if order matters: $i_1 \neq i_2$ $i_2 \neq i_3$, $i_3 \neq i_1$

Drawing 3 balls: 2nd model. We take

$S = \{$ Triple of balls, numbered from 1 to 11, non ordered $\}$

$= \{ (i_1, i_2, i_3) \in \{1, .., 11\}; \underbrace{i_1 < i_2 < i_3} \}$

since order does not matter, we fix the order

We get

$$\boxed{|S| = \binom{11}{3} = 165}$$

Probability: We assume

$$\mathbb{P}(\{(i_1, i_2, i_3)\}) = \frac{1}{165}$$

$\forall \ 1 \leq i_1 < i_2 < i_3 \leq 11$

Event

$E =$ " $2B$ & $1W$ "

Thus

$$\boxed{\mathbb{P}(E) = \frac{|E|}{165}}$$

$E =$ "2 B, 1 W"

Computing $|E|$: We decompose E as

Experiment 1: Pick 2 B among 5

$$\# \text{ outcomes} = \binom{5}{2} = 10$$

Experiment 2: Pick 1 W among 6

$$\# \text{ outcomes} = \binom{6}{1} = 6$$

Thus $\boxed{|E| = \binom{5}{2}\binom{6}{1} = 60}$

Computing $\mathbb{P}(E)$

$$\mathbb{P}(E) = \frac{|E|}{|S|} = \frac{60}{165}$$

$$\boxed{\mathbb{P}(E) = 36.4\%}$$

Example: drawing balls (2)

Model 1: We take

- $S = \{$Ordered triples of balls, tagged from 1 to 11$\}$
- $\mathbf{P} =$ Uniform probability on S

Computing $|S|$: We have

$$|S| = 11 \cdot 10 \cdot 9 = 990$$

Decomposition of E: We have

$$E = \mathrm{WBB} \cup \mathrm{BWB} \cup \mathrm{BBW}$$

Example: drawing balls (3)

Counting E:

$$|E| = |\mathrm{WBB}| + |\mathrm{BWB}| + |\mathrm{BBW}| = 3 \times (6 \times 5 \times 4) = 360$$

Probability of E: We get

$$\mathbf{P}(E) = \frac{|E|}{|S|} = \frac{360}{990} = \frac{4}{11} = 36.4\%$$

Example: drawing balls (4)

Model 2: We take

- $S = \{$Non ordered triples of balls, tagged from 1 to 11$\}$
- $\mathbf{P} =$ Uniform probability on S

Computing $|S|$: We have

$$|S| = \binom{11}{3} = 165$$

Decomposition of E: We have

$$E = \{\text{Triples with 2 B and 1 W}\}$$

Example: drawing balls (5)

Counting E:

$$|E| = \binom{5}{2} \times \binom{6}{1} = 60$$

Probability of E: We get

$$\mathbf{P}(E) = \frac{|E|}{|S|} = \frac{60}{165} = \frac{4}{11} = 36.4\%$$

Remark:

When experiment $\equiv$ draw k objects from n objects, two choices:

1. Considered the ordered set of possible draws
2. Consider the draws as unordered

Poker example.

Cards: $1, \dots, 10, \overbrace{J, Q, K}^{11 \quad 12 \quad 13}$

Colors: ♥ ♦ ♠ ♣

52 cards

Poker: We get a hand of 5 cards
There are some interesting hands:

pair, 2 pairs, triple, straight, flush, full hand, ...

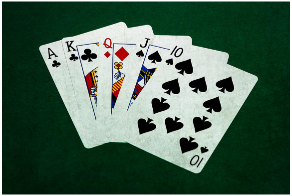
Example: poker game (1)

Situation: Deck of 52 cards and

- Hand: 5 cards
- Straight: distinct consecutive values, not of the same suit

Problem: Compute

$$\mathbf{P}(E), \quad \text{with} \quad E = \text{"Straight is drawn"}$$



Poker, sample space

$S = \{ \text{Non ordered hands of 5 cards} \}$

$$|S| = \binom{52}{5} = 2{,}598{,}960 \quad \text{(large number)}$$

Hyp: probability is uniform. Every hand has prob.

$$\boxed{\frac{1}{\binom{52}{5}}}$$

Event: E = " hand is a straight "

We have $\mathbb{P}(E) = \frac{|E|}{|S|}$

Decomposition for E: We have

$$E = \underbrace{\text{"1 2 3 4 5"}}_{E_1} \cup \underbrace{\text{"2 3 4 5 6"}}_{E_2} \cup \cdots \cup \underbrace{\text{"10 11 12 13 1"}}_{E_{10}}$$

disjoint

$$\Rightarrow \quad \mathbb{P}(E) = \mathbb{P}(E_1) + \cdots + \mathbb{P}(E_{10})$$

$$\boxed{\mathbb{P}(E) = 10\,\mathbb{P}(E_1)}$$

Counting $E_1 =$ "1 2 3 4 5"

Exp. 1: Pick 1 # outcomes $= \binom{4}{1} = 4$

Exp. 2: Pick 2 # outcomes $\binom{4}{1} = 4$

⋮

Exp. 5 Pick 5 # outcomes $= 4$

Total: 4^5 outcomes

Then we remove all "1 2 3 4 5" of the same suit: 4 to be removed

$$\Rightarrow \boxed{|E_1| = 4^5 - 4}$$

Computing probabilities

$$\mathbb{P}(E_1) = \frac{|E_1|}{|S|} = \frac{4^5 - 4}{|S|}$$

$$\mathbb{P}(E) = 10\,\mathbb{P}(E_1)$$

$$= \frac{10\,(4^5 - 4)}{\binom{52}{5}}$$

$$\boxed{\mathbb{P}(E) = 0.39\%}$$

Example: poker game (2)

Model: We take

- $S = \{\text{Non ordered hands of cards}\}$
- $\mathbf{P}$ = Uniform probability on S

Computing $|S|$: We have

$$|S| = \binom{52}{5} = 2,598,960$$

Decomposition of E: We have

$$E = \{\text{Straight hands}\}$$

Example: poker game (3)

Counting E: We have

- $\#$ possible 1,2,3,4,5: 4^5
- $\#$ possible 1,2,3,4,5 not of the same suit: $4^5 - 4$
- $\#$ possible values of straights: 10

Thus

$$|E| = 10(4^5 - 4) = 10,200$$

Probability of E: We get

$$\mathbf{P}(E) = \frac{|E|}{|S|} = \frac{10(4^5 - 4)}{\binom{52}{5}} = 0.39\%$$