Slow Wave in Fluid-Filled Fractures: What is Missing in Biot's Theory?

Valeri Korneev, (vakorneev@lbl.gov), Lawrence Berkeley National Laboratory Berkeley, CA 94720

Summary

Permeability values consistently demonstrate that measurements at field scale (hundreds of meters to kilometers) show an increase of several orders of magnitude compared to values obtained at laboratory scales. This behavior finds an explanation in a self-similar distribution of fractures in rocks, an explanation supported by the wide-scale applicability of the Gutenberg-Richter law for distribution of earthquakes and seismic emission. Dramatic changes in permeability suggest the predominant role of fractures in fluid-flow processes at field scales, and that at those scales we can neglect fluid flow in pores. This explains why classical Biot's theory, while occasionally working well at laboratory scale, fails to describe the wave properties observed in the field—the interaction among fluid-filled fractures and an elastic, embedding matrix does not have the proper description. However, incorporation of matrix elasticity for fluid-filled fracture model leads to the existence of slow Stoneley guided waves with distinct dispersive properties. Phase velocity and attenuation of these waves are described by simple formulas in a general case of a viscous fluid. Stoneley guided waves carry most of energy and are capable of producing resonances in finite fractures at low frequencies. These resonances likely cause the strong nonlinear behavior of fluid reservoirs and may be a driving mechanism for fluid mobilization in rocks.

Introduction

In comparing various cases of water and gas presence in underground reservoirs there were observed phase shifts and frequency-dependent amplitude changes (e.g. Goloshubin *et al.* (2002), Korneev *et al.* (2004), and Castagna *et al*. (2003)). Most efforts to explain the reflection data from fluid-saturated rocks have used Biot's theory. Despite the computed fields carrying the right qualitative dependencies, the amplitudes of the effect were hundreds of times smaller than the observed values over a reasonably wide range of parameters. Clearly, fluid-flow intensity is the main factor responsible for the frequency dependence. However, the flow is restricted because (1) the boundary conditions of the problem imply that at the boundary fluid moves with the same amplitude as in solid matrix (no flow across the boundary), and (2) vertical incidence results in a strictly vertical fluid-flow direction, assuming no flow in lateral direction. Attempts to apply BISQ (by Dvorkin et al., 1994) which accounts for lateral flow at grain scales also did not allow to match the data.

With respect to the frequency dependence of wave propagation in fluid-saturated rock, we might hope to use this dependency to evaluate hydrological parameters, such as permeability. Indeed, if the data obey Biot's equations, then they can be inverted, using their frequency dependence and assuming locally constant permeability. However, this last assumption is invalid at field scale. From hydrogeology results, it follows that some scaling rules need to be applied to laboratory permeability measurements before using them at a field scale (Neuman (1994). Typically, a five-orders-of-scale increase results in a correspondent 5–7 orders of permeability increase. The immediate conclusion following from the scale-dependence of permeability is that in Biot's equations, permeability should not be a constant but rather a function of frequency. Thus, the hydrological measurements of permeability suggest that at field scales the pore fluid flow is negligibly small and does not contribute to average fluid flow. The main role at field scales belongs to flow in fractures.

Fractures in rocks follow self-similarity distribution which in particular, results in the Gutenberg-Richter Law for seismic events.

Some modifications of Biot's theory (e.g. see Mavko and Nur (1975)) or scattering theory (Hudson, 1980) introduce fractures into the model. In addition, Barenblatt et al. (1960) and Pride and Berryman (2003) have proposed modifications of Biot's theory, extending it to a dual-medium model. In these modifications, fractures play a role of Biot type media with different parameters where the medium becomes heterogeneous, resulting in much stronger attenuation and dispersion of seismic waves. These theories suffer, however, from the following problems: (1) they do not allow lateral (i.e., wave propagation direction) fluid flow; (2) they formally treat 2D flow in fractures in the same manner as in Biot's theory (flow in tubes), therefore neglecting the strongest slow fluid wave-propagation effects; (3) they use certain characteristic fracture parameters, whereas in reality fractures have self-similar properties; and (4) , they skip over the problem of wave-scattering processes in heterogeneous models, even though strong attenuation and dispersion take place in such models without any fluid presence.

Biot's Fracture Model

Consider viscous fluid flow in a fracture using Biot's approach. Combining a linearized equation for compressible viscous fluid with a continuity relation, we can obtain the equation for the viscous fluid velocity motion $\mathbf{v}$, in the form:

$$\frac{\partial^2 \mathbf{v}}{\partial t^2} - \eta \nabla^2 \frac{\partial \mathbf{v}}{\partial t} - \left(\xi - \frac{\eta}{3} \right) \nabla \nabla \bullet \frac{\partial \mathbf{v}}{\partial t} - c^2 \nabla \nabla \bullet \mathbf{v} = 0 \qquad (1)$$

with viscosity coefficients η and ξ.

Consider a fracture model represented by two parallel rigid walls and separated by a viscous fluid layer. Assuming that the main bulk of motion directed along the fracture walls (OX axis) for the total flow

$$F = \frac{1}{h}\int_0^h \mathrm{v_x}\,dz \qquad (2)$$

across any x= const., we get the equation

$$\frac{\partial^2 F}{\partial t^2} + \frac{12\eta}{h^2\rho}\frac{\partial F}{\partial t} - \frac{\xi + \frac{4\eta}{3}}{\rho}\frac{\partial^2}{\partial x^2}\frac{\partial F}{\partial t} - c^2\frac{\partial^2 F}{\partial x^2} = 0 \qquad (3)$$

which has the exact same form as Equation (3.4). Note that fracture permeability $\kappa_{fr} = h^2/12$ is embedded in the denominator of the second term of Equation 3, in the same manner as in the classical Biot's equations.

Elastic Fracture Model for Nonviscous Fluid

In Biot's theory, the interaction of fluid and solid primarily describes viscous friction forces acting on the walls of the rigid matrix as a result of the relative motion. Frequency-dependent effects in such models disappear for nonviscous fluids. However, consideration of elasticity in fracture walls raises the issue of strong frequency-dependent effects in wave propagation phenomenon. Waves in a fracture filled with nonviscous fluid (Figure 1) were investigated in the classical paper by Ferrazzini and Aki (F&A) (1994).

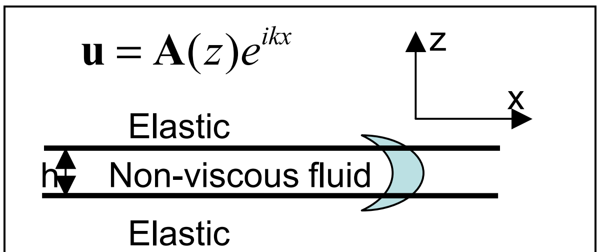

Figure 1: Fluid-filled fracture model by Ferrazzini and Aki (1987). Nonviscous fluid interacts with elastic walls of the fracture.

They showed that in such model, a symmetric mode exists that is associated with Stoneley guided waves, with phase velocities approaching zero as frequency goes to zero. Using this wave, the authors explained low-frequency volcanic tremors as the oscillations of molted lava in opened fractures. They stopped short of deriving the asymptotics for the phase velocity of this wave, which has the form

$$V_{f0} = \left(\frac{\omega h \mu}{\rho_f}(1-\gamma^2)\right)^{\frac{1}{3}}, \quad \omega h / \mathrm{v_{min}} << 1 \qquad (4)$$

and has a strong dependence on frequency and fracture thickness. Note that the low frequency asymptotics for slow waves in Biot's theory, and its modifications (such as the squirt flow model), are different and proportional to the square root of frequency. Slow fluid waves also have been studied numerically (Groenenboom and Falk, 2000) for the purpose of monitoring hydrofracture processes, demonstrating their dominantly-high amplitude. The dispersive wave represented by Equation (4) has a variety of different names (e.g., "slow fluid wave," "fluid guided wave," "Stoneley guided wave," "slow wave," "first symmetric fluid mode"), all of which have the same meaning.

Therefore, we have two competing models for describing wave propagation in the fluid-filled fracture, one (Biot's model) using viscous fluid contained between rigid walls, and the other (Ferrazzini and Aki's model) using nonviscous fluid between elastic half-spaces. Which one is more adequate for descriptions of fluid-elastic interactions in fractures? To get an answer, we expanded the Ferrazzini and Aki's model to viscous fluids.

Elastic Fracture Model for Viscous Fluid

Consider a layer filled with viscous fluid between two elastic halfspaces. In this case, fluid inside the fracture is capable of supporting two waves: a P-wave and an S-wave with velocity

$$V_{s_1}^2 = -i\frac{\omega\eta}{\rho} \qquad (5)$$

which describes a diffusion process.

Following a similar approach as described in Ferrazzini and Aki (1994), it is possible to obtain a dispersion equation for the symmetric mode (not presented here for brevity's sake). At low frequencies, this dispersion equation has the form

$$V_f^3 = V_{f0}^3\frac{\beta}{1+\beta}, \qquad \beta = -i\frac{S^2}{12}, \qquad (6)$$

with normalized skin factor

$$S = h\sqrt{\frac{\omega\rho_f}{\eta}} \qquad (7)$$

Depending on the value of *S*, we have two cases:

$$V_f^3 = -i\frac{\omega^2 h^3 \mu}{12\eta}(1-\gamma^2) \qquad (8)$$

when *S*<<1, and

$$V_f = V_{f0}\left(1 - \frac{i4}{S^2}\right) \qquad (9)$$

when *S*>>1.

It is quite clear that when S>>1, the solution of Equation (7) approaches that of Ferrazzini and Aki. To estimate the

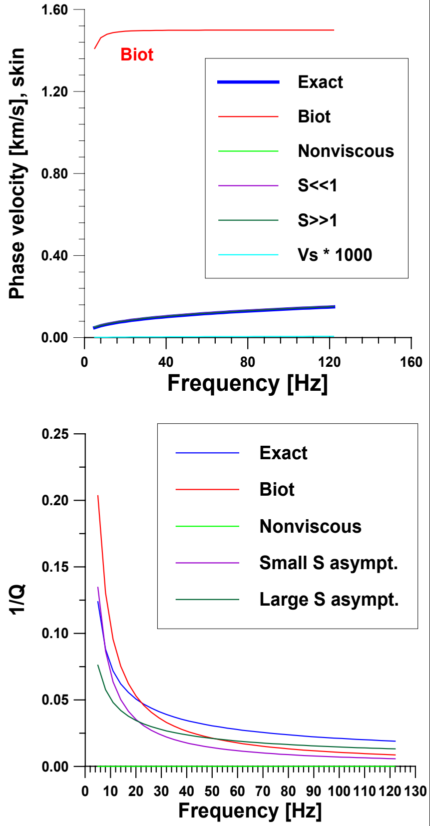

Figure 2. Comparison of solutions for a water-filled 0.5 mm thick fracture as functions of frequency. Solutions include: Biot's solution from Equation 3 (red line), the exact solution for a viscous fluid in an elastic fracture (blue line), the asymptotic (S << 1) solution from Equation 8 (purple line), the asymptotic (S >> 1) solution from Equation 9 (dark green line), the solution for nonviscous fluid by Ferrazzini and Aki (1987) from Equation 4 (light green line) (a) Phase velocities, (b) Inverse of attenuation factor Q. Note the very small propagation velocities. While all fluid-elastic solutions have about the same velocity dependencies, the difference in attenuation is quite large. Q values reach 50 at seismic frequencies.

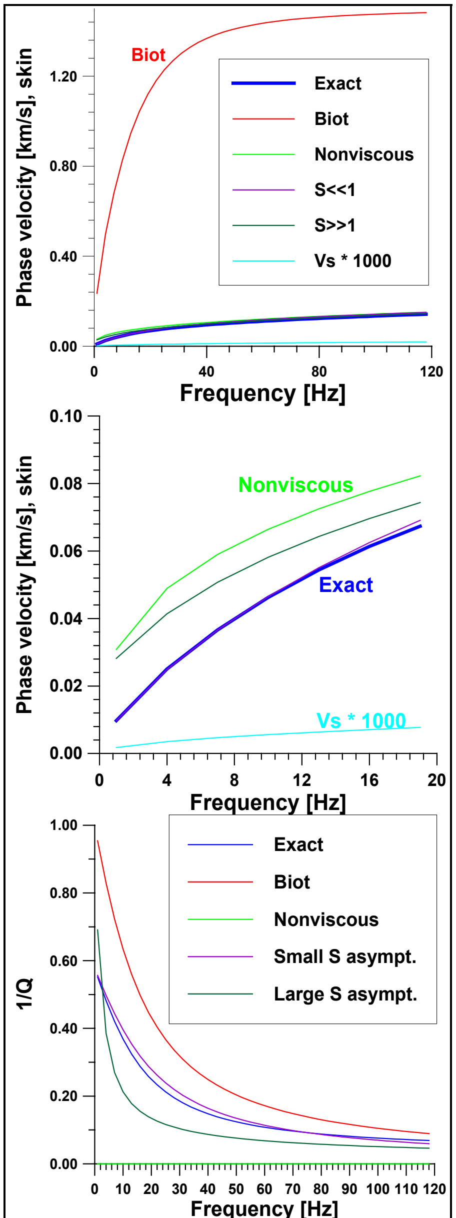

Figure 3. Same as in Figure 2, but now for oil-filled fractures. The small S asymptotic provides a good approximation for the exact solution, which differs significantly from the Ferrazzini and Aki solution. Q values reach 10 at seismic frequencies.

possible effects of different fluids on velocity and the attenuation of the Stoneley guided wave, two sets of computations were done for a 0.5 mm thick fracture filled with water and oil having viscosities of 1 centipoises and 10 centipoises, respectively. Five different solutions were compared, including Biot's solution (Equation 3). The exact solution for viscous fluid is obtained by a complex root search for the exact dispersion equation. Results are shown in Figures 2 and 3.

As we can see, in both cases, Biot's solution substantially overestimates Stoneley wave propagation velocities—predictably, since (physically) Biot's solution describes body wave propagation rather than surface-wave propagation. For water, Ferrazzini and Aki's solution gives a good approximation for the whole range of frequencies, while for oil it largely overestimates velocities. Surprisingly, the Stoneley waves at seismic frequencies are in the range of tens of meters/second, which results in 3–10 m wavelengths. These values are comparable with rock fracture lengths and therefore the resonance conditions might be satisfied in finite fractures. Obviously, Ferrazzini and Aki's solution does not estimate wave attenuation; however, the exact solution and its approximations indicate that even for a rather thin fracture, the quality factor Q can reach values of 20, supporting possibility of fluid wave resonances in fractures. Considering the statistical distribution of fracture sizes, we speculate that at any given frequency, resonant conditions will be satisfied for some fracture population.

Conclusions

Fractures play a major role in determining the permeability and direction of fluid flow in rock at field scales. This is especially true for fluid reservoirs. Fractures generally have hierarchical self-similar distribution in most rock at all scales. Fluid-filled fractures are very contrast waveguides for dispersive Stoneley fluid waves and are likely to be capable of strongly absorbing energy from passing seismic waves. Energy absorption can be tenfold more effective when resonant conditions are met, which creates favorable conditions for nonlinear fluid-flow effects. There is convincing evidence that fluid reservoirs exhibit nonlinearity by generating combination frequencies. The resonant conditions for fracture fluid waves can exist even below prospective seismic frequencies, when guided-wave velocities approach zero at low frequencies. Guided waves in intersecting fluid-filled fractures interact with each other, generating frequency-dependent reflection and transmission fields. Fracture systems also are the contrast scattering media for propagating seismic waves. All of the above phenomena lead to strong frequency-dependent propagation effects for seismic waves, which affect both velocity and attenuation. There is no existing theory that adequately describes the interaction of propagating seismic waves with systems of fluid-filled fractures in rock. A substantial effort is needed to understand the key seismic signatures of such systems.

Acknowledgments

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EDITED REFERENCES

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REFERENCES

Barenblatt, G. I., I. P. Zheltov, and I. N. Kochina, 1960, Basic concepts in the theory of seepage of homogeneous liquids in fissured rocks: Journal of Applied Mathematics, 24, 1286–1303.

Berryman, J. G., and H. F. Wang, 2000, Elastic wave propagation and attenuation in a double-porosity dual-permeability medium: International Journal of Rock Mechanics and Mining Sciences, 37, 63–78.

Biot, M. A., 1962, Mechanics of deformation and acoustic propagation in porous media: Journal of Applied Physics, 33, 1482–1498.

Castagna, J. P., S. Sun, and S. R. Wu., 2003, Instantaneous spectral analysis: detection of low-frequency shadows associated with hydrocarbons: The Leading Edge, 22, 120–127.

Dvorkin, J., and A. Nur, 1995, Squirt flow in fully saturated rocks: Geophysics, 60, 97–107.

Ferrazzini, V., and K. Aki, 1987, Slow waves trapped in a fluid-filled infinite crack: Implications for volcanic tremor: Journal of Geophysical Research, 92, 9215–9223.

Goloshubin, G. M., V. Korneev, D. Silin, V. Vingalow, and C. VanSchuyer, 2006, Reservoir imaging using low frequencies of seismic reflections: The Leading Edge, 25, 527–531.

Groenenboom, J., and J. Falk, 2000, Scattering by hydraulic fractures: Finite-difference modeling and laboratory data: Geophysics, 65, 612–622.

Hudson, J. A., 1980, Overall properties of a cracked solid: Mathematical Proceedings of the Cambridge Philosophical Society, 99, 371–384.

Korneev, V. A., G. M. Goloshubin, T.V. Daley, and D.B. Silin, 2004, Seismic low-frequency effects in monitoring of fluid-saturated reservoirs: Geophysics, 69, 522–532.

Neuman, S. P., and V. Di Federico, 2003, Multifaceted nature of hydrogeologic scaling and its interpretation: Reviews of Geophysics, 41, 1014; http://dx.doi.org/10.1029/2003RG000130.

Pride, S. R., and J. G. Berryman, 2003, Linear dynamics of double-porosity dual-permeability materials: I. Governing equations and acoustic attenuation: Physics Review E, 68, 036603.