# Synthetic wavefront generation for aero-induced turbulence using boundary layer data

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# Aero-Induced Turbulence

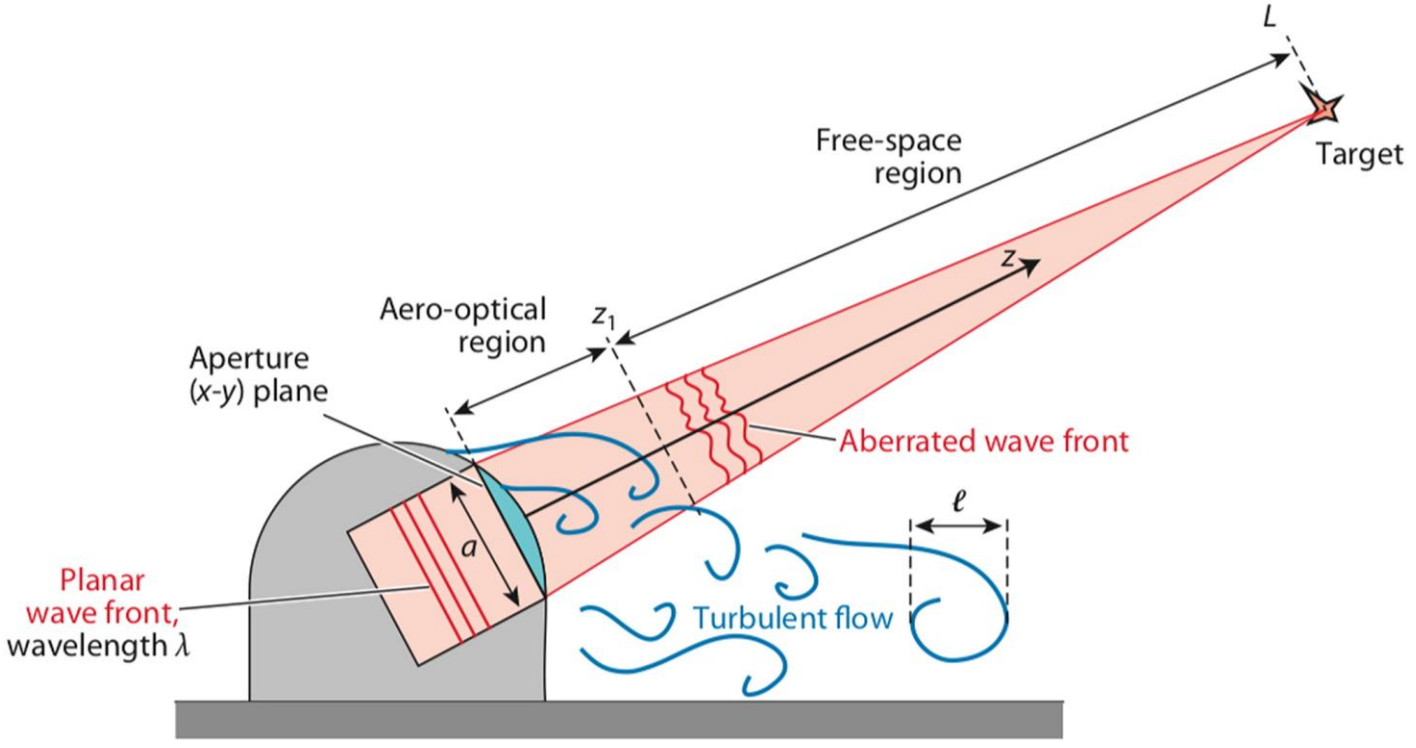
**Goal**: *Detect* and *transmit* light from the side of a high-speed aircraft.

https://aero-optics.nd.edu/research/hemispherical-turret-beam-directors/


[1] E. J. Jumper, S. Gordeyev, and M. R. Whiteley, “Aero-optical effects,” in Aero-Optical Effects, (John Wiley Sons, Incorporated, United States, 2023).

# Aero-Induced Turbulence

**Problem**: Turbulent flow near the aperture **distorts light propagation**.

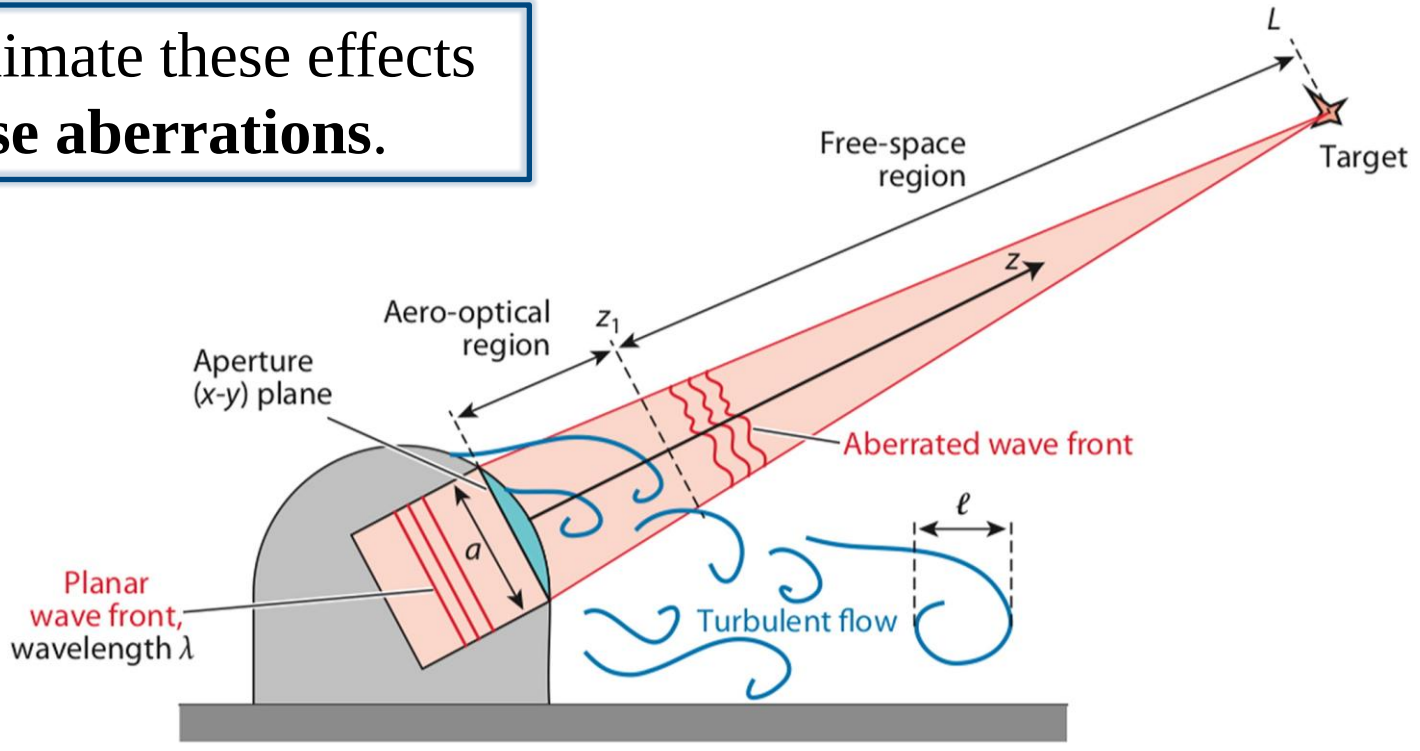



[2] M. Wang, A. Mani, and S. Gordeyev, "Physics and Computation of Aero-Optics," *Annual Review of Fluid Mechanics*, Vol. 44, No. 1, 2012, pp. 299–321.

# Aero-Induced Turbulence

**Problem**: Turbulent flow near the aperture **distorts light propagation**.

We approximate these effects as **phase aberrations**.

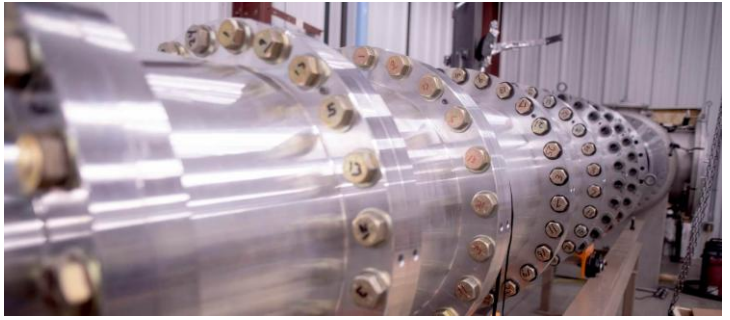



[2] M. Wang, A. Mani, and S. Gordeyev, "Physics and Computation of Aero-Optics," *Annual Review of Fluid Mechanics*, Vol. 44, No. 1, 2012, pp. 299–321.

# Adaptive Optics for Aero-Induced Turbulence

- Adaptive-optic (AO) systems can compensate for phase aberrations.
- **Problem**: AO system development *requires phase aberration data*. Existing data acquisition methods are *expensive and time-intensive*.

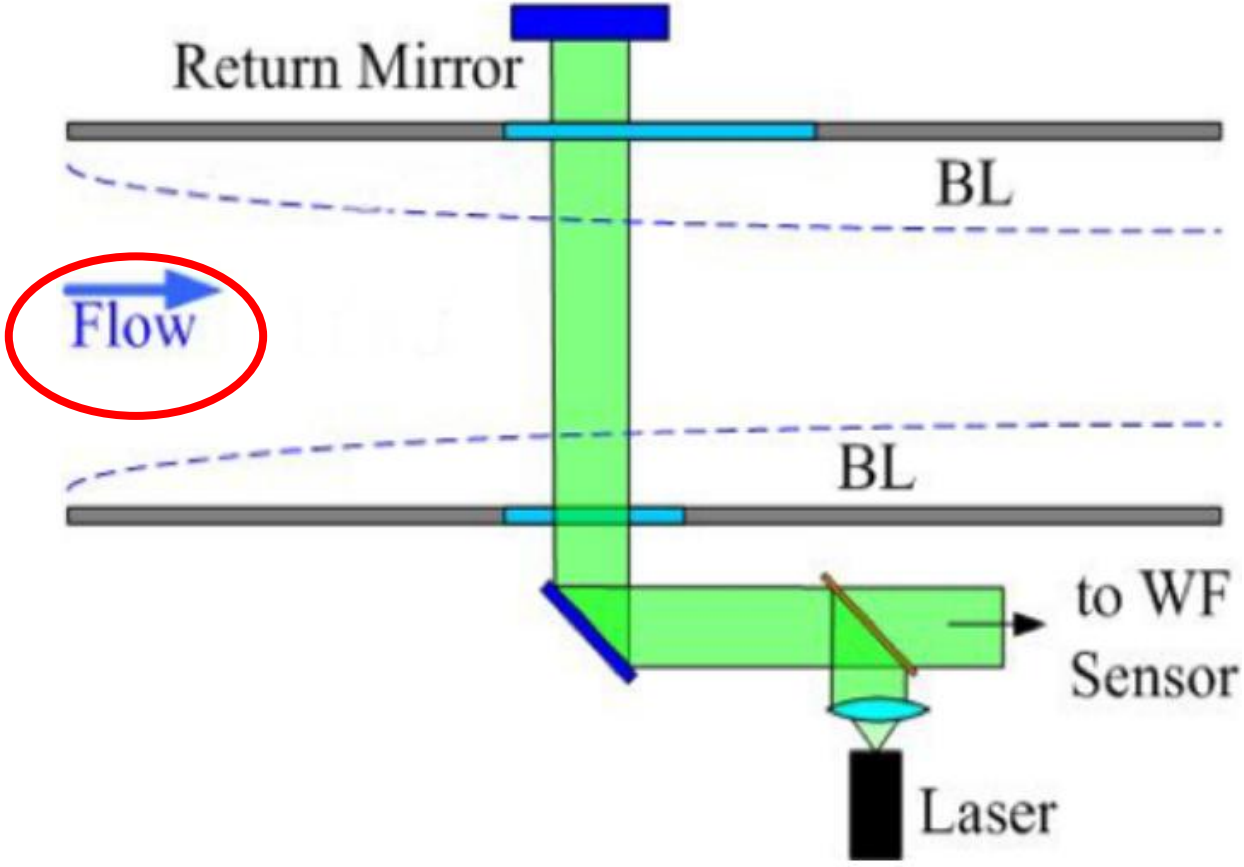

https://aero-optics.nd.edu/research/aaol/



https://hyperlab.nd.edu/facilities/

- **Solution**: We develop **ReVAR (Re-whitened Vector Auto-Regression)**, a *data-driven algorithm* that generates *synthetic time-series of phase aberration data* of arbitrary duration.

# Experimental Data: Wind Tunnel Experiment




[3] M. R. Kemnetz and S. Gordeyev, "Optical investigation of large-scale boundary-layer structures",
*54th AIAA Aerospace Sciences Meeting*, 4 - 8 Jan 2016, San Diego, California, AIAA Paper 2016-1460.

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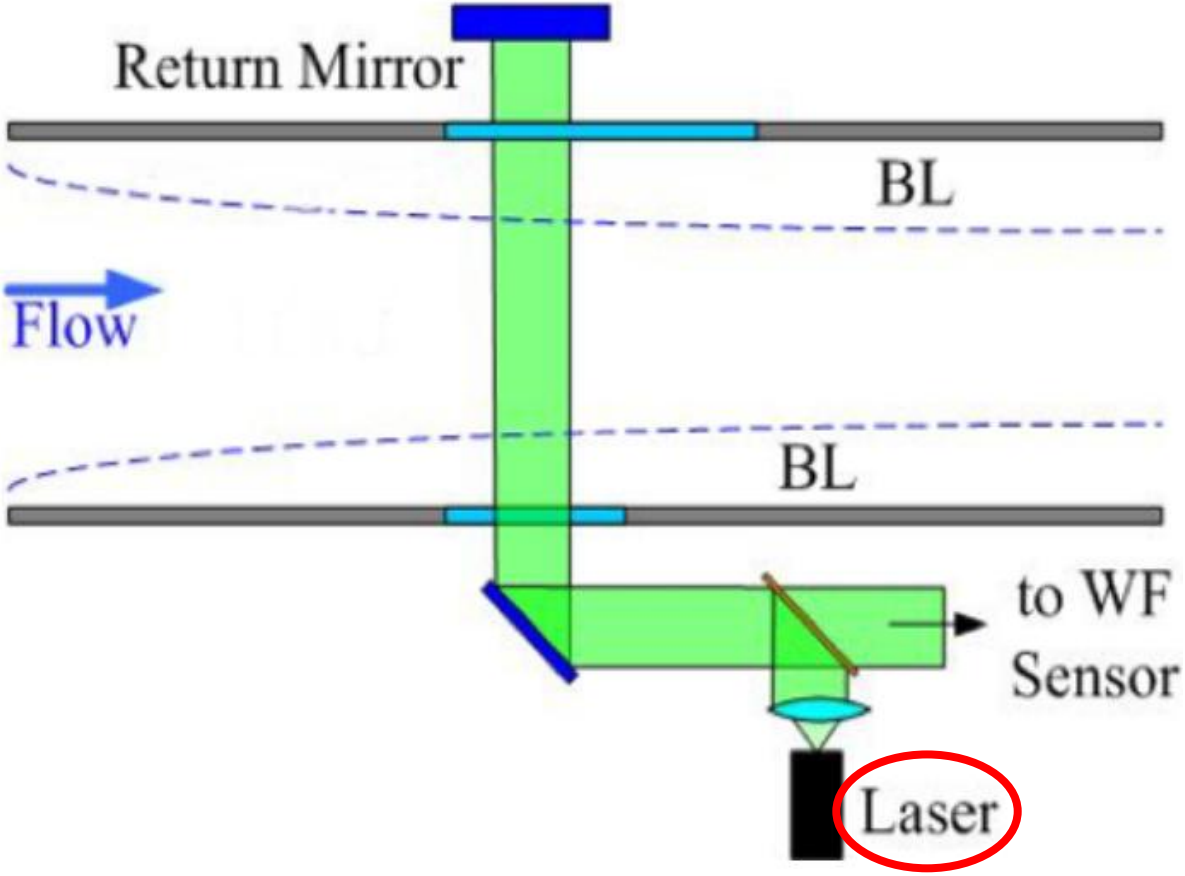


[3] M. R. Kemnetz and S. Gordeyev, "Optical investigation of large-scale boundary-layer structures",
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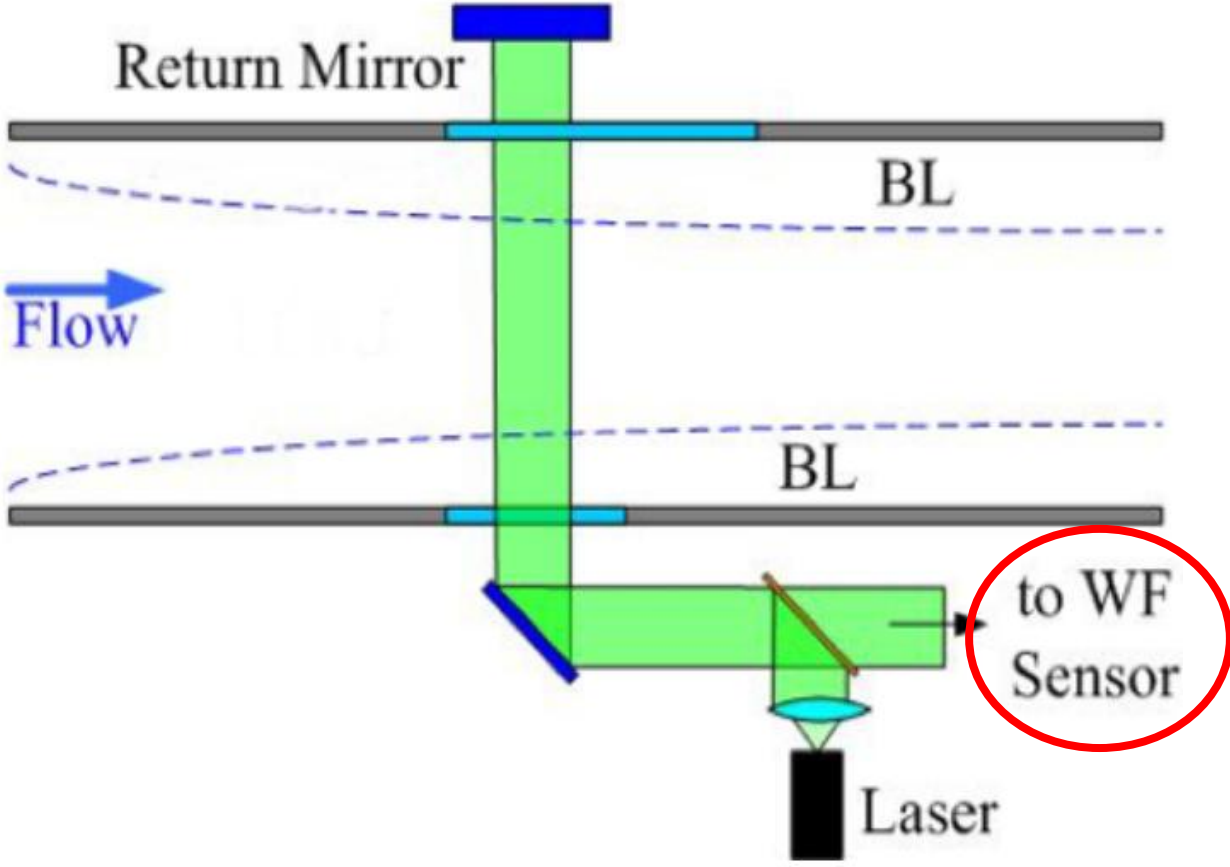
# Experimental Data: Wind Tunnel Experiment

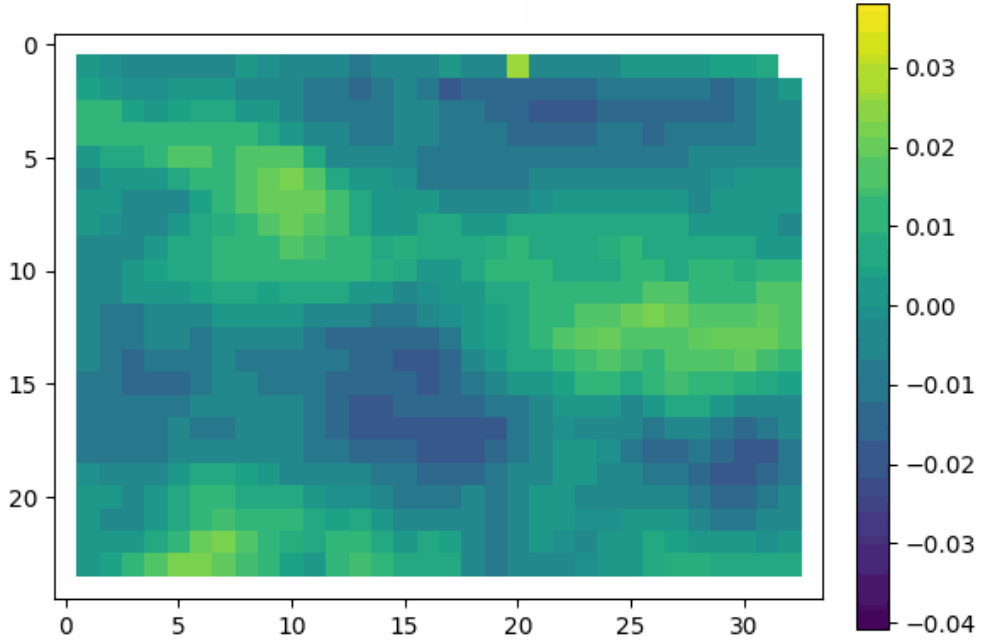
- This simulates the environment around the imaging system.
- The result of the experiment is a **time-series of phase screens**.



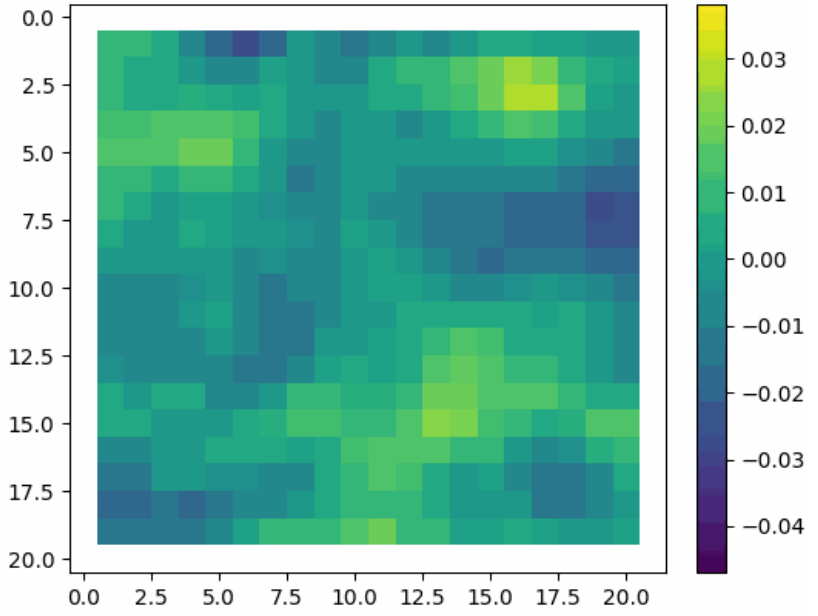

[3] M. R. Kemnetz and S. Gordeyev, "Optical investigation of large-scale boundary-layer structures", *54th AIAA Aerospace Sciences Meeting*, 4 - 8 Jan 2016, San Diego, California, AIAA Paper 2016-1460.

# Experimental Data: Visualization

Data Set F06



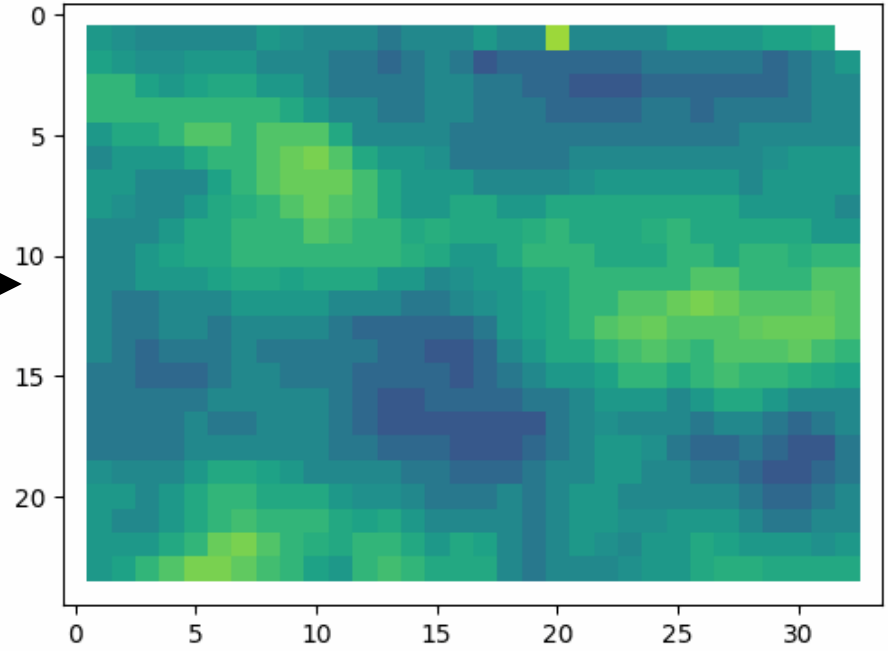

Data Set F12



# ReVAR: Re-whitened Vector AutoRegression

**ReVAR**:

- Take in a **short video** of *input data*.
- Create a **much longer** video of *synthetic data*.

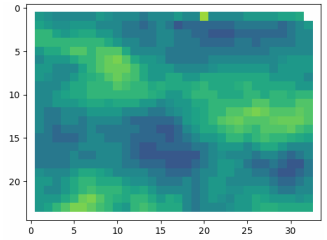



**Match** the *spatial* and *temporal* **statistics** of the input video.

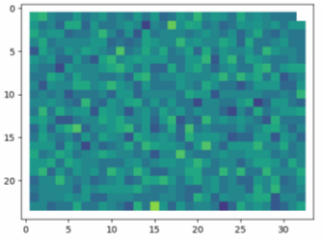
# ReVAR: Re-whitened Vector AutoRegression

*Transform* input data into **white noise** in a way that is **invertible**.

***Input Data***

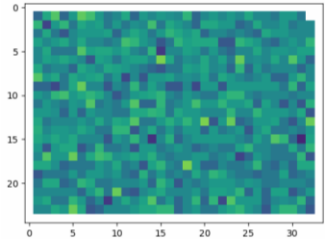


**ReVAR**

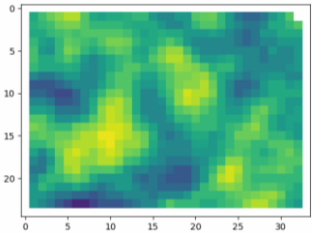


***White Noise***

***White Noise***



**$\text{ReVAR}^{-1}$**



***Synthetic Data***

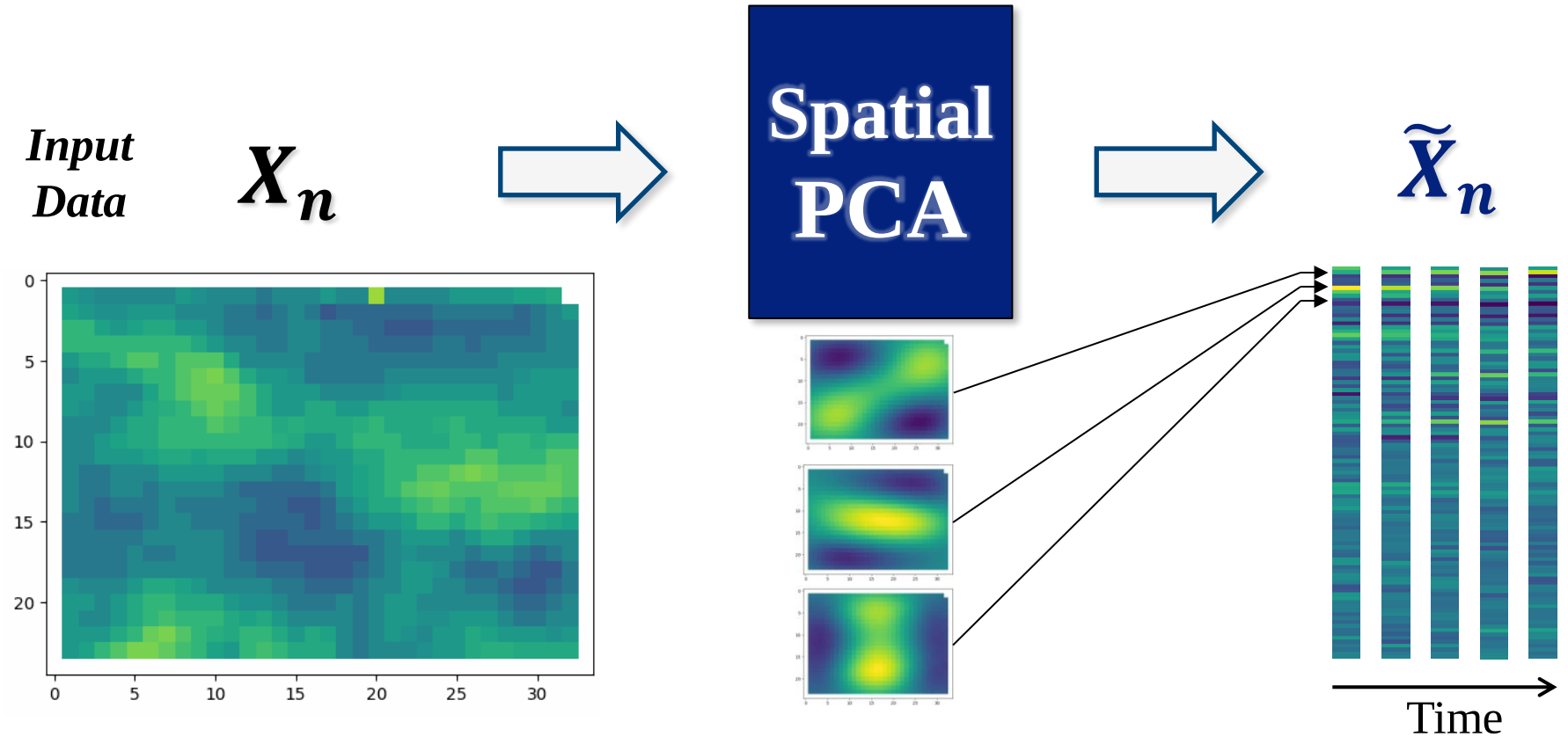
# ReVAR: Algorithm Overview

1. Parameter Estimation


2. Data Synthesis

# Parameter Estimation: PCA

Use PCA to extract the *temporal coefficients*.



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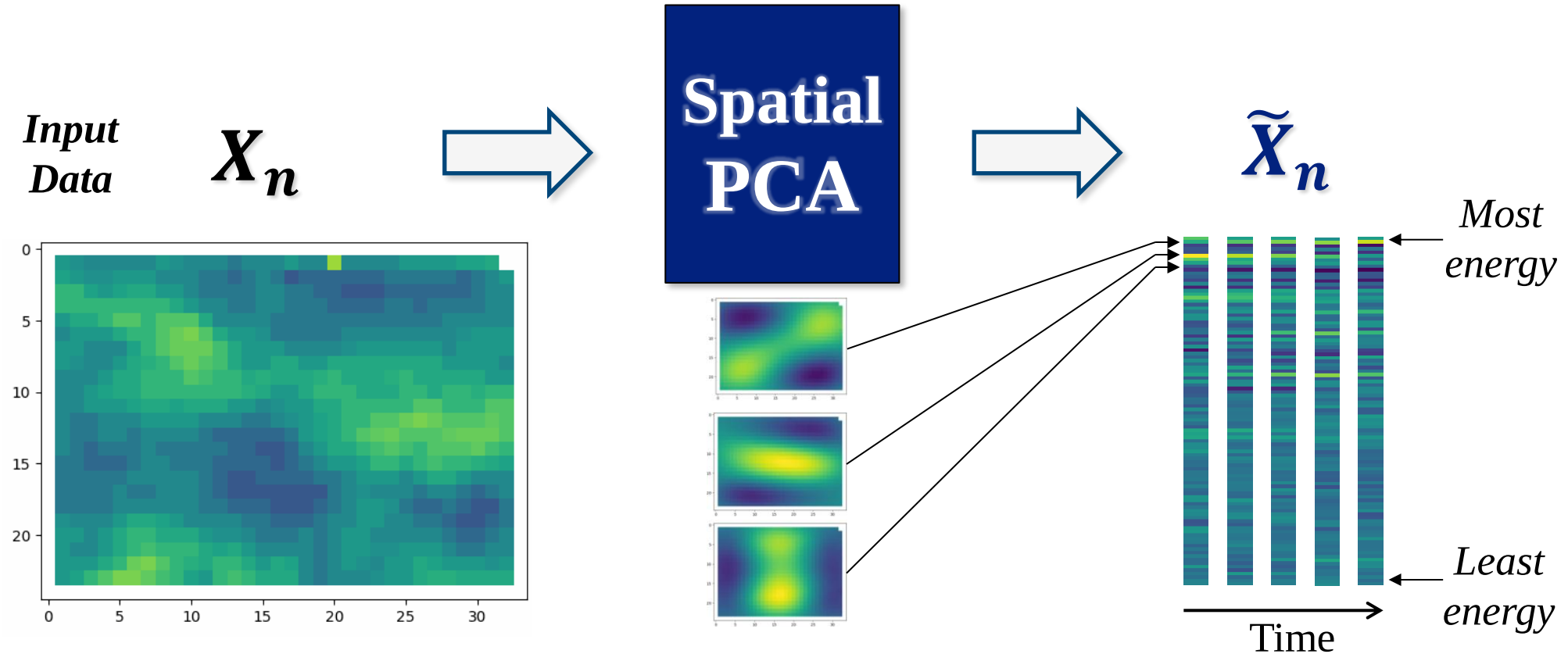
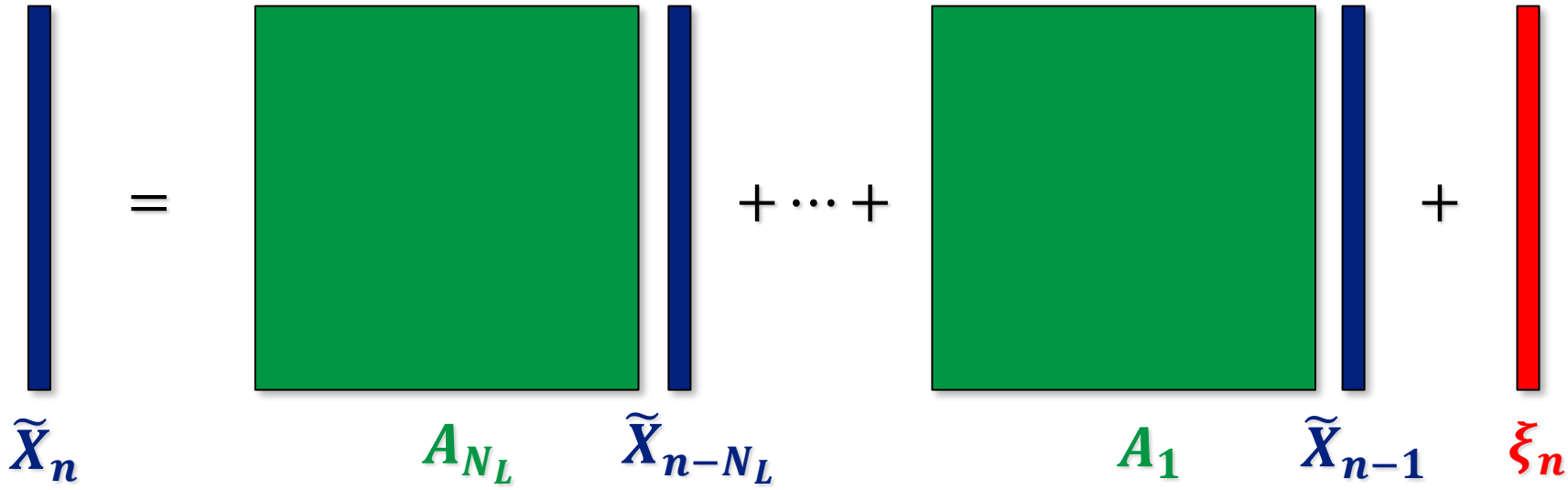


# ReVAR: Vector Auto-Regression (VAR)

$$\tilde{X}_n = A_{N_L}\tilde{X}_{n-N_L} + \cdots + A_1\tilde{X}_{n-1} + \xi_n$$

**Linear Time Prediction**

**Prediction Error**




[4] H. Lütkepohl, *New Introduction to Multiple Time Series Analysis*, 2005, Springer-Verlag Berlin, pg. 13-14, 69-71.

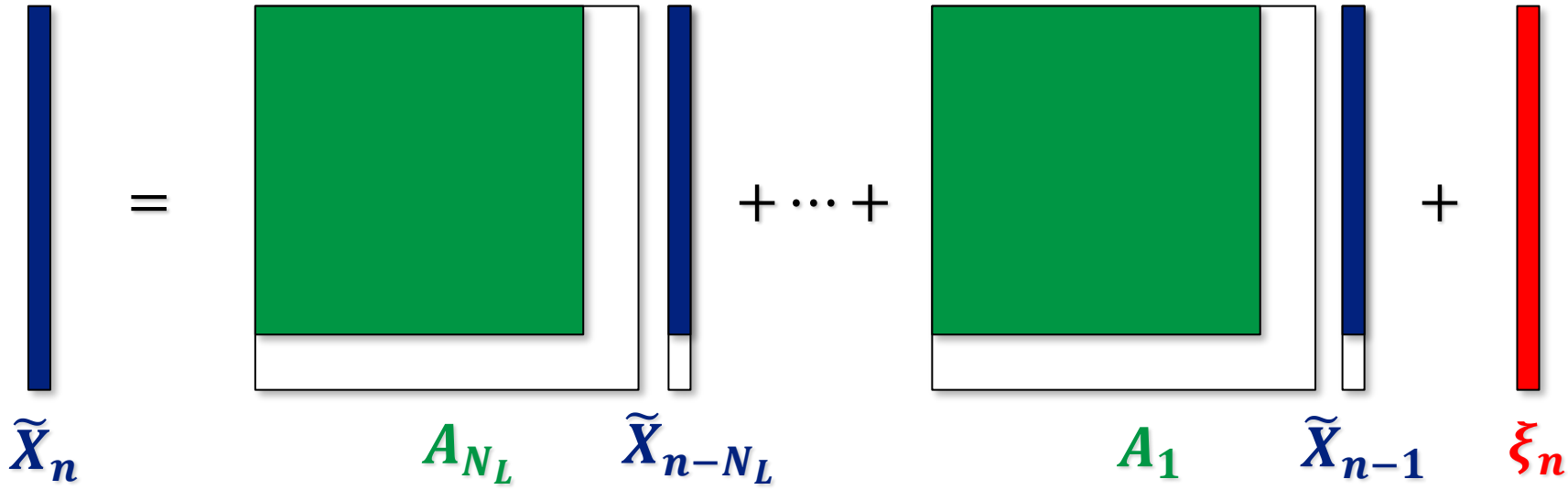
# ReVAR: Vector Auto-Regression (VAR)

$$\tilde{X}_n = A_{N_L}\tilde{X}_{n-N_L} + \cdots + A_1\tilde{X}_{n-1} + \xi_n$$

**Restrict** time prediction to the **top vector components**.

**Linear Time Prediction**

**Prediction Error**





[4] H. Lütkepohl, *New Introduction to Multiple Time Series Analysis*, 2005, Springer-Verlag Berlin, pg. 13-14, 69-71.

# ReVAR: Vector Auto-Regression (VAR)

$$\xi_n = \tilde{X}_n - \left(A_{N_L}\tilde{X}_{n-N_L} + \cdots + A_1\tilde{X}_{n-1}\right)$$

What are the statistics of the **prediction error $\xi_n$**?

$\xi_n$

# ReVAR: Vector Auto-Regression (VAR)

$$\xi_n = \tilde{X}_n - \left(A_{N_L}\tilde{X}_{n-N_L} + \cdots + A_1\tilde{X}_{n-1}\right)$$

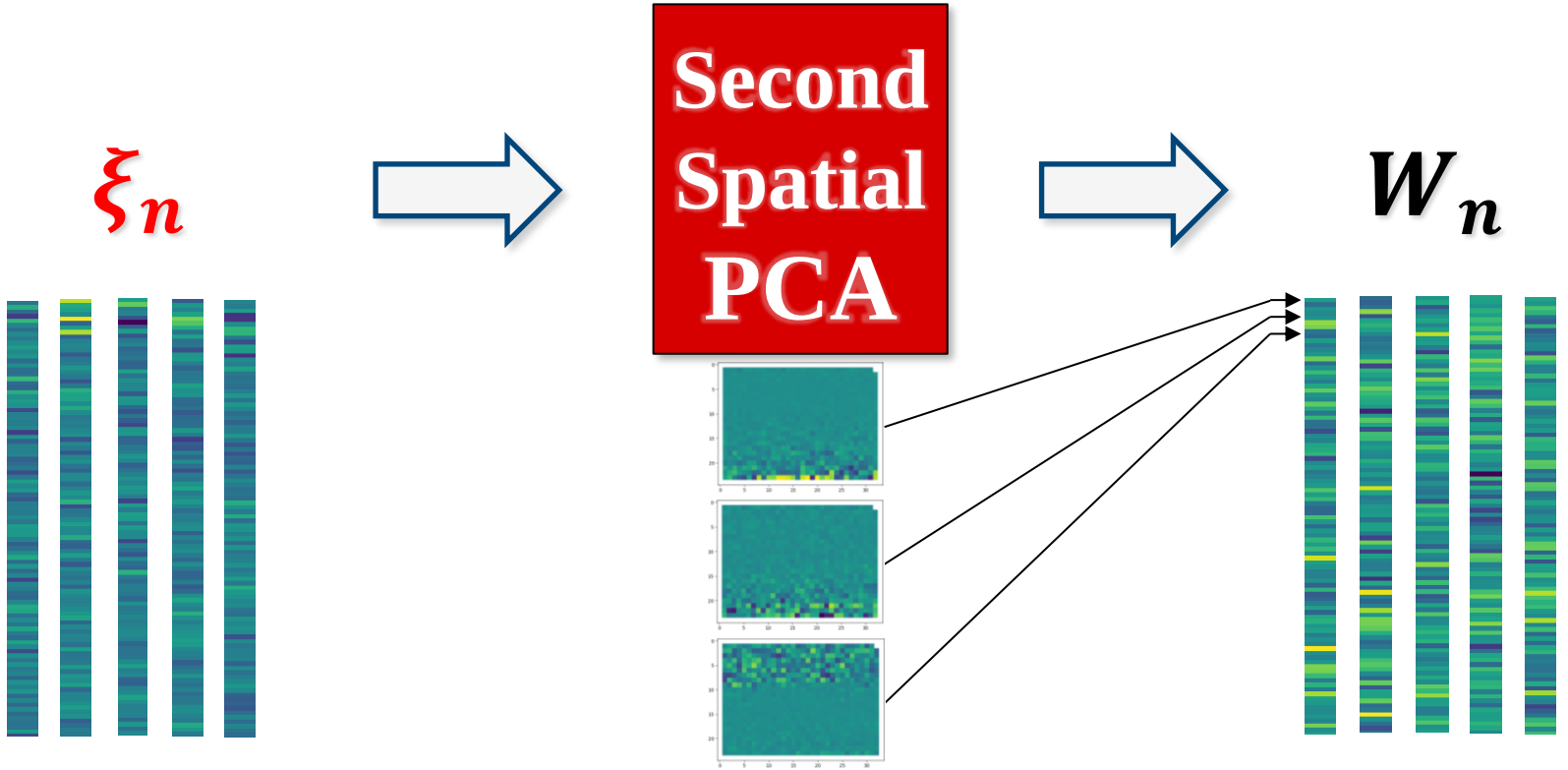
What are the statistics of the **prediction error** $\xi_n$?

$\xi_n$ is **white** in *time* but **correlated** in *space*.

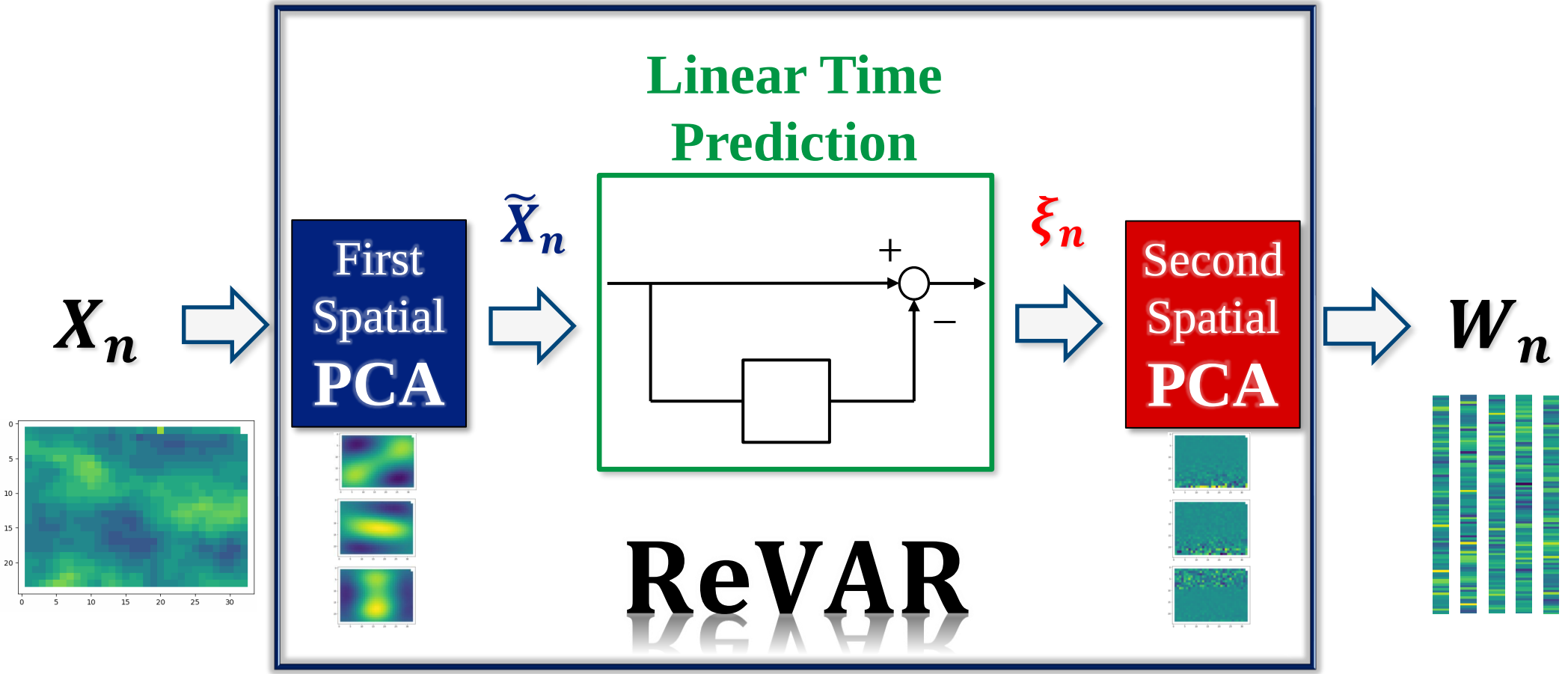
$\xi_n$

# Parameter Estimation: Re-whitening

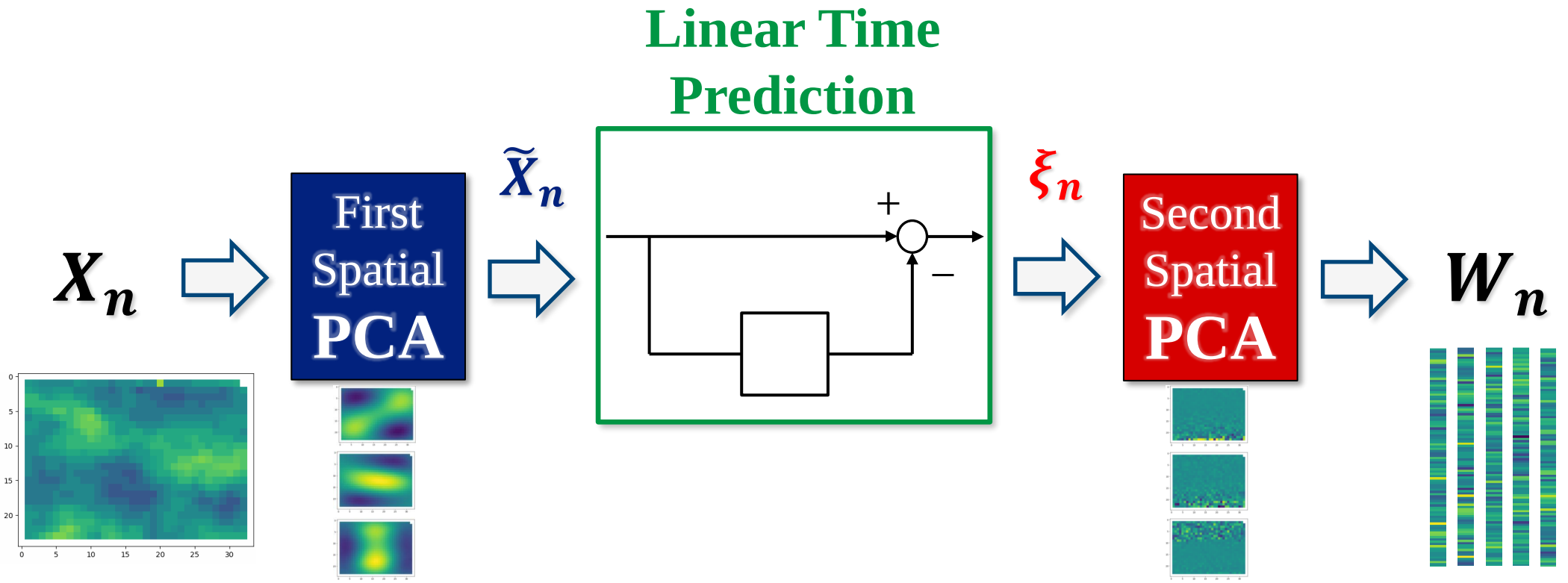
A **second PCA** *whitens* $\xi_n$ in *space*.



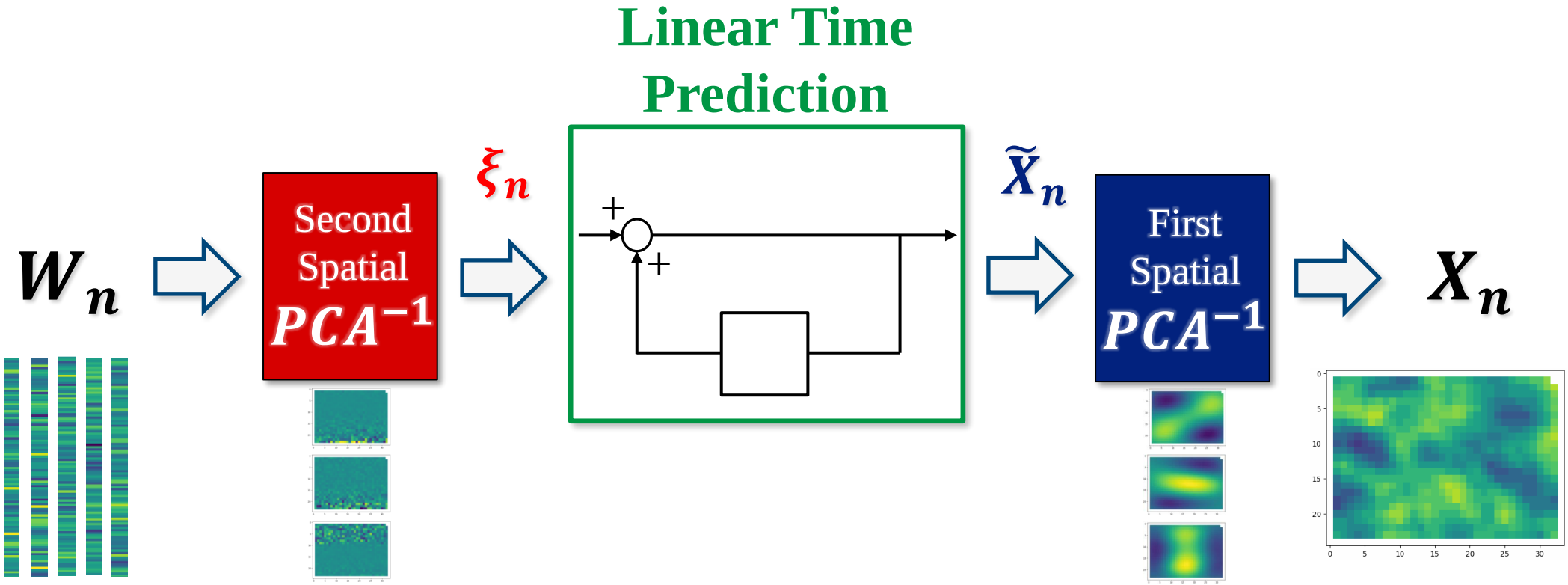
# ReVAR: Parameter Estimation



# ReVAR: Parameter Estimation



# ReVAR: Data Synthesis



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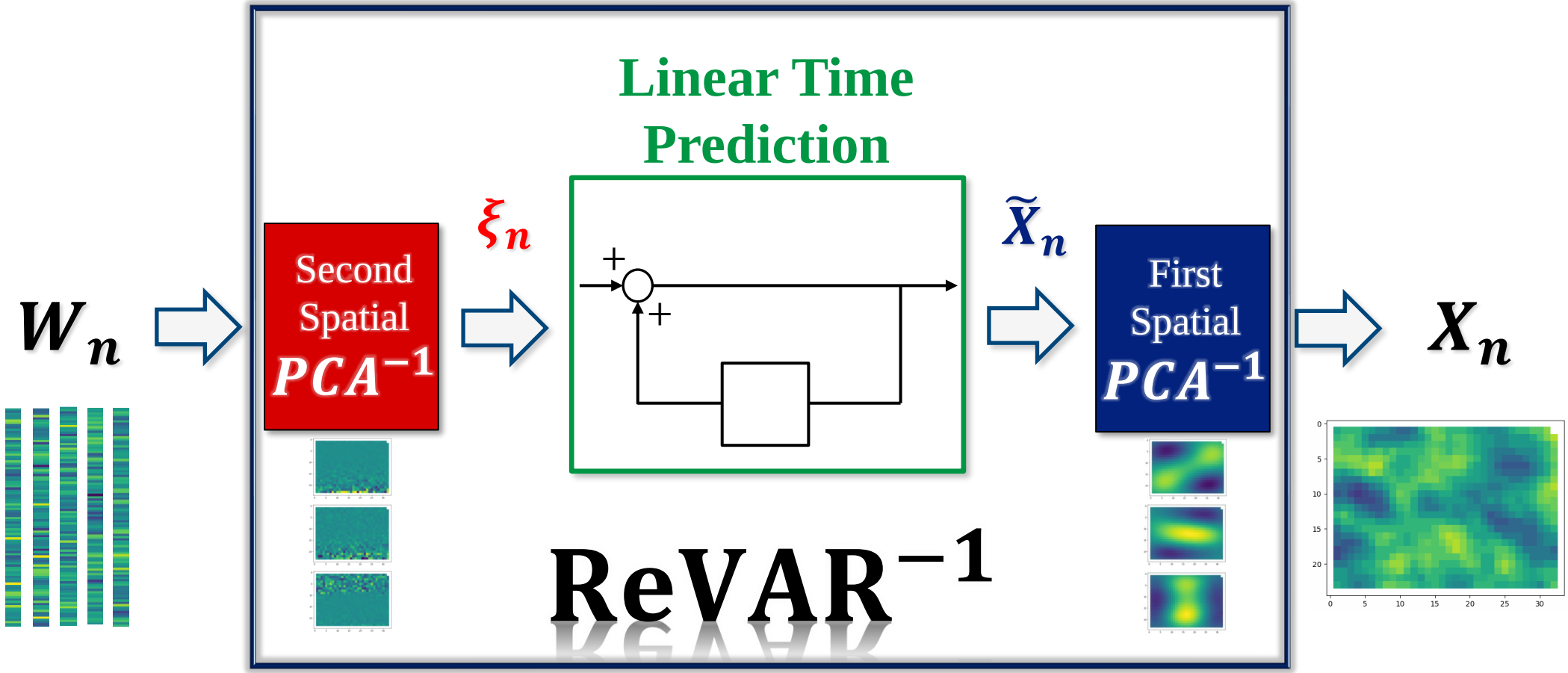

# ReVAR: Algorithm Overview

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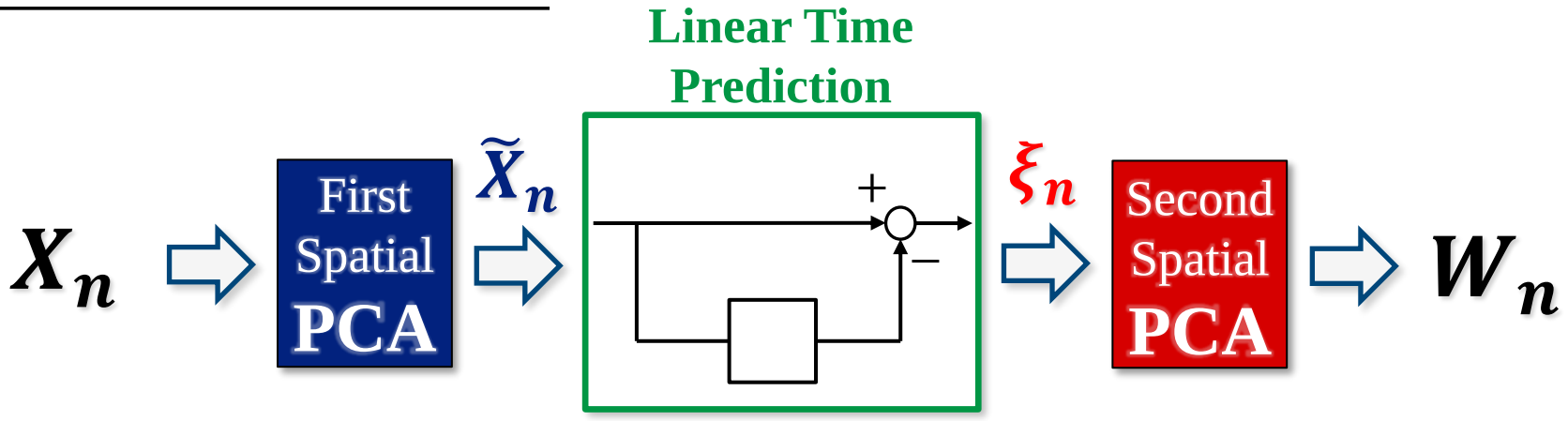
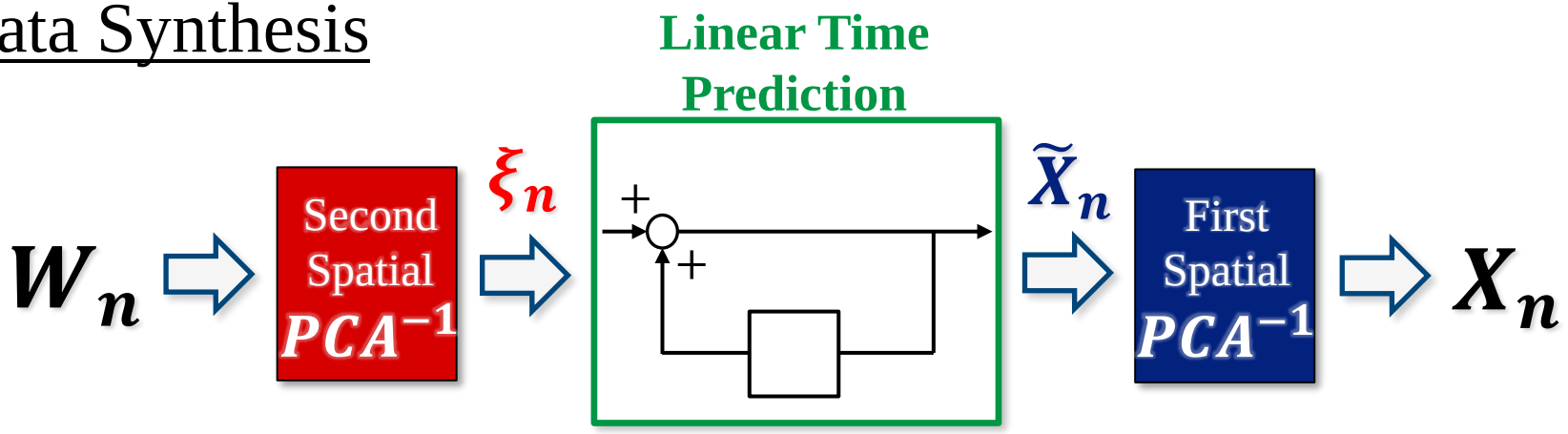
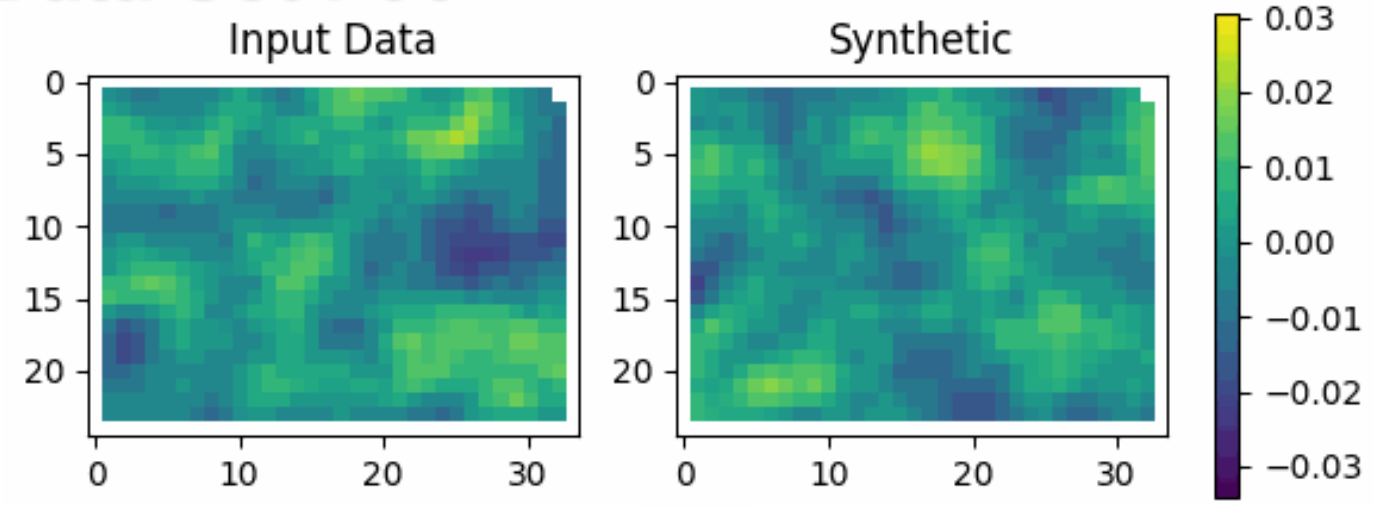
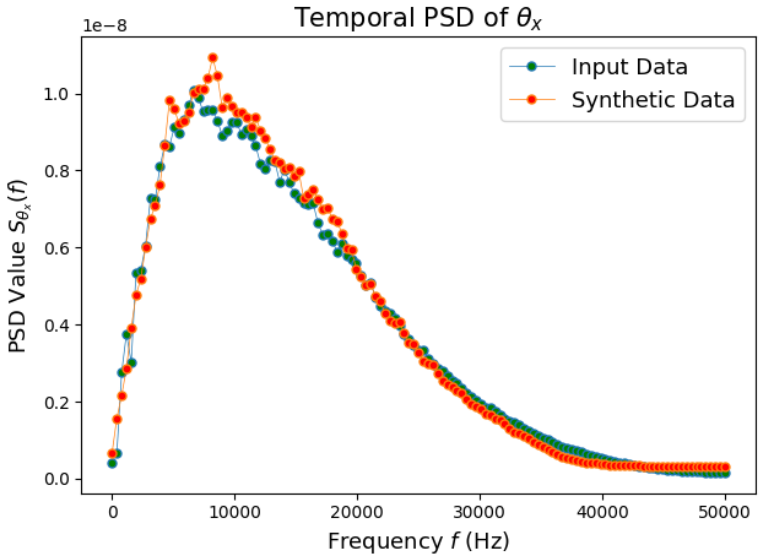


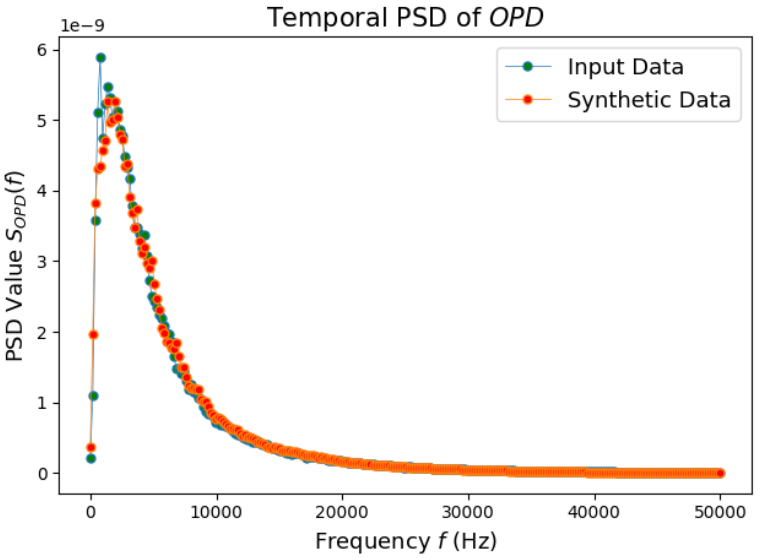

2. Data Synthesis



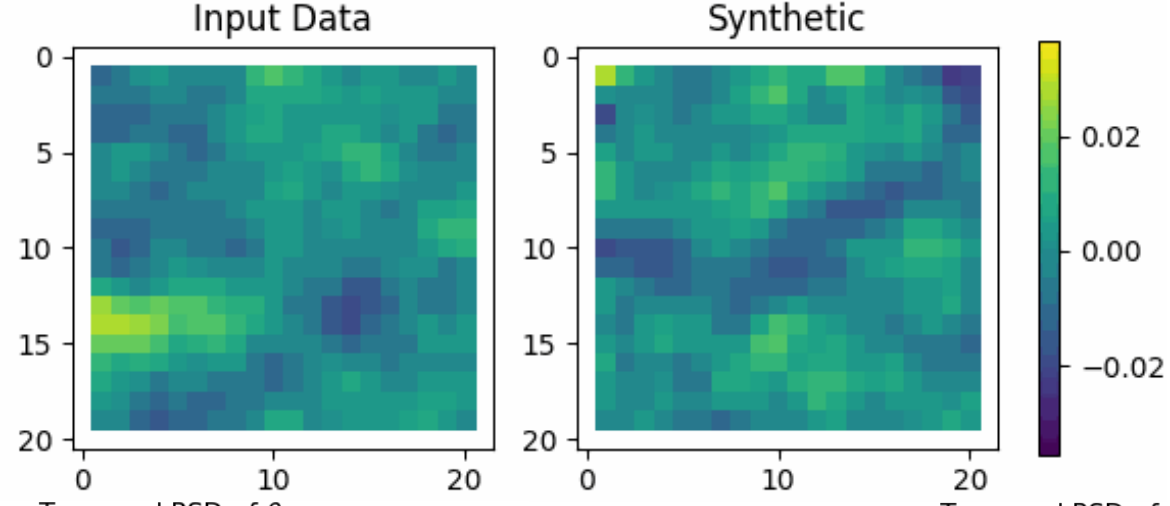
# Results: Data Set F06

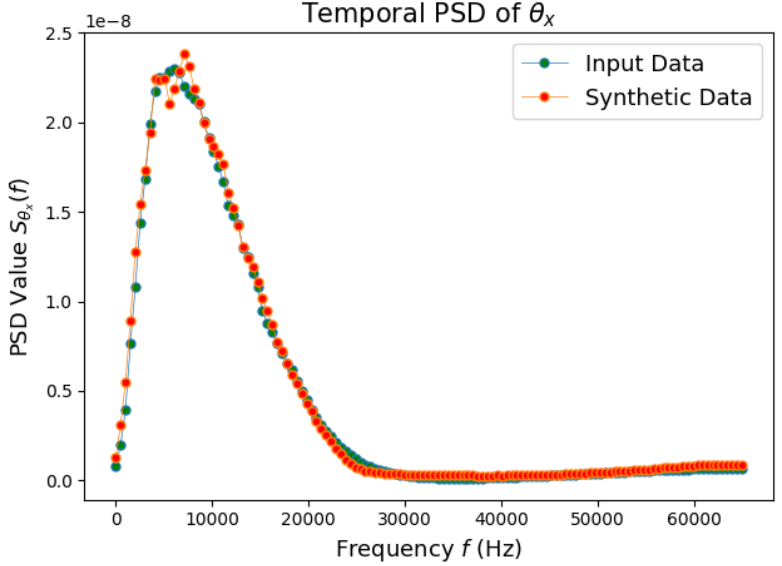


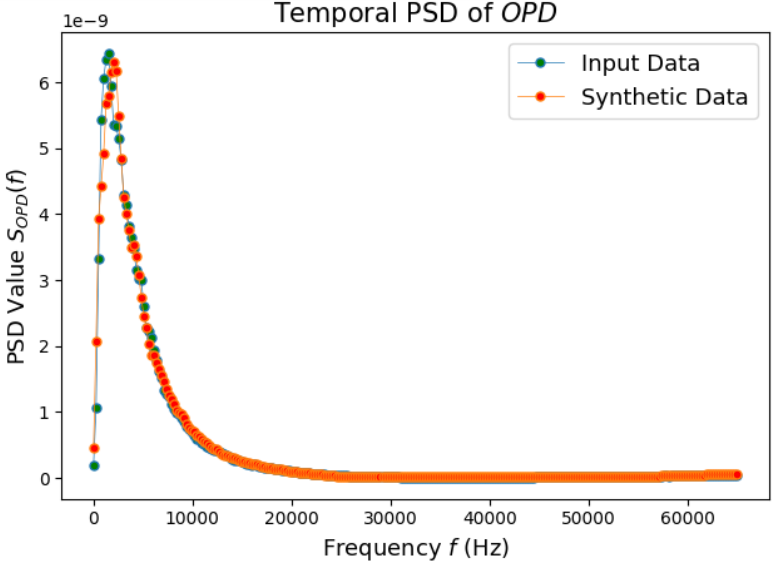



# Results: Data Set F12



# Algorithm Run-Time

Data Set F06

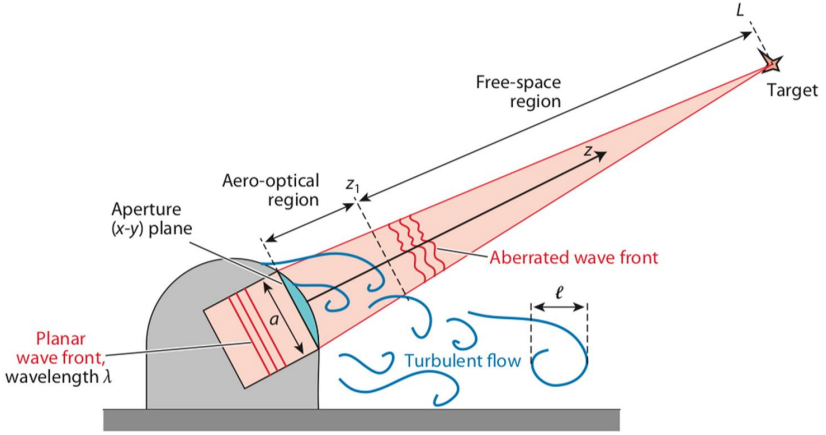
- Parameter Estimation: **5.88 mins**
- Data Synthesis: **1.33 mins**
  (1 sec of synthetic data)

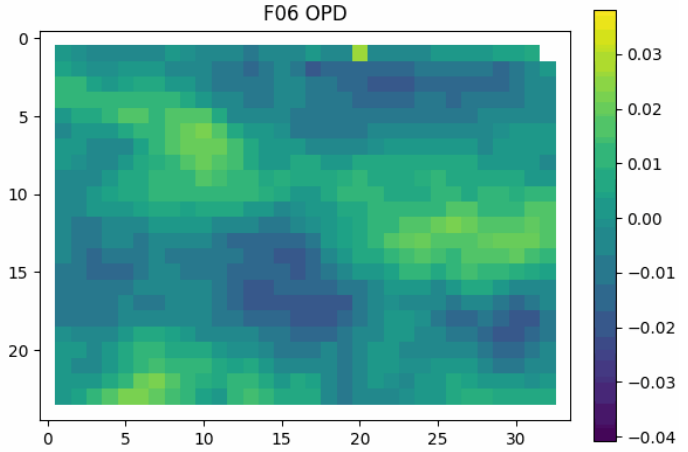
Data Set F12

- Parameter Estimation**:** **6.96 mins**
- Data Synthesis: **1.32 mins**
  (1 sec of synthetic data)

# Summary

- Aero-induced turbulence causes phase aberrations in propagating light waves, thus degrading imaging performance from a high-speed aircraft.
- AO system development requires phase aberration data to compensate for these effects.

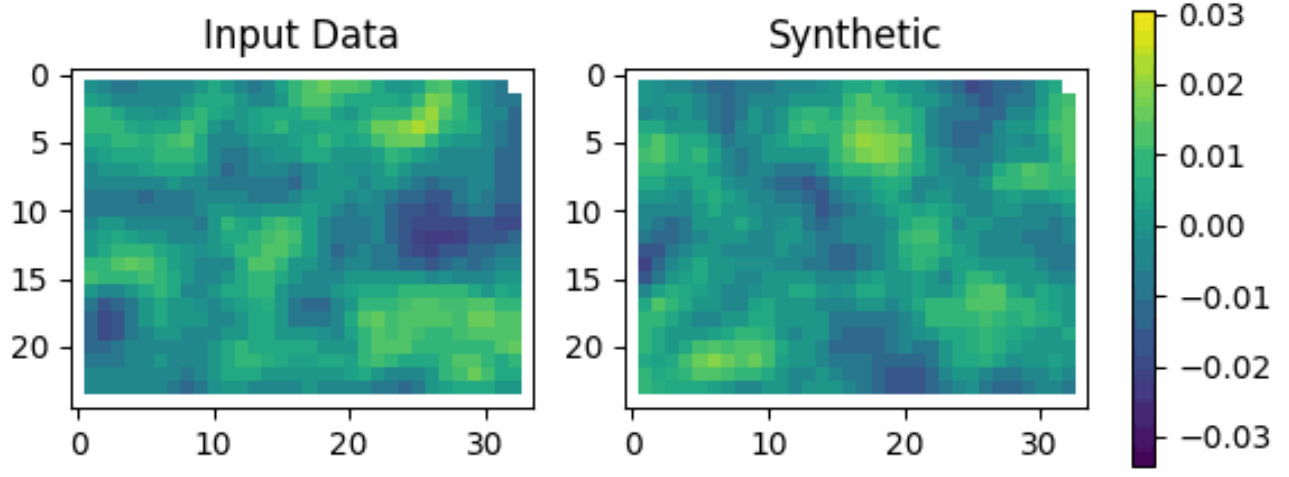



[2] M. Wang, A. Mani, and S. Gordeyev, "Physics and Computation of Aero-Optics," *Annual Review of Fluid Mechanics*, Vol. 44, No. 1, 2012, pp. 299–321.

# Conclusion

We created **ReVAR**: a computationally efficient algorithm that quickly generates high quality time-series of phase screens with arbitrary duration.



# References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