MA 416 - PROBABILITY

REVIEW PROBLEMS - FINAL

Problem 1. At Dr Gesund's office, the waiting time T is modeled by an exponential random variable with mean 10mn. Today the office proposes the following deal: if your waiting time is less than 20mn, you pay the full amount of your visit. Otherwise, you get reimbursed your waiting time minus 20. We call X the amount which is reimbursed by the office. Find the cdf of X. Then find the probability that you get reimbursed twice in 5 visits.

Problem 2. Let X_1, X_2 be two independent variables with common distribution $\mathcal{E}(\lambda)$. Find the density of $\frac{X_1}{X_1+X_2}$.

Problem 3. Let $ABCD$ be a square with the area 1. Let α, β, γ be random points on $\overline{AB}, \overline{BC}, \overline{CD}$, respectively. Let S be the area of the triangle $\alpha\beta\gamma$. Find $\mathbf{E}[S]$.

Problem 4. Let U_1, U_2 be two independent variables with common distribution $\mathcal{U}([0,1])$. Their Box-Muller transform can be written as

$$X_1 = (-2\ln(U_1))^{1/2}\cos(2\pi U_2), \qquad X_2 = (-2\ln(U_1))^{1/2}\sin(2\pi U_2).$$

Prove that X_1, X_2 are two independent variables with common distribution $\mathcal{N}(0,1)$.

Problem 5. The number of patients arriving at a hospital from 2pm to 3pm with severe symptoms follows a Poisson distribution with mean 1. The hospital ressources are enough to take care of 3 of these patients maximum. What is the probability that the hospital ressources are reached on a given day from 2pm to 3pm? What is the probability that the hospital ressources are reached more than twice on a given week from 2pm to 3pm?