Math 523 Fall 2024 Assignment 5, supplementary question

Let $\Omega \subset \mathbf{R}^n$, $n \geq 2$ be an open set and let $u \in C^2(\Omega)$. Prove that if for any sphere S belonging to Ω together with its interior, we have

$$\int_S \frac{\partial u}{\partial \nu} dS_x = 0,$$

then u is harmonic in Ω. Here ν is the outer normal to S as usual.