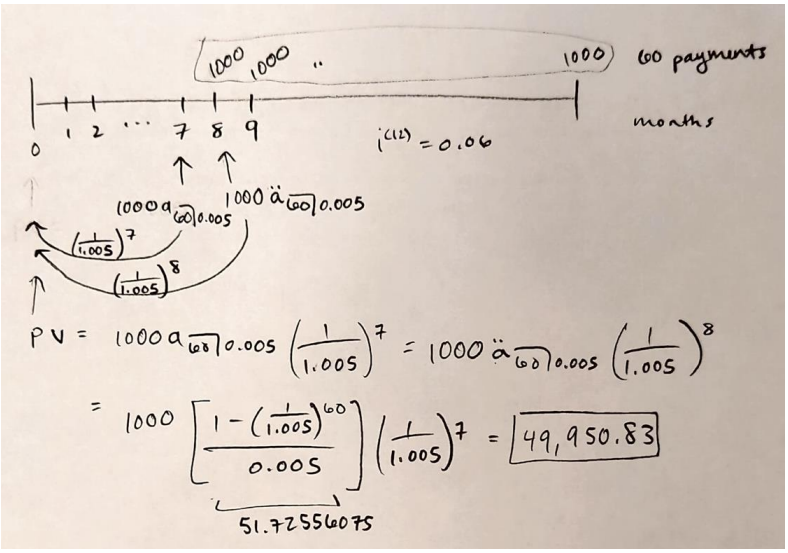
$$P_7 = \left(1,000\right)\left(\frac{1-\left(1.005\right)^{-60}}{0.005}\right) = 51,725.56075$$

Alternatively, you can use the BA-II Plus:

$$\boxed{N} \leftarrow 60;\ \boxed{PMT} \leftarrow 1000;\ \boxed{I/Y} \leftarrow 0.5\ \boxed{CPT}\ \boxed{PV}\ \Rightarrow 51,725.56075 = P_7$$

Lastly, we need to discount this to Month 0:

$$PV\left(\text{at Month } 0\right) = \left(51,725.56\right)\left(1.005\right)^{-7} = 49,950.84$$



Points	
1	Correct interest rate
1	Correct annuity setup
2	Correct handling of deferral period

7. (4 points) You received a birthday gift from your grandparents today, at age 20. However, instead of giving it to you as a lump sum, they would like to give it to you as a perpetuity due with semiannual payments (2 payments per year) of P.

   They purchased the gift today for 5,149.48 using an annual effective interest rate of 4%,.

   Calculate P to two decimal places.

   Solution:

$$\left(1+\frac{i^{(2)}}{2}\right)^2=(1+i)\Rightarrow\frac{i^{(2)}}{2}=(1.04)^{\frac{1}{2}}-1=0.019803903$$

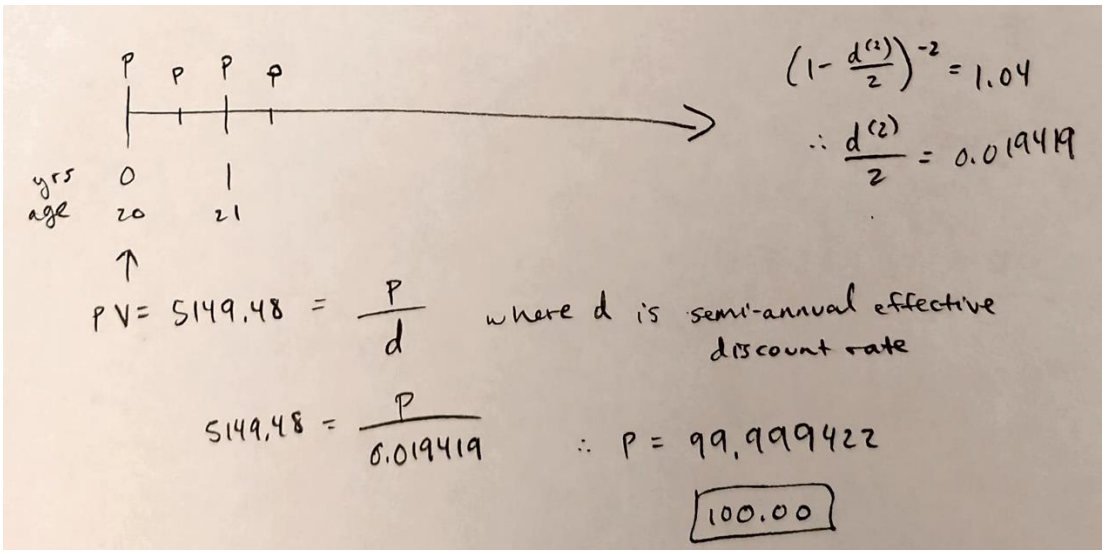
Let $i_{0.5}$ be the semiannual effective interest rate $\left(i_{0.5}=\frac{i^{(2)}}{2}\right)$

By Perpetuity Due Formula (adjusted for semiannual payments):

$$PV=P\left(\frac{1}{d_{0.5}}\right)=P\left(\frac{1}{i_{0.5}v}\right)=P\left(\frac{1}{i_{0.5}}\right)\left(v^{-1}\right)=P\left(\frac{1}{i_{0.5}}\right)(1+i_{0.5})$$

$$5{,}149.48=P\left(\frac{1}{0.019803903}\right)(1.019803903)$$

$$\Rightarrow P=5{,}149.48\left(\frac{0.019803903}{1.019803903}\right)=99.999\Rightarrow P=100.00$$

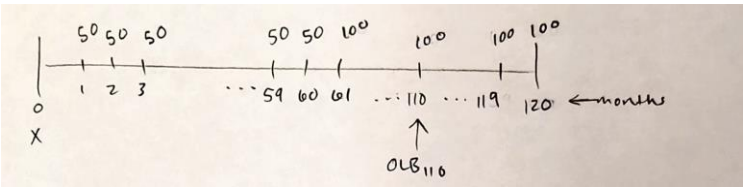


Points	
2	Correct interest rate
2	Correct equation of value and perpetuity setup

8. (4 points) You have taken out a loan of X to purchase a car. You will repay the loan with 120 monthly payments. The first 60 monthly payments are 50 each and the second 60 monthly payments are 100 each.

   Given that the interest rate on the loan is 12% compounded monthly, find the outstanding loan balance right after the 110th payment to two decimal places.

   Solution:



$$N = 120 - 110 = 10\ ;\ \frac{i^{(12)}}{12} = \frac{0.12}{12} = 0.01\ ;\ PMT = 100$$

Using formula:

$$OLB_{110} = 100a_{\overline{10}|\,i=0.01} = 100\left(\frac{1-(1.01)^{-10}}{0.01}\right) = 947.13$$

Using BA-II Plus:

$\boxed{N} = 10$

$\boxed{I/Y} = 1$

$\boxed{PMT} = -100$

$\boxed{CPT}\ \boxed{PV} \Rightarrow 947.13$, so $OLB_{110} = 947.13$

Points	
1	Correct interest rate
3	Correct OLB setup, prospective or retrospective, by hand or on the calculator

SECTION 2

30 Points Total

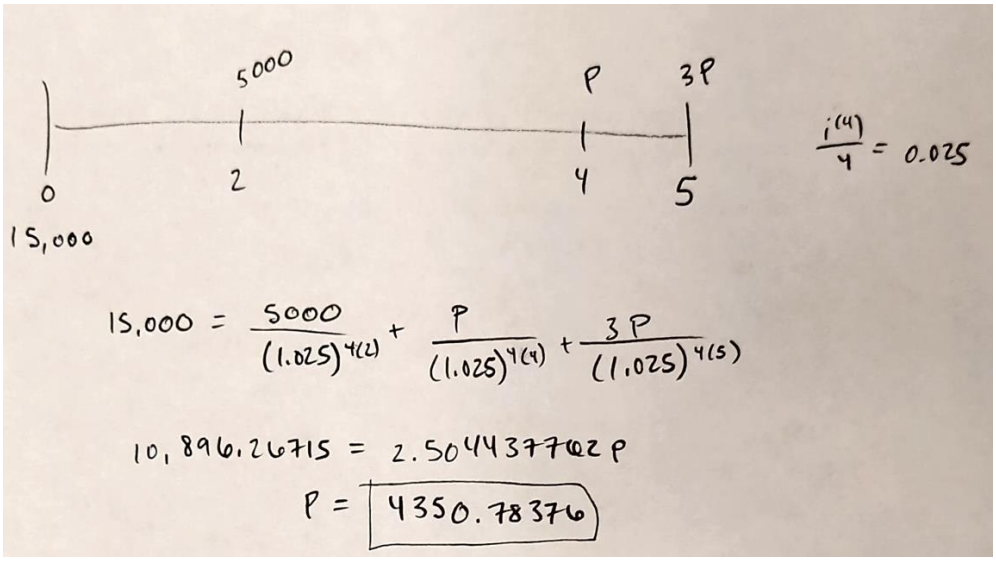
5 Questions (6 points each)

1. (6 points) You borrow 15,000 from the bank at an effective quarterly interest rate of 2.5%.

   You repay the bank with 3 payments. There is a payment of 5,000 at the end of 2 years, a payment of P at the end of 4 years, and a payment of $3P$ at the end of 5 years.

   Determine P to two decimal places.

   Solution:



$or\ \mathrm{i} = (1.025)^4 - 1 = 0.103812891$

$$15,000 = 5,000(1+0.103812891)^{(-2)} + P(1+0.103812891)^{(-4)} + 3P(1+0.103812891)^{(-5)}$$

$$15,000 - 5,000(1.103812891)^{-2} = P\left[(1.103812891)^{-4} + 3(1.103812891)^{-5}\right]$$

$$P = \frac{15,000 - 5,000(1.103812891)^{-2}}{(1.103812891)^{-4} + 3(1.103812891)^{-5}}$$

$$\Rightarrow P = 4,350.78$$

Points	
2	Correct interest rate (convert to annual effective rate or use proper exponents)
4	Correct equation of value (discounting to 0 or accumulating to time 5) • Note: if marked for incorrect interest, as long as the incorrect rate is used properly for discounting, credit should be given for this part

2. (6 points) XYZ Corporation has the option to invest in one of two projects. The two projects have equal net present values when calculated using a 5% annual effective interest rate. They are expected to produce the following cash flows:

Time	Project 1	Project 2
0	-10,000	X
1	-3,000	10,000
2	7,000	-20,000
3	12,000	45,000
4	10,000	30,000

At the end of 4 years, no further cash flows are expected.

Calculate the internal rate of return for Project 2 to four decimal places.

Solution:

$NPV_1 = -10,000 - 3,000(1.05)^{-1} + 7,000(1.05)^{-2} + 12,000(1.05)^{-3} + 10,000(1.05)^{-4} = 12,085.14$

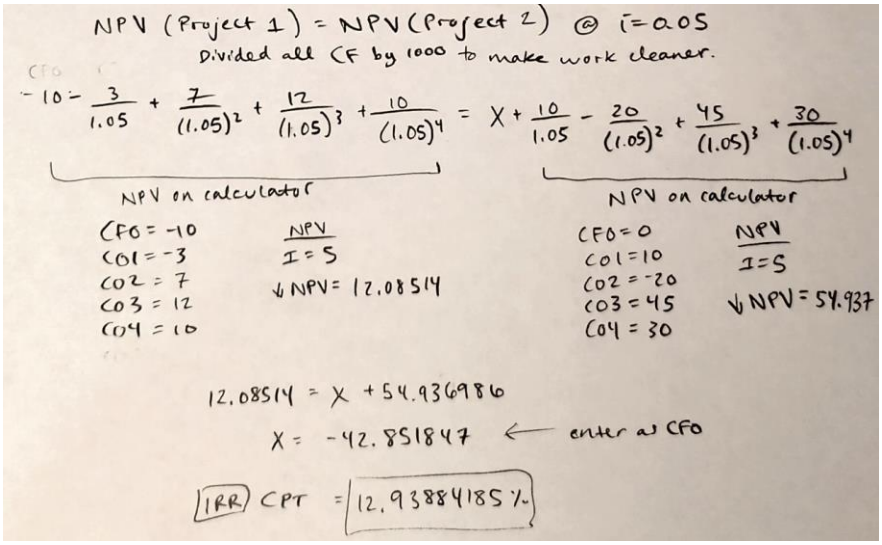
NPV for Project 2:

$12,085.14 = X + 10,000(1.05)^{-1} - 20,000(1.05)^{-2} + 45,000(1.05)^{-3} + 30,000(1.05)^{-4}$

$\Rightarrow X = -42,851.85$

Enter X into CF0 with Project 2 Cashflows in calculator

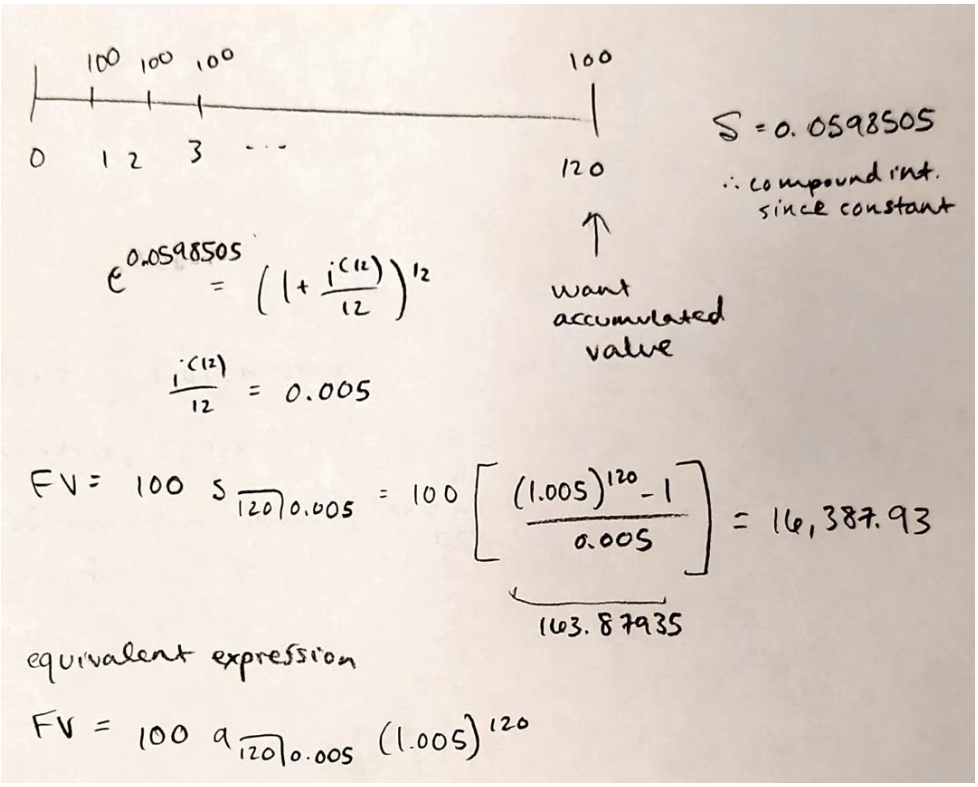
IRR CPT on calculator, IRR=12.93884185%



Points	
4	Correct setup to find X
2	Correct method to find IRR with X solved for in Project 2

3. (6 points) Given $\delta = 0.0598505$, calculate the accumulated value of 100 paid at the end of each month for 10 years to two decimal places.

Solution:



Points	
2	Correct conversion to monthly effective interest rate
4	Correct setup for accumulated value of stream of end of period payments • 2 points for setting up as stream of payments rather than single point • 2 points for finding accumulated value (rather than present value) Note: If setup correctly for accumulated value of a single payment of 100, give 2 points for demonstrating understanding of using constant force of interest

4. (6 points) You have won the lottery. You have the choice of the following three options:

   a. A lump sum of 5,000,000.

   b. An annuity immediate with 10 years of quarterly payments of 166,142.13.

   c. A perpetuity immediate with level annual payments of P that begin 5 years from today.

All payments have the same present value at an annual interest rate of i.

Determine P, rounded to the nearest dollar.

Solution:

$PV(a) = PV(b) = 5,000,000 \Rightarrow$ use your BA-II+ Calculator to find $\frac{i^{(4)}}{4}$:

$\boxed{N} \leftarrow (10)(4) = 40; \boxed{PV} \leftarrow 5,000,000; \boxed{PMT} \leftarrow -166,142.13$

$\boxed{CPT}\boxed{I/Y} \rightarrow 1.467384731\% = \frac{i^{(4)}}{4}$

$i = \left(1 + \frac{i^{(4)}}{4}\right)^4 - 1 = (1.01467384731)^4 - 1 = 0.06$

$PV(a) = PV(c) = 5,000,000$

$5,000,000 = P\left(\frac{1}{0.06}\right)\left(\frac{1}{1.06}\right)^4$, so $P = 378,743$ if interpreted as first payment is at t=5

*if interpreted as first payment at t=6, then $5,000,000 = P\left(\frac{1}{0.06}\right)\left(\frac{1}{1.06}\right)^5$ and $P = 401,467.67$

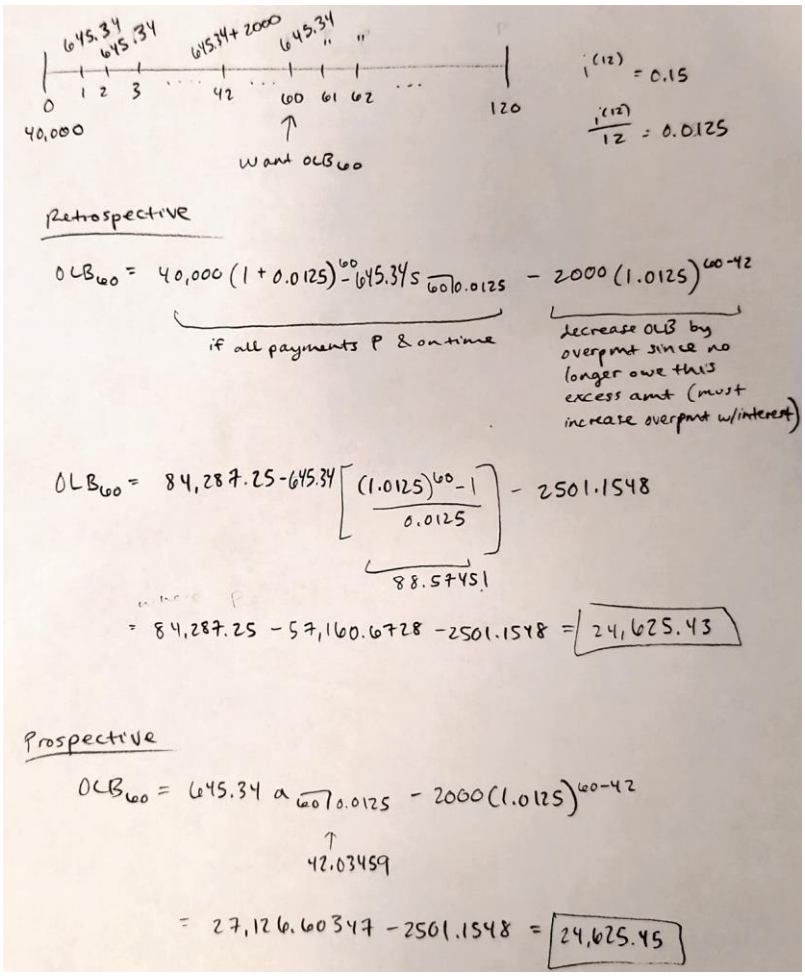
Points ***2 answers accepted for this problem due to unclear wording***	
3	Correct setup to solve for quarterly effective interest rate • 2 points for correct setup of equation of value or calc input for ann immediate • 1 point for recognizing interest rate from calculator as quarterly effective rate
3	Correct setup for deferred perpetuity • 1 point for perpetuity immediate formula (P/i) • 1 point for correct interest rate (converted quarterly effective to annual) • 1 point for correctly adjusting for deferral period *Note: subtracted 1 for any incomplete calculation since many didn't solve (floor 1 if received any credit for setup), subtracted 0.5 if math error

5. (6 points) You borrow 40,000 to buy a car. You will repay the loan with 120 level monthly payments of 645.34. The interest rate on the loan is 15% compounded monthly.

   You make the first 60 payments on time. Additionally, at the end of the 42nd month, you make an additional payment of 2,000 over and above the normal payment.

   Determine OLB_{60} to two decimal places.

   Solution:



Points	
2	Correct handling of overpayment • 1 point for subtracting amount of overpayment from OLB • 1 point for correct interest accumulation
1	Correct effective monthly interest rate
3	Correct setup for OLB (retrospective or prospective) before overpayment