MA 59500 Topics in Model Theory

Spring Semester 2025

updated January 7th 2025

As a Boilermaker pursuing academic excellence, I pledge to be honest and true in all that I do. Accountable together – we are Purdue.

Course Information

Course Number	MA 59500
Course Title	Topics in Model Theory
Course Section	179
CRN	29565
Course Credit	3 credit hours
Prerequisites	Key background from a first course in logic (such as MA 58500) will be assumed. Concepts from abstract algebra and analysis will be used to illustrate key ideas. See Brightspace for a list of topics with which you will be expected to be familiar.
Lecture Times	Monday, Wednesday, Friday 10:30am
Meeting Location	SCHM 123
Instructional Modality	Face-to-Face
Office Hours	TBA

Instructor

Name	Prof. Margaret E. M. Thomas
Email Address	memthomas at purdue.edu (you must use your official Purdue email; emails **MUST** include **MA59500** in the subject line)
Office	MATH 638

Learning Outcomes

Students completing the course will

(i) acquire an understanding of fundamental concepts and techniques in model theory;
(ii) learn how to apply these techniques to analyse key properties of mathematical theories and structures, such as completeness, quantifier elimination and categoricity;
(iii) be prepared to undertake more advanced studies in model theory.

Course Description

This course serves as an introduction to model theory (a branch of mathematical logic), which is the study of mathematical structures in terms of their logical properties. Some of the central questions that one can investigate from this perspective include the following. Given a mathematical structure, such as the real or complex field (often enriched with additional operations), one can analyse its class of definable sets (that is, the sets that can be defined in that structure using particular kinds of logical expressions). In particular, one can study the algebraic/geometric/combinatorial nature of these definable sets in terms of the logical complexity of their defining expressions, or seek to understand whether or not certain interesting sets/functions can be realised as definable sets therein. Alternatively, one can study the class of all structures in which certain logical expressions are true, considering, for example, how many such structures there can be of any given size, up to isomorphism. By bringing a foundational perspective to core mathematical ideas, model theory can be (and has been!) applied to many other

areas of mathematics (including algebra, combinatorics, algebraic geometry, number theory, operator theory, dynamical systems, …) and beyond.

The goal of this course is to cover a variety of central concepts in model theory, in particular motivated by various areas of application. Such concepts could include elementary extensions, ultraproducts, complete theories, categoricity, model completeness, quantifier elimination, elimination of imaginaries, types, saturated and homogeneous models, indiscernibles and stability. The aim will also be to discuss some of the key examples of model-theoretic structures and their properties, which could include algebraically closed fields (and more generally strongly minimal and stable theories) and real closed fields (and more generally o-minimal and NIP theories), with a view to modern applications of model theory (including applications of o-minimal structures to diophantine geometry, Hodge theory, and dynamical systems; and the role of NIP and related tame structures in combinatorics).

Course Webpages

Official Course Page:

https://www.math.purdue.edu/academic/courses/semester/202520/ma59500/index.html

The main website for the course will be in Brightspace, which you will be expected to check regularly. Course information, announcements and problem sheets for assignments will be posted there (see **Assessment Methods** below).

Some information may also be found on the instructor's course website as follows (in particular, in the event that Brightspace is not accessible, the syllabus will be available at the following page):

https://www.math.purdue.edu/~thoma922/teaching/ma59500/ma59500mtss25.html

Textbook

There is no one specific book recommended for this class. Here is a list of resources suitable for various aspects of the course. More suggestions of suitable reading may appear here in due course.

Marker, David – An Invitation to Mathematical Logic, Graduate Texts in Mathematics (no. 301), Springer (2024) Link to Purdue Libraries catalogue entry.

Marker, David – Model Theory: An Introduction, Graduate Texts in Mathematics (no. 217), Springer (2002) Link to Purdue Libraries catalogue entry.

Chang C. C. and Keisler H. J. – Model Theory, 3rd edition, Studies in Logic and the Foundations of Mathematics (vol. 73), North-Holland (1990) Link to Purdue Libraries catalogue entry.

The above can be downloaded through Purdue Libraries. In the corresponding catalogue entry, select "Online Access Available", which links to the publisher's website (Career Account username and password required) from which the corresponding book may be downloaded.

Hodges, W. – A Shorter Model Theory, CUP (1997)

Hodges, W. – Model Theory, CUP (1993)

Technology

- Brightspace – for course resources and announcements, including details/instructions for the assignments; **you are expected to check announcements on Brightspace regularly**.
- Gradescope (course links within Brightspace) – for submission of assignments. (See Brightspace for instructions on using Gradescope, as well as **Assessment Methods → Assignments → Submitting to Gradescope** below.)

- Zoom (meeting links, if they are ever needed, will appear within Brightspace) – not to be used as a rule, but available in case of unexpected developments, e.g. if it happens that a class has to pivot online, then the expectation is that this would take place on Zoom.

Use of AI

You are **not allowed** to use any kind of AI tools in connection with this course. There are two main reasons for this. First, if you provide AI tools with information about this course and its materials, for example by asking questions incorporating course material, then this is arguably in breach of copyright law and thus strictly prohibited. Note that there is course content that was developed specifically for this course and can be found only within the materials of this course. (See also **Copyright** below.) Second, the central objective of this course is to acquire an understanding of and appreciation for the mathematics involved, in particular (since this is a class in logic) for the underlying mechanics of the mathematical reasoning process itself. Therefore, it is essential that you engage actively with the process, and not seek to circumvent this, say with the use of shortcuts such as AI tools (use of which is moreover very possibly going to be inaccurate or misleading). In other words, use of AI tools is likely greatly to undermine your learning in this course!

Assessment Methods

Six or seven (6-7) written assignments, approximately once every two weeks.

The assignment problem sheets will be posted to Brightspace. You must submit your work for each assignment before the **deadline** given on the problem sheet (unless you have an extension). Answers should be submitted to Gradescope (see "**Submitting to Gradescope**" below for instructions on how to submit your work).

Extensions: Contact the instructor **by email** as early as possible if you would like to request an extension for an assignment. If the extension is granted, then the deadline in Gradescope will be adjusted for you. (See also policies regarding **Attendance** below.)

Late submissions: **The "late deadline" listed in Gradescope is only there in case of emergencies**, such as technical issues, and you should still aim to adhere to the stated deadlines.

If you end up uploading your work after the regular deadline but before the "late deadline", it will be stamped as **late,** and will be only be graded at the discretion of the instructor and may incur a late penalty. Please **email the instructor as soon as possible with an explanation** in this case.

If you are too late to submit through Gradescope at all (i.e. the "late deadline" has passed), then email the instructor to explain the situation; it is at the instructor's discretion whether or not to accept your work or impose a late penalty at that stage.

Notes on the assignments and on using Gradescope to submit your work

You are allowed – indeed, you are encouraged – to work together with other students from the class on the assignments. You are also encouraged to come to office hours to ask for help with assignments (see also **Use of AI** above). However, it is important that you understand the material well enough to write up solutions in your own words, and you **must do so. You must also state clearly** (a) the names of anyone with whom you collaborated on the work, and (b) any resources (other than class notes or the books listed above under **Textbook**) that you used.

Presenting work that is not your own as though it were your own is cheating, and could result in sanctions and referral to the OSRR (see also **Academic Integrity** below).

Submitting to Gradescope: You need to **finish (not just start) submitting by the deadline**, so please be sure to start the submission process in Gradescope early enough that you finish in time!

You will find in Brightspace the Gradescope link for submitting each assignment. If you have not used Gradescope before, then the first time you use such a link to connect to Gradescope you may need to create an account with Gradescope, with which you may then login to Gradescope directly thereafter. (If you already have a Gradescope account, you might nevertheless need to wait until the roster is set up in Gradescope before you see the course available to you there, or else use an assignment link from Brightspace to add the course to Gradescope yourself.)

Please note that **there is no "course entry code"** mechanism for this course.

Basic instructions for using Gradescope and the Gradescope Mobile App (including preparing your work in a suitable manner for submission) can be found on Brightspace. **Please note:**

- You can submit your work using the Gradescope Mobile App or the Gradescope website.
- If you handwrite your solutions on paper, then you should scan them (ideally as a **.pdf file**) for uploading, e.g. by using the Gradescope Mobile App. If you write your solutions in electronic form (typed or handwritten), then please save them as a **.pdf file** for uploading.
- Please make every effort to ensure that your scans are **readable;** in particular, they should be good quality scanned documents (and not, say, photos of your notebook from a strange angle)!
- While submitting your work, you are asked to "**assign pages**" to all your solutions so that they can be located correctly. **You should do this. You might score 0 if you do not do this, as otherwise it might not be possible to locate your solution to a question**. If you forget to do this or are in a hurry to submit, then **you may still assign pages after the deadline**.
- Please always review your submission after uploading to make sure that everything you intended to submit was successfully submitted, and assign pages at that point if still needed.

Course Evaluation

Problem sets will be offered regularly throughout the semester (see **Assignments** above). The expectation is that students demonstrate engagement with the course by submitting written solutions to a reasonable subset of these problems (with the lowest assignment score dropped) and by engaging actively in class discussions.

Students who get at least 97% of the total points in this course are guaranteed an A+, 93% guarantees an A, 90% an A-, 87% a B+, 83% a B, 80% a B-, 77% a C+, 73% a C, 70% a C-, 67% a D+, 63% a D, and 60% a D-; for each of these grades, it's possible that at the end of the semester a somewhat lower percentage will be enough to get that grade.

The instructor reserves the right to adjust this grading scheme at the end of the course; this adjustment will be applied uniformly and in a manner which only increases individual grades.

Boilerplate Information for Boilermakers

Please read the below even though it is likely similar to the information that you will receive relating to other courses, as you should be aware of the policies that are specific to this course. Please also remember to consult this syllabus in the event that anything arises during the semester in relation to these topics.

Attendance

Students need to inform the instructor of any conflict that can be anticipated and will affect the timely submission of an assignment. Only the instructor can excuse a student from a course requirement or responsibility.

Under academic regulations, excused absences may be granted for cases of grief/bereavement, military service, jury duty, parenting leave, or emergent or urgent care medical care. In such cases, you or your representative should contact or go to the Office of the Dean of Students website to complete appropriate forms for instructor notification. For details, see the Academic Regulations & Student Conduct section of the University Catalog website.

In other situations, when conflicts or absences can be anticipated, such as for many University-sponsored activities and religious observations, the student should inform the instructor of the situation as far in advance as possible **in writing, by email**. Please note that attending personal events (such as weddings or other family occasions) would **not** qualify.

For **unanticipated or emergency absences** when advance notification to the instructor is not possible, the student should contact the instructor as soon as possible thereafter **by email**.

Learning Resources

The Helen Bass Williams Academic Success Center, provides a variety of proactive, practical and approachable academic support services for you to strengthen your approaches and strategies for learning, including study skills consultations, peer coaching, workshops, and online handouts. Visit the ASC website for more information and to access resources.

Academic Integrity

Academic integrity is one of the highest values that Purdue University holds. Individuals are encouraged to alert university officials to potential breaches of this value by either emailing integrity@purdue.edu, or by calling 765-494-8778, or by contacting the Office of the Dean of Students. While information may be submitted anonymously, the more information that is submitted provides the greatest opportunity for the university to investigate the concern. More details are available on the course Brightspace under University Policies and Statements.

Purdue prohibits "dishonesty in connection with any University activity. Cheating, plagiarism, or knowingly furnishing false information to the University are examples of dishonesty" (Section B.2.a of the Student Regulations concerning Conduct).

Furthermore, the University Senate has stipulated that "the commitment of acts of cheating, lying, and deceit in any of their diverse forms (such as the use of ghostwritten papers, the use of substitutes for taking examinations, the use of illegal cribs, plagiarism, and copying during examinations) is dishonest and must not be tolerated. Moreover, knowingly to aid and abet, directly or indirectly, other parties in committing dishonest acts is in itself dishonest."

Incidents of academic misconduct in this course will be addressed by the course instructor and referred to the Office of Student Rights and Responsibilities (OSRR) for review at the university level. Any violation of course policies as it relates to academic integrity will result minimally in a

failing or zero grade for that particular assignment or test, and at the instructor's discretion may result in a failing grade for the course. In addition, all incidents of academic misconduct will be forwarded to OSRR, where university penalties, including removal from the university, may be considered.

Commercial Note Taking in Classes

Notes taken in class are generally considered to be "derivative works" of the instructor's presentations and materials, and they are thus subject to the instructor's copyright in such presentations and materials. **No individual is permitted to sell or otherwise barter notes, either to other students or to any commercial concern, without the express written permission of the course instructor.** To obtain permission to sell or barter notes, the individual wishing to sell or barter the notes must be registered in the course or must be an approved visitor to the class. Course instructors may choose to grant or not grant such permission at their own discretion, and may require a review of the notes prior to their being sold or bartered. If they do grant such permission, they may revoke it at any time, if they so choose.

See the Regulations on Student Conduct: Miscellaneous Conduct Regulations (point 10).

Copyright

See the University Policies and Statements section of Brightspace for guidance on Use of Copyrighted Materials. Effective learning environments provide opportunities for students to reflect, explore new ideas, post opinions openly, and have the freedom to change those opinions over time. Students and instructors are the authors of the works they create in the learning environment. As authors, they own the copyright in their works subject only to the university's right to use those works for educational purposes. Students may not copy, reproduce, or post to any other outlet (e.g., YouTube, Facebook, or other open media sources or websites, **including AI tools**) any work in which they are not the sole author, or the joint author having obtained the permission of the other author(s). In particular, disseminating notes, assignments, solutions, textbook extracts, exams, handouts or other course materials (**including to AI tools**) is **strictly prohibited**.

Nondiscrimination

Purdue University is committed to maintaining a community which recognizes and values the inherent worth and dignity of every person; fosters tolerance, sensitivity, understanding, and mutual respect among its members; and encourages each individual to strive to reach his or her own potential. In pursuit of its goal of academic excellence, the University seeks to develop and nurture diversity. The University believes that diversity among its many members strengthens the institution, stimulates creativity, promotes the exchange of ideas, and enriches campus life. Purdue's nondiscrimination policy: https://www.purdue.edu/purdue/ea_eou_statement.php

Academic Accommodation of Students with Disabilities

Purdue University strives to make learning experiences accessible to all participants. If you anticipate or experience physical or academic barriers based on disability, you are encouraged to contact the Disability Resource Center at: drc@purdue.edu or by phone: 765-494-1247, as soon as possible.

If the Disability Resource Center (DRC) has determined reasonable accommodations that you would like to utilize in this class, you must send your Course Accommodation Letter to the instructor. Instructions on sharing your Course Accommodation Letter can be found by visiting: https://www.purdue.edu/drc/students/course-accommodation-letter.php. **Additionally, you are**

strongly encouraged to contact the instructor as soon as possible to discuss implementation of your accommodations.

Mental Health

If you find yourself beginning to feel some stress, anxiety and/or feeling slightly overwhelmed, try Therapy Assistance Online (TAO), a web and app-based mental health resource available courtesy of Purdue Counseling and Psychological Services (CAPS). TAO is available to all students at any time by creating an account on the TAO Connect website, or downloading the app from the App Store or Google Play. It offers free, confidential well-being resources through a self-guided program informed by psychotherapy research and strategies that may aid in overcoming anxiety, depression and other concerns. It provides accessible and effective resources including short videos, brief exercises, and self-reflection tools.

If you need support and information about options and resources, please see the Office of the Dean of Students. Call 765-494-1747. Hours of operation are M-F, 8am- 5pm.

If you find yourself struggling to find a healthy balance between academics, social life, stress, etc. sign up for free one-on-one virtual or in-person sessions with a Purdue Wellness Coach at RecWell. Student coaches can help you navigate through barriers and challenges toward your goals throughout the semester. Sign up is free and can be done on BoilerConnect.

If you're struggling and need mental health services: Purdue University is committed to advancing the mental health and well-being of its students. If you or someone you know is feeling overwhelmed, depressed, and/or in need of mental health support, services are available. For help, such individuals should contact Counseling and Psychological Services (CAPS) at 765-494-6995 during and after hours, on weekends and holidays, or by going to the CAPS offices in West Lafayette or Indianapolis.

Basic Needs Security

Any student who faces challenges securing their food or housing and believes this may affect their performance in the course is urged to contact the Dean of Students for support. There is no appointment needed and Student Support Services is available to serve students 8 a.m.-5 p.m. Monday through Friday.

Major Campus Emergency

In the event of a major campus emergency, course requirements, deadlines, and grading are subject to change that may be necessitated by a revised semester calendar or other circumstances beyond the instructor's control. Relevant changes to this course will be posted on course websites or sent by email. You are expected to read your @purdue.edu email on a frequent basis.

A link to Purdue's Information on Emergency Preparation and Planning is located in Brightspace under "University Policies and Statements." This website covers topics such as Severe Weather Guidance, Emergency Plans, and a place to sign up for the Emergency Warning Notification System. You are encouraged to download and review the Emergency Preparedness for Classrooms document, which is also included at the end of this syllabus.

The first day of class, the instructor will review the Emergency Preparedness plan for the specific classroom, following Purdue's required Emergency Preparedness Briefing. Please make note of items such as:

- The location to where we will proceed after evacuating the building if we hear a fire alarm.
- The location of our Shelter in Place in the event of a tornado warning.
- The location of our Shelter in Place in the event of an active threat such as a shooting.

Current information for the building for this class, which is SCHM, is also given below:

Fire Alarm: Immediately evacuate the building and gather at the John Purdue Statue near Memorial Oval. In the event of inclement weather, gather inside the Class of 1950 Building.

Shelter in Place Location:

Tornado: If a tornado warning has been issued for campus, move to the lowest level possible away from exterior doors and windows. Seek more information on storm conditions from National Weather Service weather radio or application on mobile device.

Major hazardous materials release: Shelter in nearest building or classroom, shutting any open doors and windows.

Active Shooter etc.: If one cannot get away, shelter in a room that is securable preferably without windows.

If you are directed to shelter in place, but you are unaware of the specific reason, proceed to the lowest level of the building but continue to seek additional information by all possible means to determine the type of incident.

EMERGENCY PREPAREDNESS for Classrooms

EMERGENCY NOTIFICATION PROCEDURES are based on a simple concept – if you hear a fire alarm inside, proceed outside. If you hear a siren outside, proceed inside.

- **Indoor Fire Alarms** mean to stop class or research and immediately **evacuate** the building.
  - Proceed to your Emergency Assembly Area away from building doors. **Remain outside** until police, fire, or other emergency response personnel provide additional guidance or tell you it is safe to leave.

- **All Hazards Outdoor Emergency Warning Sirens** mean to immediately seek shelter (**Shelter in Place**) in a safe location within the closest building.
  - "Shelter in place" means seeking immediate shelter inside a building or University residence. This course of action may need to be taken during a tornado, an active threat such as a shooting or release of hazardous materials in the outside air. Once safely inside, find out more details about the emergency*. **Remain in place** until police, fire, or other emergency response personnel provide additional guidance or tell you it is safe to leave.

**In both cases, you should seek additional clarifying information by all means possible…Campus Emergency Status page, text message, email alert, TV, radio, etc…review the Purdue Emergency Warning Notification System multi-communication layers at http://www.purdue.edu/ehps/emergency_preparedness/warning-system.html*

EMERGENCY RESPONSE PROCEDURES:

- Review the **Emergency Procedures Guidelines** https://www.purdue.edu/emergency_preparedness/flipchart/index.html
- Review the **Building Emergency Plan** (available on the Emergency Preparedness website or from the building deputy) for:
  - evacuation routes, exit points, and emergency assembly area
  - when and how to evacuate the building.
  - shelter in place procedures and locations
  - additional building specific procedures and requirements.

EMERGENCY PREPAREDNESS AWARENESS VIDEO

- **"Run. Hide. Fight.®"** is a 6-minute active shooter awareness video that illustrates what to look for and how to prepare and react to this type of incident. See: https://www.youtube.com/watch?v=5mzI_5aj4Vs

MORE INFORMATION

Reference the Emergency Preparedness web site for additional information: http://www.purdue.edu/ehps/emergency_preparedness