Student Name:_____

Purdue ID:_____

STAT 472 – Spring 2025
Quiz 5

MTHW 304 12:50 – 1:15 PM
Tuesday, April 1st, 2025

INSTRUCTIONS

- Do not open this quiz until you are told to do so.
- There are 20 points possible on this quiz.
- You have 25 minutes to complete this quiz.
- Be sure you have filled in your name and Purdue ID in the slots at the top of the page.
- Show all work to maximize partial credit.
- Be sure all cell phones are silenced and put away out of view. This policy applies to smart watches as well.
- Headphones are not permitted unless prior approval was granted by your instructor.
- Formula sheets are not permitted.
- You are only permitted to use calculator(s) from the following list:
    - BA II Plus
    - BA II Plus Professional
    - BA-35
    - TI–30Xa or TI–30XA (same model just different casing)
    - TI-30X II (IIS solar or IIB battery)
    - TI-30XS MultiView (or XB battery)
- When time expires, put your pencil down and close your exam. Failure to do so will result in automatic disqualification from obtaining University-Earned Credit.

PURDUE HONORS PLEDGE

"As a boilermaker pursuing academic excellence, I pledge to be honest and true in all that I do. Accountable together - we are Purdue."

STUDENT AGREEMENT

By signing below,

- I agree with the Purdue Honors Pledge stated above.
- I will not give or receive any assistance on this exam, and I will report any infractions of the honors pledge.
- I acknowledge that I only used calculator(s) from the above list.
- I am claiming all work in this exam as my own.

X_____

1. (5 points) Let Y be the present value random variable for a whole life annuity due with annual payments of 500 issued to (70). You are given:

   i. Mortality follows the Standard Ultimate Life Table
   ii. $i = 0.05$

   a. (2 points) Define the present value random variable, Y.

   Solution:

$$Y = 500\,\ddot{a}_{\overline{K_{70}+1|}} \quad \text{or} \quad 500\left(\frac{1-v^{K_{70}+1}}{d}\right)$$

Points	
1	Annuity payment coefficient and use of (Kx+1)
1	Certain annuity (either as a or the formula for it)

   b. (3 points) Calculate $Var(Y)$.

   Solution:

$$Var(Y) = (500)^2\,\frac{{}^2A_{70} - (A_{70})^2}{d^2}$$

$$= (500)^2\,\frac{(0.21467) - (0.42818)^2}{\left(\frac{0.05}{1.05}\right)^2}$$

$$= 3,454,340.608$$

Points	
1	Include death benefit squared as coefficient
1	Correct Var(Z) as numerator
1	Discount rate squared as denominator

2. (5 points) You are given:

   i. $\ddot{a}_{36} = 20.940$

   ii. $\ddot{a}_{65} = 13.052$

   iii. ${}_{30}E_{35} = 0.27225$

   iv. ${}_{30}p_{35} = 0.883$

   v. ${}_{29}p_{36} = 0.884$

   vi. $i = 0.04$

   Calculate $\ddot{a}^{(2)}_{35:\overline{30}|}$ using the two-term Woolhouse approximation.

   Solution:

$Y = 500\,\ddot{a}_{\overline{k_{70}+1}|}$ or $500\left(\frac{1-v^{k_{70}+1}}{d}\right)$

$\ddot{a}^{(2)}_{35:\overline{30}|} = \ddot{a}^{(2)}_{35} - {}_{30}E_{35}\,\ddot{a}^{(2)}_{65}$

$= \overset{①}{20.86183867} - (0.27225)\overset{②}{(12.802)} = \boxed{17.37649}$

① $\ddot{a}^{(2)}_{35} \approx \ddot{a}_{35} - \frac{2-1}{2(2)} \approx \ddot{a}_{35} - \frac{1}{4} \approx 20.86183867$

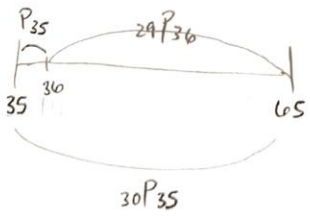
② $\ddot{a}^{(2)}_{65} \approx \ddot{a}_{65} - \frac{1}{4}$

$\approx 13.052 - \frac{1}{4} \approx 12.802$

$\ddot{a}_{35} = 1 + \left(\frac{1}{1.04}\right) p_{35}\, \overset{20.940}{\ddot{a}_{36}} = 21.11183867$

$= \frac{{}_{30}p_{35}}{{}_{29}p_{36}} = \frac{0.883}{0.884}$

$= 0.998868778$



Points	
1	Setup for finding p_{35}
2	Setup for $\ddot{a}^{(2)}_{35:\overline{30}|}$ as combination of whole life or other correct form
1	Setup for finding $\ddot{a}_{35}$
1	Setup for two-term woolhouse formula adjustment

3. (10 points) You (60) won the lottery. You will receive <u>both</u> of the following:

   i. A deferred term life annuity due that makes monthly payments of 600 for 10 years, starting at age 65.

   ii. An annuity due with 5 years of guaranteed monthly payments of 600, starting now (age 60).

   You are given that $i = 0.05$, deaths are uniformly distributed between integral ages, and mortality follows the Standard Ultimate Life Table.

   Calculate the actuarial present value of the payments to two decimal places.

   <u>Solution:</u>

$$APV = \overset{(1)}{600(12)\left[{}_{5}E_{60}\,\ddot{a}^{(12)}_{65:\overline{10}|}\right]} + \overset{(2)}{600(12)\,\ddot{a}^{(12)}_{\overline{5}|}} = 74,176.1453$$

(1) $7200\left[{}_{5}E_{60}\left(\ddot{a}^{(12)}_{65} - {}_{10}E_{65}\,\ddot{a}^{(12)}_{75}\right)\right]$

where $\ddot{a}^{(12)}_{65} = \alpha(12)\,\ddot{a}_{65} - \beta(12) = (1.0002)(13.5498) - 0.46651 = 13.085999996$

$\ddot{a}^{(12)}_{75} = \alpha(12)\,\ddot{a}_{75} - \beta(12) = (1.0002)(10.3178) - 0.46651 = 9.853353356$

$= 7200(0.766687)\left[13.086 - (0.55305)(9.85335)\right] = 42,165.2273$

(2) $7200\left(\dfrac{1-\left(\frac{1}{1.05}\right)^5}{d^{(12)}}\right) = 7200\left(\dfrac{1-\left(\frac{1}{1.05}\right)^5}{0.04869}\right) = 7200(4.4459608)$

$= 32,010.918$

Points	
6	Correct setup for (i) • 2 points for correct treatment of deferral (v and p) • 2 points for correct conversion to monthly payments • 2 points for correct formula for finding term life annuity due
3	Correct setup for (ii)
1	Correct coefficient of 12 for both (i) and (ii)