

High Order Reconstruction Methods for Piecewise Smooth Functions

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It is well known that while spectral methods yield exponentially convergent approximations for smooth functions, the results for piecewise smooth functions are poor, with spurious oscillations developing near the discontinuities and a much reduced overall convergence rate. This behavior, known as the Gibbs phenomenon, is considered as one of the major drawbacks in the application of spectral methods. Recently spectral reprojection reconstruction methods have been developed to reconstruct piecewise smooth images in their smooth subintervals and restore the exponential convergence properties of spectral methods. Specifically, unlike standard filtering, the convergence does not deteriorate as the discontinuities are approached. The most familiar and easily analyzed spectral reprojection method is the Gegenbauer reconstruction method. However it is apparent that the Gegenbauer reconstruction method is not robust and in particular it suffers from round off error. Methods to alleviate these difficulties have been recently developed.

All high order reconstruction methods require a-priori knowledge of the jump discontinuity locations, since these edges determine the intervals of smoothness in which the spectral reprojection reconstruction method can be applied. The local edge detection method has been recently been developed to determine the location of edges on scattered grid point data. It can also be applied to detect the discontinuities in the derivatives of functions.

In this talk I discuss recent advances in edge detection and high order reconstruction methods. Examples include applications from medical imaging, where noise is an additional impediment to the reconstruction. Other spectral reprojection methods are briefly discussed.