

Student Name: _____

Purdue ID: _____



STAT 472 – Spring 2025 Quiz 3
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MTHW 304 12:55 – 1:15 PM
Tuesday, February 18th, 2025

INSTRUCTIONS

- Do not open this quiz until you are told to do so.
- There are 20 points possible on this quiz.
- You have 20 minutes to complete this quiz.
- Be sure you have filled in your name and Purdue ID in the slots at the top of the page.
- Show all work to maximize partial credit.
- Be sure all cell phones are silenced and put away out of view. This policy applies to smart watches as well.
- Headphones are not permitted unless prior approval was granted by your instructor.
- Formula sheets are not permitted.
- You are only permitted to use calculator(s) from the following list:
 - BA II Plus
 - BA II Plus Professional
 - BA-35
 - TI-30Xa or TI-30XA (same model just different casing)
 - TI-30X II (IIS solar or IIB battery)
 - TI-30XS MultiView (or XB battery)
- When time expires, put your pencil down and close your exam. Failure to do so will result in automatic disqualification from obtaining University-Earned Credit.

PURDUE HONORS PLEDGE

“As a boilermaker pursuing academic excellence, I pledge to be honest and true in all that I do.
Accountable together - we are Purdue.”

STUDENT AGREEMENT

By signing below,

- I agree with the Purdue Honors Pledge stated above.
- I will not give or receive any assistance on this exam, and I will report any infractions of the honors pledge.
- I acknowledge that I only used calculator(s) from the above list.
- I am claiming all work in this exam as my own.

X _____

1. (5 points) You are given the following incomplete mortality table.

(x)	l_x	d_x	q_x	p_x
97	--	--	--	0.44
98	--	--	0.20	--
99	--	100	1.00	--

You are also given:

- $l_{92} = 1000$ and $l_{95} = 800$
- Assume constant force of mortality between integral ages.

Calculate ${}_{3|2.6}q_{92}$.

Solution:

$$\begin{aligned}
 {}_{3|2.6}q_{92} &= \frac{l_{95} - l_{97.6}}{l_{92}} \\
 &= \frac{800 - l_{97.6}}{1000} \\
 &= \boxed{0.6264061}
 \end{aligned}$$

$$\begin{aligned}
 l_{99} &= 100 \\
 l_{98}(1 - 0.20) &= l_{99} \quad \downarrow 100 \\
 \therefore l_{98} &= 125 \\
 l_{97}(0.44) &= l_{98} \quad \downarrow 125 \\
 l_{97} &= 284.09 \\
 \text{CFM} \\
 l_{97.6} &= (l_{97})^{0.4} (l_{98})^{0.6} \\
 &= 173.5939164
 \end{aligned}$$

Points	
2	Correct setup for ${}_{3 2.6}q_{92}$
3	Correct setup for $l_{97.6}$ 1 point for geometric interpolation for CFM 1 point for l_{98} 1 point for l_{97} *note that if both l bounds are incorrect, if method for flowing from one to the next is correct give 1 point out of the 2

2. (5 points) You are given that the mortality for all Purdue students follow the Standard Ultimate Life Table. There are 15,000 students who are age 20 at Purdue University.

Let L_{15} be the random variable representing the number who will still be alive at age 35.

Calculate the $Var(L_{15})$.

Solution:

$${}_{15}p_{20} = \frac{l_{35}}{l_{20}} = \frac{99,556.70}{100,000} = 0.995567$$

$$Var[L_{15}] = 15,000({}_{15}p_{20})({}_{15}q_{20}) = 15,000(0.995567)(1 - 0.995567) = 66.20022767$$

Points	
2	Correct setup for ${}_{15}p_{20}$
2	Correct setup for variance (recognize binomial)

3. For a 2-year select and ultimate table, you are given:

- i. $q_{96} = 0.350$, $q_{97} = 0.475$, $q_{98} = 0.675$
- ii. $q_{[x]} = 0.5q_x$ for all x .
- iii. $q_{[x]+1} = 0.4q_{x+1}$ for all x .
- iv. $l_{[96]} = 10000$

a. (6 points) Find $l_{[97]}$.

Solution:

$l_{[96]} = 10,000$, find $l_{[97]}$

* First ultimate age in common : 99
or 98 since beyond 2-yr select period

$$\underbrace{l_{[96]} p_{[96]} p_{[96]+1} p_{[96]+2}}_{\substack{\text{advances underwritten} \\ @ 96 \text{ to ultimate age } 99}} = \underbrace{l_{[97]} p_{[97]} p_{[97]+1}}_{\substack{\text{advances underwritten} \\ @ 97 \text{ to ultimate age } 99}}$$

$$10,000 \left[1 - \overbrace{(0.5)(0.35)}^{0.175} \right] \left[1 - \overbrace{(0.4)(0.475)}^{0.19} \right] (1 - 0.675) = l_{[97]} \left[1 - \overbrace{(0.5)(0.475)}^{0.2375} \right] \left[1 - \overbrace{(0.4)(0.675)}^{0.27} \right]$$

$$10,000 (0.825)(0.81)(0.325) = l_{[97]} (0.7625)(0.73)$$

$$l_{[97]} = \boxed{3901.751628}$$

Points	
3	Correct determination of corresponding $p_{[x]}$ and $p_{[x]+1}$ values
3	Correct equation of value containing $l_{[96]}$ and $l_{[97]}$

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3. [Problem Restated] For a 2-year select and ultimate table, you are given:

- i. $q_{96} = 0.350, q_{97} = 0.475, q_{98} = 0.675$
- ii. $q_{[x]} = 0.5q_x$ for all x .
- iii. $q_{[x]+1} = 0.4q_{x+1}$ for all x .
- iv. $l_{[96]} = 10000$

b. (4 points) Given $A_{[97]+1} = 0.80$ and $v = 0.90$, calculate $A_{[97]}$.

Solution:

$$\begin{aligned}
 A_{[97]+1} &= 0.8 \\
 v &= 0.9 \\
 A_{[97]} &= vq_{[97]} + vP_{[97]}A_{[97]+1} \\
 &= (0.9)(0.5)(0.475) + (0.9)(1 - (0.5)(0.475))(0.8) \\
 &\quad \underbrace{\hspace{1.5cm}}_{q_{[97]} = 0.2375} \quad \underbrace{\hspace{1.5cm}}_{P_{[97]} = 0.7625} \\
 &= \boxed{0.76275}
 \end{aligned}$$

Points	
1	Correct setup for $p_{[x]}$
3	Correct setup for recursive formula • 1 point for proper use of v terms • 1 point for correct placement of p and q • 1 point for treatment of A's