MA 49500-001 N AND 59500-180 INT INTRODUCTION TO NUMBER THEORY, CRN 29558 AND 29604

TREVOR D. WOOLEY

Class Meeting Times: Tuesdays and Thursdays 16:30 - 17:45
Class location/modality: Face-to-face in PHYS 333.
Credit Hours: 3 hours
Course web page: https://www.math.purdue.edu/∼twooley/2025nt/2025nt.html
Course Brightspace page: https://purdue.brightspace.com/d2l/home/1222730

Textbook: You may wish to consult a number of texts to assist you in this class. The class will follow the instructor's LaTeXed notes available from the course web-page, primarily assisted by:
An Introduction to the Theory of Numbers, I. Niven, H. S. Zuckerman and H. L. Montgomery, 5th edition, Wiley, 1991.

You may also find useful:
A concise introduction to the theory of numbers, A. Baker, Cambridge University Press, 1984.

A friendly introduction to number theory, J. H. Silverman, 3rd edition, Prentice Hall, 2005.

Prerequisites: This course is intended for third- or fourth-year undergraduate students and beginning graduate students, who have taken and obtained a grade of B- or better in MA 35301 (Linear Algebra II).

Instructor: Prof. Trevor Wooley, twooley@purdue.edu
Location: 4.22 Math, Tel. 765-496-6439
Office Hours: Tu 14:00-15:00, W 13:30-14:30, Th 14:30-15:30

***** ALL *** information for this course will be available from the course web-page. The use of Brightspace is only as a portal to this web-page. Use email to contact me, *NOT* Brightspace.**

TA/Grader: Boqiang Zhang, zhan4329@purdue.edu

Course Description and Learning Outcomes: This course serves as an introductory exploration of number theory, without an abstract algebra prerequisite, so that final year students without a pure mathematics background will find this accessible. Connections with abstract algebra will, however, be noted for interested students, and the material should provide reinforcement and preparation for abstract algebra for those with ambitions in this direction.

After completing the course, students should expect to: 1. Know the basic tools of number theory, including appropriate proofs and methods for computing basic number theoretic functions; 2. Be conversant with the concepts of congruences, the Chinese Remainder Theorem, primitive roots, quadratic residues and quadratic reciprocity; 3. Know

about prime numbers and multiplicative functions; 4. Know about Diophantine approximation and continued fractions; 5. Know about Pell's equation and binary quadratic forms.

Course Content: We will cover, roughly speaking, most of the first four chapters, and much of chapters 5 to 9, of the book by Niven, Zuckerman and Montgomery, though there will be some additional material (as well as some omitted material) to provide a modern comprehensive introduction to the subject. There will be advance notice of the topics to be covered in the course roughly 3 weeks prior to being covered, though you can easily interpolate the likely topics from the opening sentence of this paragraph given that there are 44 classes total. Interpolation is a mathematical skill that is a prerequisite for the course! Comprehensive notes for the class will be provided on the course website. We start with a discussion of divisibility, the definition of primes, and then a discussion of congruences. This leads us to the Chinese Remainder Theorem and properties of polynomial congruences to prime and prime power moduli, including primitive roots and Hensel's lemma. Then we consider quadratic congruences in detail, introducing quadratic residue symbols and quadratic reciprocity. The remaining part of the course is devoted to a number of topics stemming from this foundation: Diophantine approximation, arithmetic functions, binary quadratic forms, Pell's equation and continued fractions, quadratic fields.

Assessment: Course credit will be based on:
HW: Weekly homeworks, the top 10 scores are totalled
MT1: First in-class midterm Tuesday 18th February, 2025
MT2: Second in-class mid-term Thursday 27th March, 2025
FE: Final exam in Finals Week, date and time to be confirmed.

The total score available for the course is, whichever is higher:
35% (HW) + 20% (MT1) + 10% (MT2) + 35% (FE) = 100% (max possible)
or
35% (HW) + 10% (MT1) + 20% (MT2) + 35% (FE) = 100% (max possible)

Homeworks will be posted on the course web-page:

https://www.math.purdue.edu/∼twooley/2025nt/2025nt.html

and are to be submitted via Gradescope (more on this below, and later once arrangements are finalised with our grader). Each homework is worth an equal amount of credit. Late homework will almost certainly not be graded. If you miss the deadline one week, give yourself a deadline 24 hours earlier the next week so that you are more likely to meet the deadline. That missed homework can be treated as a learning experience – only the top 10 homework scores count towards the final grade. There is a limited grading capacity for this course. You are nonetheless encouraged to attempt non-graded questions, and you are welcome to bring these to office hours for discussion.

Homework is intended to be an individual effort. By all means feel free to discuss the problems with your fellow students (there is much to be learned from discussion), but afterwards formulate your own answers and write them up individually. You may find that solutions to problems are available at various online sites. Although online searching skills are valuable, they are not intended to gain credit in this class. Resorting to such sites for homework answers will degrade any benefit to be had from this class. If you use such a site, clearly state that this is the case to obtain partial credit. If I become

aware that your solutions are derived from an online resource rather than having been formulated through your own efforts, and you do not state this on your answers, you should expect that the credit available for these answers will be made negligible.

Purdue mandates that I include the following:
"Students who get at least 97% of the total points in this course are guaranteed an A+, 93% guarantees an A, 90% an A-, 87% a B+, 83% a B, 80% a B-, 77% a C+, 73% a C, 70% a C-, 67% a D+, 63% a D, and 60% a D-; for each of these grades, it's possible that at the end of the semester a somewhat lower percentage will be enough to get that grade".

This is an undergraduate topics and a beginning graduate course, and you should imagine that a more nuanced approach will be taken in determining grades. In particular, it is likely that the numerical scores required to achieve the grades listed will be lower than those indicated in the mandated text by 5-10%.

Attendance at lectures: Not mandatory, but strongly advised.

Homework submission and return: We will use Gradescope for homework submission, return and as a portal for you to receive grades and feedback on your homeworks. If you run into problems with homework submission, please just drop me an email at twooley@purdue.edu and we will find a workaround. The expectation is that pdfs of homeworks can be submitted directly, and that hand-written solutions can be scanned with a smartphone or via computer, and the scanned images submitted. There will be more information once everything is fully established with our grader.

Boilerplate Notes for Boilermakers:

This course will adhere to all of Purdue's standard regulations and requirements found here:

https://catalog.purdue.edu/content.php?catoid=17&navoid=22202).

These regulations are frequently updated, and what follows is a digest of the main points that may differ from the most recent updates in detail, but not in spirit. We will of course respect the most recent updates, as is required.

Academic guidance in the event you are quarantined/isolated. If you must miss class at any point in time during the semester, please reach out to me via email so that we can communicate about how you can maintain your academic progress. If you find yourself too sick to progress in the course, notify your adviser and notify me via email. Be smart about COVID infections! If you are COVID positive, or suspect that to be the case, avoid spreading it around the class and contact me **by email** to make arrangements based on your particular situation.

Attendance: This course follows Purdue's academic regulations regarding attendance, which states that students are expected to be present for every meeting of the classes in which they are enrolled. When conflicts or absences can be anticipated, such as for many University-sponsored activities and religious observations, the student should inform the instructor of the situation as far in advance as possible. For unanticipated or emergency absences when advance notification to the instructor is not possible, the student should contact the instructor as soon as possible by email or phone. When the student is unable to make direct contact with the instructor and is unable to leave word with the instructor's department because of circumstances beyond the student's control, and in cases falling under excused absence regulations, the student or the student's representative should contact or go to the Office of the Dean of Students website to complete appropriate forms for instructor notification. Under academic regulations, excused absences may be granted for cases of grief/bereavement, military service, jury duty, and parenting leave. There is now a Medically Excused Absence Policy for Students. For details, see the Academic Regulations and Student Conduct section of the University Catalog website.

Academic Integrity: Academic integrity is one of the highest values that Purdue University holds. Individuals are encouraged to alert university officials to potential breaches of this value by either emailing integrity@purdue.edu, or by calling 765-494-8778, or by contacting the Office of the Dean of Students (https://www.purdue.edu/odos/). While information may be submitted anonymously, the more information that is submitted provides the greatest opportunity for the university to investigate the concern. Purdue prohibits "dishonesty in connection with any University activity. Cheating, plagiarism, or knowingly furnishing false information to the University are examples of dishonesty" (Section B.2.a of the Student Regulations

https://catalog.purdue.edu/content.php?catoid=17&navoid=21802).

Furthermore, the University Senate has stipulated that "the commitment of acts of cheating, lying, and deceit in any of their diverse forms (such as the use of ghostwritten papers, the use of substitutes for taking examinations, the use of illegal cribs, plagiarism, and copying during examinations) is dishonest and must not be tolerated. Moreover, knowingly to aid and abet, directly or indirectly, other parties in committing dishonest acts is in itself dishonest." Incidents of academic misconduct in this course will be addressed

by the course instructor and referred to the Office of Student Rights and Responsibilities (OSRR) for review at the university level. Any violation of course policies as it relates to academic integrity will result minimally in a failing or zero grade for that particular assignment or test, and at the instructor's discretion may result in a failing grade for the course. In addition, all incidents of academic misconduct will be forwarded to OSRR, where university penalties, including removal from the university, may be considered.

Boilermaker Honor Pledge: "As a Boilermaker pursuing academic excellence, I pledge to be honest and true in all that I do. Accountable together – we are Purdue."

https://www.purdue.edu/odos/osrr/honor-pledge/about.html.

Nondiscrimination: Purdue University is committed to maintaining a community which recognizes and values the inherent worth and dignity of every person; fosters tolerance, sensitivity, understanding, and mutual respect among its members; and encourages each individual to strive to reach his or her own potential. In pursuit of its goal of academic excellence, the University seeks to develop and nurture diversity. The University believes that diversity among its many members strengthens the institution, stimulates creativity, promotes the exchange of ideas, and enriches campus life. More details are available on our course Brightspace table of contents, under University Policies.

Purdue's nondiscrimination policy can be found at

https://www.purdue.edu/purdue/ea_eou_statement.php.

Basic Needs Security: Any student who faces challenges securing their food or housing and believes this may affect their performance in the course is urged to contact the Dean of Students for support. There is no appointment needed and Student Support Services is available to serve students 8 a.m.-5 p.m. Monday through Friday.

Academic Accommodation of Students with Disabilities: Purdue University strives to make learning experiences accessible to all participants. If you anticipate or experience physical or academic barriers based on disability, you are encouraged to contact the Disability Resource Center at: drc@purdue.edu or by phone: 765-494-1247, as soon as possible.

If the Disability Resource Center (DRC) has determined reasonable accommodations that you would like to utilize in this class, you must send your Course Accommodation Letter to the instructor. Instructions on sharing your Course Accommodation Letter can be found by visiting:

https://www.purdue.edu/drc/students/course-accommodation-letter.php

Additionally, you are strongly encouraged to contact the instructor as soon as possible to discuss implementation of your accommodations.

Mental Health: If you find yourself beginning to feel some stress, anxiety, and/or feeling slightly overwhelmed, try WellTrack at https://purdue.welltrack.com/. Sign in and find information and tools at your fingertips, available to you at any time. If you need support and information about options and resources, please see the Office of the Dean of Students, http://www.purdue.edu/odos, for drop-in hours (M-F, 8 am-5 pm). If you find yourself struggling to find a healthy balance between academics, social life, stress, etc. sign up for free one-on-one virtual or in-person sessions with a Purdue Wellness Coach at RecWell. Student coaches can help you navigate through barriers and challenges toward your goals throughout the semester. Sign up is completely free and can

be done on BoilerConnect. If you have any questions, please contact Purdue Wellness at evans240@purdue.edu.If you're struggling and need mental health services: Purdue University is committed to advancing the mental health and well-being of its students. If you or someone you know is feeling overwhelmed, depressed, and/or in need of mental health support, services are available. For help and to speak with a clinician, contact Counseling and Psychological Services (CAPS) at 765-494-6995 or by going to CAPS' office on the second floor of the Purdue University Student Health Center (PUSH). For urgent situations after hours, on weekends and holidays, call 765-494-6995 to speak with a clinician. Please see http://www.purdue.edu/caps/ for further information.

Commercial Note Taking in Classes: Notes taken in class are generally considered to be "derivative works" of the instructor's presentations and materials, and they are thus subject to the instructor's copyright in such presentations and materials. No individual is permitted to sell or otherwise barter notes, either to other students or to any commercial concern, for a course without the express written permission of the course instructor. See the Regulations on Student Conduct: Miscellaneous Conduct Regulations:

https://catalog.purdue.edu/content.php?catoid=17&navoid=21802

Use of AI: Use of Artificial Intelligence tools such as ChatGPT will not benefit you in this class, so engage and enhance your own intelligence instead. In particular, do not feed homework problem sets into any AI tools – this is a violation of copyright laws.

Major Campus Emergency: In the event of a major campus emergency, course requirements, deadlines and grading percentages are subject to changes that may be necessitated by a revised semester calendar or other circumstances beyond the instructor's control. Relevant changes to this course will be posted onto the course website or can be obtained by contacting the instructors or TAs via email or phone. You are expected to read your @purdue.edu email on a frequent basis.

EMERGENCY PREPAREDNESS LECTURE

As we begin this semester I want to take a few minutes and discuss emergency preparedness. Purdue University is a very safe campus and there is a low probability that a serious incident will occur here at Purdue. However, just as we receive a "safety briefing" each time we get on an aircraft, we want to emphasize our emergency procedures for evacuation and shelter in place incidents. Our preparedness will be critical IF an unexpected event occurs!

Emergency preparedness is your personal responsibility. Purdue University is actively preparing for natural disasters or human-caused incidents with the ultimate goal of maintaining a safe and secure campus. Let's review the following procedures:

For any emergency text or call 911.

There are more than 300 Emergency Telephones (aka blue lights) throughout campus that connect directly to the Purdue Police Department (PUPD). If you feel threatened or need help, push the button and you will be connected right away.

If we hear a fire alarm we will immediately evacuate the building and proceed to:

Primary Location: in order not to interfere with emergency personnel, occupants are to proceed to the south side of the Physics Building and assemble in the area between PHYS and MSEE. DO NOT assemble between the building and Northwestern Avenue or between the building and Forney Hall, Hampton Hall, Armstrong Hall. In case of inclement weather assemble in MSEE Atrium, ground floor. **Do not use the elevator.**

If we are notified of a Shelter in Place requirement for a tornado warning we will stop classroom or research activities and shelter in the lowest level of this building away from windows and doors.

Our preferred location is: basement corridors, basement offices, basement restrooms or the lowest level of the building (stay away from windows and doors)

If we are notified of a Shelter in Place requirement for a hazardous materials release we will shelter in our classroom shutting any open doors and windows.

If we are notified of a Shelter in Place requirement for an active threat such as a shooting we will shelter in a room that is securable preferably without windows. Our preferred location is inside this classroom with doors and windows secured.

Attached to the syllabus is an "Emergency Preparedness for Classrooms" sheet that provides additional preparedness information. Please review the sheet and the Emergency Preparedness website for additional emergency preparedness information.

EMERGENCY PREPAREDNESS SYLLABUS ATTACHMENT

EMERGENCY NOTIFICATION PROCEDURES are based on a simple concept – if you hear a fire alarm inside, proceed outside. If you hear a siren outside, proceed inside.

- **Indoor Fire Alarms** mean to stop class or research and immediately **evacuate** the building.
  - Proceed to your Emergency Assembly Area away from building doors. **Remain outside** until police, fire, or other emergency response personnel provide additional guidance or tell you it is safe to leave.
- **All Hazards Outdoor Emergency Warning Sirens** mean to immediately seek shelter (**Shelter in Place**) in a safe location within the closest building.
  - "Shelter in place" means seeking immediate shelter inside a building or University residence. This course of action may need to be taken during a tornado, an active threat including a shooting or a release of hazardous materials in the outside air. Once safely inside, find out more details about the emergency*. **Remain in place** until police, fire, or other emergency response personnel provide additional guidance or tell you it is safe to leave.

**In both cases, you should seek additional clarifying information by all means possible…Purdue Emergency Status page, text message, Twitter, Desktop Alert, Albertus Beacon, digital signs, email alert, TV, radio, etc.…review the Purdue Emergency Warning Notification System multi-communication layers at http://www.purdue.edu/ehps/emergency_preparedness/warning-system.html*

EMERGENCY RESPONSE PROCEDURES:

- Review the **Emergency Procedures Guidelines** https://www.purdue.edu/emergency_preparedness/flipchart/index.html
- Review the **Building Emergency Plan** (available on the Emergency Preparedness website or from the building deputy) for:
  - evacuation routes, exit points, and emergency assembly area
  - when and how to evacuate the building.
  - shelter in place procedures and locations
  - additional building specific procedures and requirements.

EMERGENCY PREPAREDNESS AWARENESS VIDEOS

- **"Run. Hide. Fight.®"** is a 6-minute active shooter awareness video that illustrates what to look for and how to prepare and react to this type of incident. See: https://www.youtube.com/watch?v=5mzI_5aj4Vs (Link is also located on the EP website)

MORE INFORMATION

Reference the Emergency Preparedness web site for additional information: https://www.purdue.edu/ehps/emergency_preparedness/