Student Name:_____

Purdue ID:_____

STAT 472 – Spring 2025
Quiz 2

MTHW 304 12:55 – 1:15 PM
Tuesday, February 4th, 2025

INSTRUCTIONS

- Do not open this quiz until you are told to do so.
- There are 20 points possible on this quiz.
- You have 20 minutes to complete this quiz.
- Be sure you have filled in your name and Purdue ID in the slots at the top of the page.
- Show all work to maximize partial credit.
- Be sure all cell phones are silenced and put away out of view. This policy applies to smart watches as well.
- Headphones are not permitted unless prior approval was granted by your instructor.
- Formula sheets are not permitted.
- You are only permitted to use calculator(s) from the following list:
    - BA II Plus
    - BA II Plus Professional
    - BA-35
    - TI–30Xa or TI–30XA (same model just different casing)
    - TI-30X II (IIS solar or IIB battery)
    - TI-30XS MultiView (or XB battery)
- When time expires, put your pencil down and close your exam. Failure to do so will result in automatic disqualification from obtaining University-Earned Credit.

PURDUE HONORS PLEDGE

"As a boilermaker pursuing academic excellence, I pledge to be honest and true in all that I do. Accountable together - we are Purdue."

STUDENT AGREEMENT

By signing below,

- I agree with the Purdue Honors Pledge stated above.
- I will not give or receive any assistance on this exam, and I will report any infractions of the honors pledge.
- I acknowledge that I only used calculator(s) from the above list.
- I am claiming all work in this exam as my own.

X_____

1. (5 points) You are given that $S_0(x) = \frac{8100 - x^2}{8100}$ for $0 \le x \le 90$.

   Calculate ${}_{3|2}q_{50}$.

   Solution:

$$ {}_t p_{50} = \frac{S_0(50+t)}{S_0(50)} = \frac{8100-(50+t)^2}{8100-50^2} = \frac{5600-100t-t^2}{5600} $$

$$ {}_{3|2}q_{50} = {}_3 p_{50} - {}_5 p_{50} = \frac{5600-100(3)-3^2}{5600} - \frac{5600-100(5)-5^2}{5600} $$

$$ = 0.944821429 - 0.90625 = 0.038571429 $$

Points	
2	Correct setup for tpx
3	Correct setup with ${}_3 p_{50} - {}_5 p_{50}$ or $\left({}_3 p_{50}\right)\left({}_2 q_{53}\right)$

2. (5 points) You are given the following mortality table:

(x)	q_x
60	0.0012
61	0.0029
62	0.0034
63	0.0041
64	0.0055

Calculate $e_{61:\overline{3}|}$.

Solution:

$$e_{61:\overline{3}|} = \sum_{k=1}^{3} {}_k p_{61} = p_{61} + {}_2 p_{61} + {}_3 p_{61}$$

$$= (1-0.0029) + (1-0.0029)(1-0.0034) + (1-0.0029)(1-0.0034)(1-0.0041)$$

$$= 0.9971 + 0.99371 + 0.98964 = 2.98045$$

Points	
2	Correct equation for capped curtate expectation of life
1	Start summation at k=1
2	Correct terms in summation when expanding • 1 point for correct number of terms • 1 point for setup to calculate each ${}_k p_x$

3. (10 points) You are given that ${}_t q_{80} = 0.0025t^2$ for $0 \le t \le 20$

a) (5 points) Calculate $\overset{\circ}{e}_{80}$.

Solution:

$${}_t p_{80} = 1 - {}_t q_{80} = 1 - 0.0025t^2$$

$$e^{\circ}_{80} = \int_0^{20} {}_t p_{80}.dt = \int_0^{20} 1 - 0.0025t^2 dt = t - \frac{0.0025t^3}{3}\Bigg|_0^{20} = 13.33333$$

Points	
5	Correct setup for complete expectation of life • 2 point for correct integration limits • 2 point for correct integrand • 1 point for correct integration

b) (5 points) Calculate $Var(T_{80})$.

Solution:

$$E\left[T_{80}^2\right] = 2\int_0^{20} t({}_t p_{80}).dt = 2\int_0^{20} t - 0.0025t^3.dt = 2\left[\frac{t^2}{2} - \frac{0.0025t^4}{4}\Bigg|_0^{20}\right] = 200$$

$$E\left[T_{80}\right] = e^{\circ}_{80} = 13.3333$$

$$Var(T_{80}) = E\left[T_{80}^2\right] - E\left[T_{80}\right]^2 = 200 - 13.3333^2 = 22.2222$$

Points	
3	Correct setup for expected value of $E[T^2]$ • 1 point for correct integration limits • 1 point for correct integrand and coefficient • 1 point for correct integration
2	Correct setup for calculating variance (using (a) as part of calculation