MA 266 - DIFFERENTIAL EQUATIONS

REVIEW PROBLEMS - MIDTERM 2

Exercice 1. Solve the initial value problem

$$y'' + 2y' + 2y = 0, \qquad y\left(\frac{\pi}{4}\right) = 2,\ y\left(\frac{\pi}{4}\right) = -2.$$

Exercice 2. Determine the longest interval in which the following initial value problem is certain to have a unique twice-differentiable solution:

$$(x-2)y'' + y' + (x-2)\tan(x)y = 0, \qquad y(1) = 0, y'(1) = 1.$$

Exercice 3. Consider the initial value problem

$$y'' + 2ay' + \left(a^2+1\right)y = 0, \qquad y(0) = 1,\ y'(0) = 0.$$

3.1. Find the solution $y(t)$ of this problem.

3.2. For $a = 1$, find the smallest T such that $|y(t)| < 0.1$ for $t > T$.

Exercice 4. Solve the initial value problem

$$9y'' + 6y' + 82y = 0, \qquad y(0) = -1,\ y'(0) = 2.$$

Exercice 5. Consider the initial value problem:

$$9y'' - 12y' + 4y = 0, \qquad y(0) = a,\ y'(0) = -1.$$

5.1. Find the solution $y(t)$ of this problem.

5.2. Find the critical value of the parameter a that separates solutions that become negative from those that are always positive.

Exercice 6. Use the reduction of order method in order to find the second solution of

$$(x-1)y'' - xy' + y = 0, \qquad x > 1, \qquad y_1(x) = e^x.$$

Exercice 7. Consider the following initial value problem:

$$y'' + 2y' + 5y = 4e^{-t}\cos(2t), \qquad y(0) = 1,\ y'(0) = 0.$$

7.1. Solve the homogeneous equation.

7.2. Find a particular solution by means of the method of undetermined coefficients.

Exercice 8. Find the correct form of a particular solution of

$$y'' + 4y = t^2\sin(2t) + (6t+7)\cos(2t).$$

Exercice 9. Use the variation of parameters to find the general solution of:

$$x^2y'' - 3xy' + 4y = x^2\ln(x), \qquad x > 0, \qquad y_1(x) = x^2,\ y_2(x) = x^2\ln(x).$$

Exercice 10. A spring is stretched 10cm by a force of 3N. A mass of 2 kg is hung from the spring and is also attached to a viscous damper that exerts a force of 3N when the velocity of the mass is 5m/s. If the mass is pulled down 5cm below its equilibrium position and given an initial downward velocity of 10cm/s, determine its position u at any time t.