Simplification for uniform probabilities

General situation: We assume

•
$$S = \{s_1, ..., s_N\}$$
 finite.

•
$$\mathbf{P}(\{s_i\}) = \frac{1}{N}$$
 for all $1 \le i \le N$

Alert:

This is an important but very particular case of probability space

Conditional probabilities in this case: Reduced sample space, i.e

Conditional on F, all outcomes in F are equally likely

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Reduced sample space - I dive roll

S= {1,..., 6}, R(11)= =

F = "even outcome" = 42,4,65

E = " outcome is $4'' = \langle 4 \rangle (\Rightarrow \mathbb{P}(E \succ \frac{1}{6})$

If we undition on F, the reduced sample space becomes

 $\tilde{S} = \{2, 4, 6\}$, with every outcome having prob. = $\frac{1}{3}$

According to reduced sample space, since E = 445 C F = 42,4,65, we have R(EIF)= = Bmk we could have computed $\frac{P(E|F)}{P(F)} = \frac{P(E \cap F)}{P(F)} = \frac{P(245 \cap 22,4,65)}{R(22,4,65)}$ R 12, 4, 65) $\frac{P(345)}{P(32,4,65)} = \frac{16}{12}$ = 1 reduced 3 sample sque simpler

Example: family distribution (1)

Situation:

The Popescu family has 10 kids

Questions:

- If we know that 9 kids are girls
 - \hookrightarrow find the probability that all 10 kids are girls

Intuition: conditional

prabability is not 0.5!

Example: family distribution (1)

Situation:

The Popescu family has 10 kids

Questions:

- If we know that 9 kids are girls
 → find the probability that all 10 kids are girls
- If we know that the first 9 kids are girls
 → find the probability that all 10 kids are girls

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Family of 10 kids example

 $S = \left\{ (i_1, \dots, i_{10}) ; i_{\delta} \in \left\{ G, B \right\} \right\}$

realistic = {G, By"

Hyp: every our come equally likely

 $\mathbb{P}(L(i_{1},...,i_{10})) = \frac{1}{20} = \frac{1}{1024}$



First conditioning

F. = "at least 9 kids are girls"

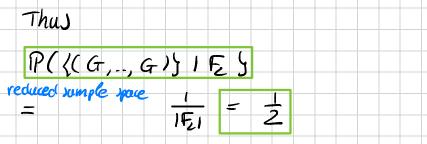
- $= \left\{ (G, G, G, ..., G); (G, ..., G, B); (G, ..., G, B, G) \right\}$
 - ... $(B, G, ..., G) = |F_i| = |I|$

We get reduced sample space $\mathbb{P}(\{(G,..,G\}) \mid IF,) \stackrel{?}{=} \frac{1}{|F,1|} = \frac{1}{|1|} \neq \frac{1}{|2|}$

second conditioning

Fz = "First nine kids are girls"

 $= \langle (G, ..., G, G); (G, ..., G, B) \rangle$



Example: family distribution (2)

Model:

- $S = \{G, B\}^{10}$
- Uniform probability: for all $s \in S$,

$$\mathsf{P}(\{s\}) = \frac{1}{2^{10}} = \frac{1}{1024}$$

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Image: A matrix

Example: family distribution (3)

First conditioning: We take

$$F_1 = \{(G, \ldots, G); (G, \ldots, G, B); (G, \ldots, G, B, G); \cdots; (B, G, \ldots, G)\}$$

Reduced sample space: Each outcome in F_1 has probability $\frac{1}{11}$

Conditional probability:

$$P(\{(G,...,G)\}|F_1) = \frac{1}{11}$$

Image: Image:

Example: family distribution (4)

Second conditioning: We take

$$F_2 = \{(G, \ldots, G); (G, \ldots, G, B)\}$$

Reduced sample space: Each outcome in F_2 has probability $\frac{1}{2}$

Conditional probability:

$$P(\{(G,...,G)\}|F_2) = \frac{1}{2}$$

Image: A matrix

Example: bridge game (1)

Bridge game:

- 4 players, E, W, N, S
- 52 cards dealt out equally to players -> 13 cards each

Conditioning: We condition on the set

 $F = \{N + S \text{ have a total of 8 spades}\}$

Question: Conditioned on F, Probability that E has 3 of the remaining 5 spades

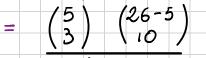
Bridge example, sample space S = "Assignements of 52 cards into 13+ 13+ 13+ 13 " $\Rightarrow |5| = (52) = (13 |3| |3|) = 5 \times 10^{28}$ We conclibur on Assignments of 26 cards = into 13+13, with 5 for F= { N+3 have 8 spaces } $|F| = \begin{pmatrix} 26 \\ 13 \end{pmatrix}$

We wish to compute

IP(E has=3 of the ramaining 5 \$ IF)

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reduced sample space





Example: bridge game (2)

Model: We take

 $S = \{$ Divisions of 52 cards in 4 groups $\}$

and we have

• Uniform probability on S

•
$$|S| = {52 \choose 13,13,13,13} \simeq 5.36 \ 10^{28}$$

Reduced sample space: Conditioned on F,

 $\tilde{S}=\{\mbox{Combinations of 13 cards among 26 cards with 5 spades}\}$ and $|\tilde{S}|=10,400,600$

Example: bridge game (3)

Conditional probability:

P(E has 3 of the remaining 5 spades| F) = $\frac{\binom{5}{3}\binom{21}{10}}{\binom{26}{13}} \simeq .339$

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Intersection and conditioning

Situation:

- Urn with 8 Red and 4 White balls
- Draw 2 balls without replacement

Question: Let

- $R_1 = 1$ st ball drawn is red
- $R_2 = 2$ nd ball drawn is red

Then find $P(R_1R_2)$

12 balls, 8 R, 4W Recall: $P(R_2 | R_1) = P(R_2 | R_1)$ P(R) $P(R_2 nR_1) = P(R_2 | R_1) P(R_1)$ Here $P(R_2 \cap R_1) = P(R_1) P(R_2 \mid R_1)$ $\frac{8}{12} \times \frac{7}{11} \simeq 42$ very \hat{x} mple computation

Intersection and conditioning (2)

Recall:

- Urn with 8 Red and 4 White balls
- Draw 2 balls without replacement

Computation: We have

 $\mathbf{P}(R_1R_2) = \mathbf{P}(R_1)\mathbf{P}(R_2|R_1)$

Thus

$$\mathbf{P}(R_1R_2) = \frac{8}{12} \frac{7}{11} = \frac{14}{33} \simeq .42$$