Student Name:_____

Purdue ID:_____

STAT 472 – Spring 2025
Quiz 4

MTHW 304 12:50 – 1:15 PM
Tuesday, March 11th, 2025

INSTRUCTIONS

- Do not open this quiz until you are told to do so.
- There are 20 points possible on this quiz.
- You have 25 minutes to complete this quiz.
- Be sure you have filled in your name and Purdue ID in the slots at the top of the page.
- Show all work to maximize partial credit.
- Be sure all cell phones are silenced and put away out of view. This policy applies to smart watches as well.
- Headphones are not permitted unless prior approval was granted by your instructor.
- Formula sheets are not permitted.
- You are only permitted to use calculator(s) from the following list:
    - BA II Plus
    - BA II Plus Professional
    - BA-35
    - TI–30Xa or TI–30XA (same model just different casing)
    - TI-30X II (IIS solar or IIB battery)
    - TI-30XS MultiView (or XB battery)
- When time expires, put your pencil down and close your exam. Failure to do so will result in automatic disqualification from obtaining University-Earned Credit.

PURDUE HONORS PLEDGE

"As a boilermaker pursuing academic excellence, I pledge to be honest and true in all that I do. Accountable together - we are Purdue."

STUDENT AGREEMENT

By signing below,

- I agree with the Purdue Honors Pledge stated above.
- I will not give or receive any assistance on this exam, and I will report any infractions of the honors pledge.
- I acknowledge that I only used calculator(s) from the above list.
- I am claiming all work in this exam as my own.

X_____

1. (5 points) You are given:

   i. Mortality follows the Standard Ultimate Life Table
   ii. $i = 0.05$

   A 15-year term insurance policy that pays 75,000 at the end of the year of death is issued to (36).

   Calculate the actuarial present value of this policy to two decimal places.

   Solution:

$$APV = 75{,}000\, A^{1}_{36:\overline{15}|}$$

$$= 75{,}000\left[A_{36} - \left({}_{15}E_{36}\right)A_{51}\right] = 75{,}000\left[0.10104 - (0.17589)(0.19780)\right]$$

$$= \boxed{515.91}$$

$${}_{15}E_{36} = \left({}_{5}E_{36}\right)\left({}_{10}E_{41}\right) = \left({}_{10}E_{36}\right)\left({}_{5}E_{46}\right) = 0.17589$$

(0.7817, 0.60879, 0.61046, 0.77956)

$$\text{or} \quad v^{15}\left(\frac{l_{51}}{l_{36}}\right) = \left(\frac{1}{1.05}\right)^{15}\left(\frac{98457.2}{99517.80}\right)$$

$$= 0.475890711$$

Points	
3	Correct setup for term insurance in terms of whole life policies
2	Correct setup for ${}_{15}E_{36}$

2. (5 points) You are given the following mortality table:

(x)	l_x	q_x
60	1000.00	0.0012
61	998.80	0.0029
62	995.90	0.0034
63	992.52	0.0041
64	988.45	0.0055

You are also given:

i. $i = 0.05$

ii. Deaths are uniformly distributed between integral ages.

iii. ${}_2E_{60} = 0.9033107$

Calculate $5000\bar{A}_{60:\overline{2}|}$ to two decimal places.

Solution:

$5000\ \bar{A}_{60:\overline{2}|}$ where $v = \frac{1}{1.05}$ & $\frac{i}{\delta} = 1.0248$ from table

$\bar{A}_{60:\overline{2}|} = \bar{A}^{1}_{60:\overline{2}|} + {}_2E_{60}$

$= \frac{i}{\delta}\left(A^{1}_{60:\overline{2}|}\right) + {}_2E_{60}$

where $1000\,A^{1}_{60:\overline{2}|} = (1000 - 998.8)v + (998.8 - 995.9)v^2$ (1.2; 2.9)

$A^{1}_{60:\overline{2}|} = 0.0037732$

$\therefore 5000\,\bar{A}_{60:\overline{2}|} = 5000\left[(1.0248)(0.0037732) + 0.9033107\right]$

0.9071775

$= \boxed{4535.89}$

Points	
2	Correct setup for endowment insurance (EOY of death)
3	Correct adjustment for moment of death payment • 1 point for i/delta • 2 points for applying only to term insurance piece

3. (10 points) For a special whole life insurance policy issued on (40), you are given:

   i. Death benefits are payable at the end of the year of death.

   ii. The amount of the benefit is 2 if death occurs within the first 20 years and is 1 thereafter.

   iii. Z is the present value random variable for the payments under this insurance.

   iv. $i = 0.03$

   v. You are given the following table of information:

x	A_x	$_{20}E_x$
40	0.36987	0.51276
60	0.62567	0.17878

   vi. $E(Z^2) = 0.24954$

a. (3 points) Define the present value random variable Z for this insurance.

Solution:

$Z = 2V^{K_x+1}$, when $0 \le K_x < 20$

$Z = V^{K_x+1}$, when $K_x \ge 20$

Points	
1	Refers to K rather than T and defines correct ranges • Only half point deducted if minor error in Kx range inequality
1	Includes DB as coefficient
1	Correct expression for discount

b. (7 points) Calculate $Var(Z)$.

Solution:

$E(Z) = 2A_{40} - {}_{20}E_{40}\left(A_{60}\right) = 2(0.36987) - (0.51276)(0.62567) = 0.41892$

$Var(Z) = E(Z^2) - [E(Z)]^2 = 0.24954 - (0.41892)^2 = 0.074046$

Points	
4	Correct setup for expected value (2 points for each term)
3	Correct setup for variance