CLT: Consider a sequence $\{X_i \,;\, i \geq 1\}$ of i.i.d random var. with $\mathbb{E}[X_i] = \mu$, $Var(X_i) = \sigma^2$

Set $\bar{X}_n = \frac{1}{n} \sum_{i=1}^{n} X_i$. Then

$$\boxed{\sqrt{n}\, \frac{(\bar{X}_n - \mu)}{\sigma} \approx Z \sim \mathcal{N}(0,1)}$$

Problem 8.14 (1)

Situation: We consider

- A certain component, which is critical to the operation of an electrical system and must be replaced immediately upon failure
- For the critical component

$$\text{Mean} = 100 \text{ h}, \text{ and Standard deviation} = 30\text{h}$$

Question:

How many of these components must be in stock so that the probability that the system is in continual operation for the next 2000 hours is at least .95?

Rmk Similar to the overbooking problem

Model For $i \geq 1$, set

X_i = lifetime of i-th component

Hyp The X_i' are i.i.d and

$\mathbb{E}[X_i] = 100 = \mu$ $\sqrt{V(X_i)} = 30 = \sigma$

We wish to find n minimal s.t.

$$\boxed{\mathbb{P}\left(\sum_{i=1}^{n} X_i \geqslant 2000\right) \geq 0.95}$$

$\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i \qquad \sqrt{n}\,\frac{(\bar{X}_n - \mu)}{\sigma} \approx Z$

Aim: Find an approximation of

$$P\left(\sum_{i=1}^{n} X_i \geq 2000\right)$$ → we want this $\geq .95$

$$= P\left(\bar{X}_n \geq \frac{2000}{n}\right)$$

$$= P\left(\sqrt{n}\,\frac{(\bar{X}_n - \mu)}{\sigma} \geq \frac{(2000/n - \mu)\sqrt{n}}{\sigma}\right)$$

$$\overset{CLT}{\approx} P\left(Z \geq \frac{(2000/n - \mu)\sqrt{n}}{\sigma}\right)$$

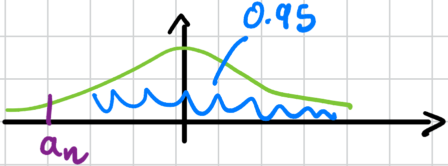
$$= P\left(Z \geq \frac{20\cancel{0}0/n - 10\cancel{0}}{3\cancel{0}}\sqrt{n}\right)$$

$$= P\left(Z \geq \frac{(200/n - 10)\sqrt{n}}{3}\right)$$ → we want this to be $\geq .95$

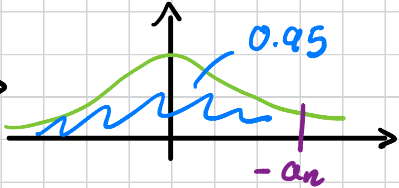
Summary We want to find n s.t.

$$\mathbb{P}\left(Z \geq a_n\right) \geq .95$$

with $a_n = \dfrac{(200/n - 10)\sqrt{n}}{3}$, $Z \sim \mathcal{N}(0,1)$



$\Leftrightarrow$



$\Leftrightarrow$ $\boxed{a_n \leq -1.64}$

Summary we want n s.t.

$$a_n = \frac{(200/n - 10)}{3}\sqrt{n} \leq -1.64$$

$$\Leftrightarrow \quad \frac{200}{n} - 10 \leq -\frac{\overbrace{3 \times 1.64}^{4.92}}{\sqrt{n}}$$

$$\Leftrightarrow \quad \frac{200}{n} + \frac{4.92}{\sqrt{n}} - 10 \leq 0$$

cv: set $x = \frac{1}{\sqrt{n}}$. We get

$$200x^2 + 4.92x - 10 \leq 0$$

We find $\boxed{n \geq 23}$

Conclusion : If we want to have 95% chances that the system operates during 1000 h or more, we need to buy at least $n = 23$ components

Rmk : Based on $\mu = 100$ only, we would say : buy $n' = 20$ components