Math 523/Fall 2024, Assignment 6

1. (12 pts)

(a) Show that the general **radial** solution to the wave equation $(\partial_t^2 - \Delta)u = 0$ in $\mathbf{R}^3$ is given by

$$u(x,t) = \frac{F(r+t) + G(r-t)}{r}, \qquad r = |x|,$$

where F and G are functions of one variable. Do not worry about the regularity of F and G.

(b) Find the solution of the Cauchy problem with $u|_{t=0} = f(r)$, $u_t|_{t=0} = g(r)$. Hint: use even or odd extensions of f and g; you decide which one.

(c) With f, g as above, show that

$$u(0,t) = f(t) + tf'(t) + tg(t),$$

therefore, when $f \in C^{k+1}$, $g \in C^k$, we have $u \in C^k$ but no better than that (i.e., not as regular as f).

2. (10 pts) p. 125, #1(a).

3. (10 pts) p. 125, #4.

4. (10 pts) p. 125, #6.

5. (10 pts) p. 125, #9.