# Math 523 Fall 2024 Assignment 4

**Name:**

**1.** Let $f$ be a function on $\mathbf{R}$ so that $f$ is $C^1$ for $x < 0$, up to $x = 0$; and also $C^1$ for $x > 0$, again up to $x = 0$ (i.e., the limits of $f$ and $f'$ from left and right exist). Let $[f]$ be the jump at $x = 0$. Find a formula for $f'$, where the derivative is in sense of distributions.

**2.** Following p. 72, #13.
(a) Prove that $F_1 = H(\mu)H(\eta)$ is a fundamental solution of $L = \partial_\mu \partial_\eta$. Here, $H$ is the Heaviside function.
(b) Making the change of variables $\mu = x + ct$, $\eta = x - ct$, find a fundamental solution of $\Box = \partial_t^2 - c^2\partial_x^2$.

**3.** p. 72, #16. Show if the operator $L = \sum_{|\alpha|\le m} a_\alpha(x)\partial^\alpha$ with real coefficients is elliptic, then $m$ is even.

**4.** (p. 82. #1(b)) Solve

$$u_{tt} - c^2 u_{xx} = 2t,$$
$$u|_{t=0} = x^2, \quad u_t|_{t=0} = 1.$$

**5.** (p. 82. #6) Solve

$$u_{tt} - c^2 u_{xx} = 1, \quad 0 \le x \le \pi, t > 0,$$
$$u|_{t=0} = u_t|_{t=0} = 0,$$
$$u|_{x=0} = 0, \quad u|_{x=\pi} = -\pi^2/2.$$

**6.** Solve the following IBVP for the wave equation on the half-line $x > 0$:

$$u_{tt} - c^2 u_{xx} = 0, \quad x \ge 0,\ t > 0,$$
$$u|_{t=0} = g(x), \quad u_t|_{t=0} = h(x),$$
$$u|_{x=0} = 0.$$

**Hint:** Odd or even extensions of some functions might be useful. You may assume the compatibility conditions if needed.

**7.** Prove that the solution of the problem

$$u_{tt} - c^2 \Delta u + q(x)u = 0, \quad x \in \Omega,\ t > 0$$

in a bounded domain $\Omega \subset \mathbf{R}^n$, with the Dirichlet boundary condition on $\partial\Omega$, is unique. Here, $q \ge 0$ is a smooth potential in $\Omega$. You can assume that $u$ is a classical solution.

Is there uniqueness assuming the Neumann boundary condition on $\partial\Omega$?

You may want to compare this question to the other ones on p. 94.