Math 523 Fall 2024 Assignment 3

Name:

1. (14 pts) (1989) Given the equation

$$xu_{xx} + 2yu_{xy} + u_{yy} + e^x u_x + 2u = 0.$$

(a) Write down the equation for the characteristic curves.
(b) Describe the regions in the plane where the equation is hyperbolic, parabolic, and elliptic (sketch the regions).

2. (14 pts) (1987) Classify the operator

$$P = x\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

at each point in $\mathbf{R}^2$ and determine the characteristic curves in any hyperbolic region.

3. (12 pts) (1984) For the operator

$$P = 4\frac{\partial^2}{\partial x^2} - 4\frac{\partial^2}{\partial x \partial y} + \frac{\partial^2}{\partial y^2} + 8\frac{\partial}{\partial x}$$

(a) Find the principal part.
(b) Find the principal symbol and the equation for the characteristic curves.
(c) Classify the operator.
(d) Find the characteristic curves.

4. (14 pts) (1998) Consider the problem

$$u_{xx} + xu_{xy} + (2y - x^2)u_{yy} - u_y = 0, \tag{1}$$

$$u(x, 0) = x^2, \quad u_y(x, 0) = x. \tag{2}$$

(a) Determine the regions in the xy plane in which (1) is hyperbolic, parabolic, or elliptic.
(b) Let A denote the x-axis in the plane. Find all points P_0 in A such that there exists a unique real analytic solution of (1), (2) in a sufficiently small neighborhood of P_0 in $\mathbf{R}^2$. State carefully a theorem which justifies your answer.
(c) Let u be a solution of (1), (2) in some neighborhood of $(1, 0)$. Is $u_{xyy}(1, 0)$ uniquely determined? If so, compute its value.

5. (12 pts) Classify the following PDE

$$u_{xx} + 6u_{xy} - 16u_{yy} = 0.$$

Find a change of coordinates that reduces it to its canonical form and use it to find the general solution (in the original coordinates).

6. (12 pts) Classify the PDE

$$u_{xx} + 2u_{xy} + u_{yy} = 0.$$

Find a change of coordinates that simplifies the equation and use it to find the general solution in the original coordinates.