Modeling Beetle Infestation Throughout a Forest as a Spatial Stochastic Process

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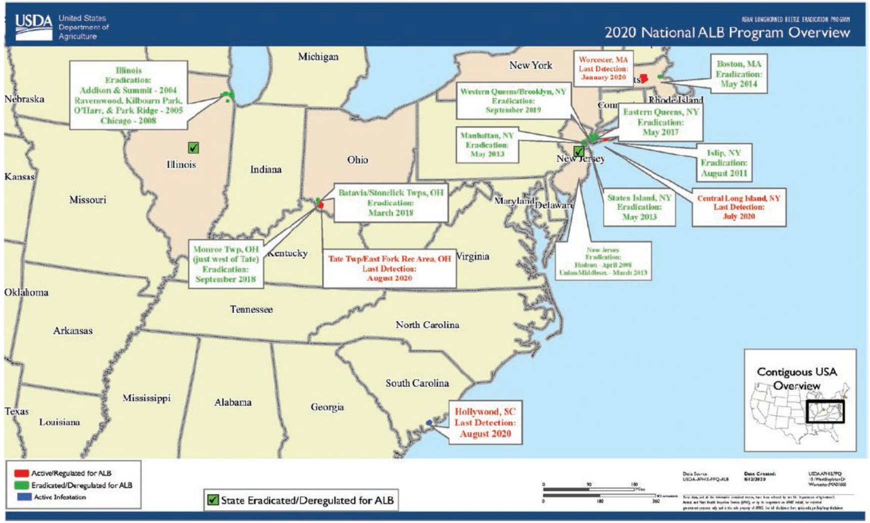
Overview

Problem Statement:

How do we predict the spread of infection caused by Asian Long-horned Beetles in a forest?

Motivation

- Asian Long-horned Beetle (ALB) is...
  - Native to asia
  - Discovered in Massachusetts in 1996
- Causes a disease to trees that **no known cure exists**



Data Analysis per Year

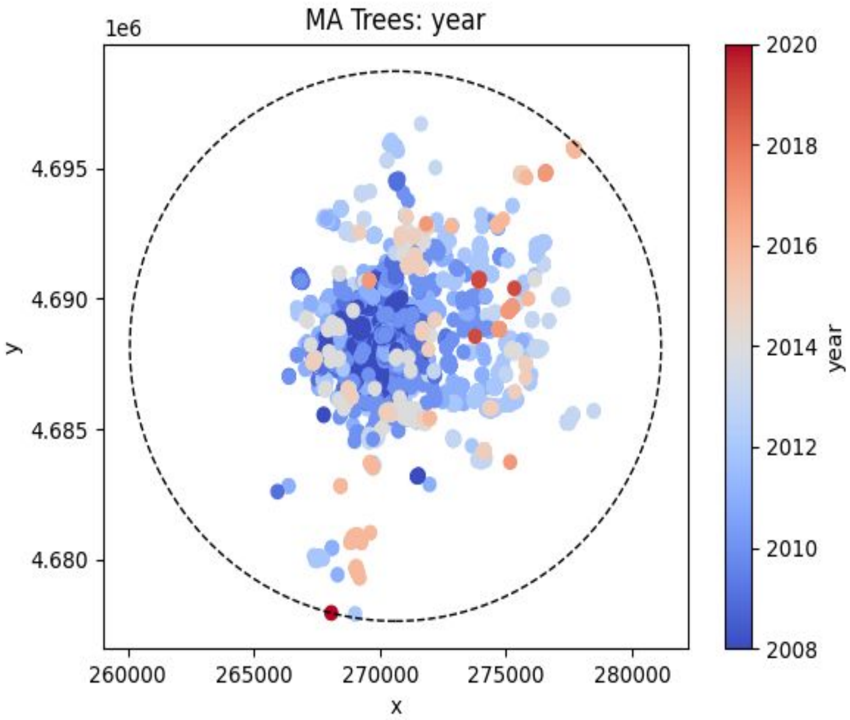
Overview

Motivation

Progress
- **Data Analysis**
- Model Construction

Next Steps

- Our data contains 2-D points of locations of infected trees in Massachusetts.
- The data set includes 2 parameters:
    - Year
    - Species



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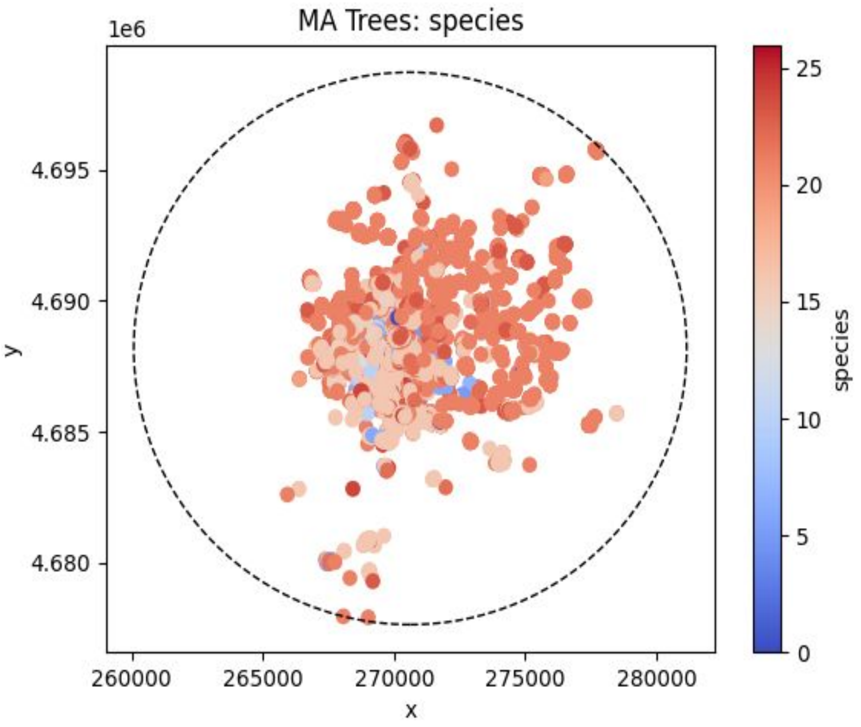
Data Analysis per Species

- Our data contains 2-D points of locations of infected trees in Massachusetts.
- The data set includes 2 parameters:
  - Year
  - **Species**



Data Analysis per Infestation Level

Overview

Motivation

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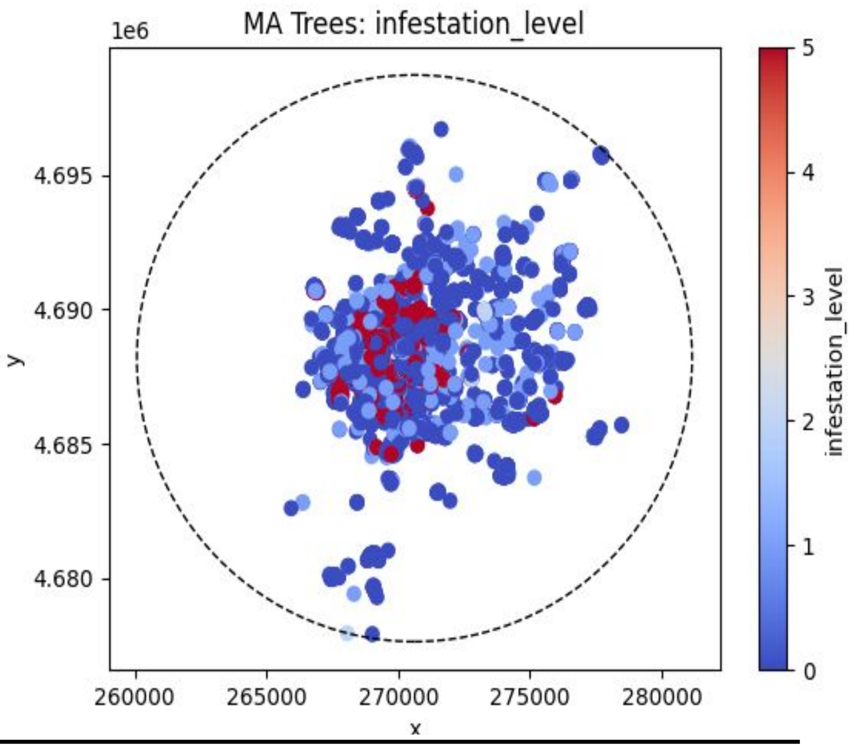
- **Data Analysis**
- Model Construction

Next Steps

- Our data contains 2-D points of locations of infected trees in Massachusetts.
- The data set includes 3 parameters:
  - Year
  - Species
  - Infestation Level



Data Analysis per Year - Grouped

Overview

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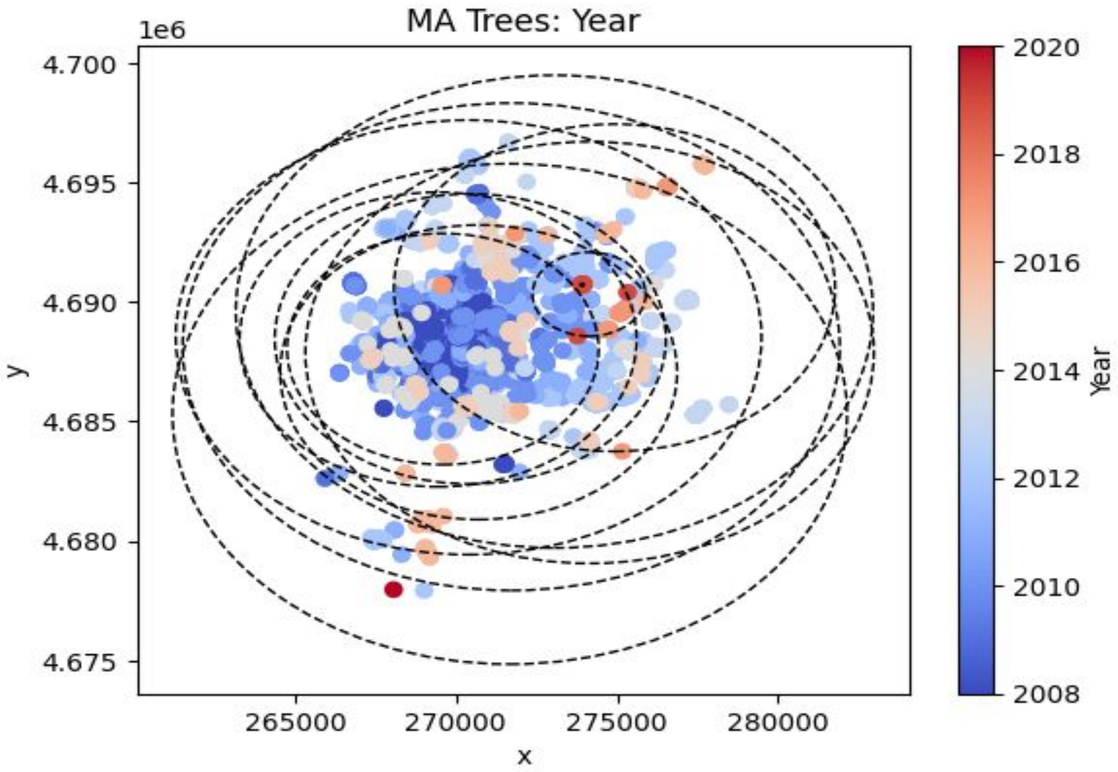


Methods Employed

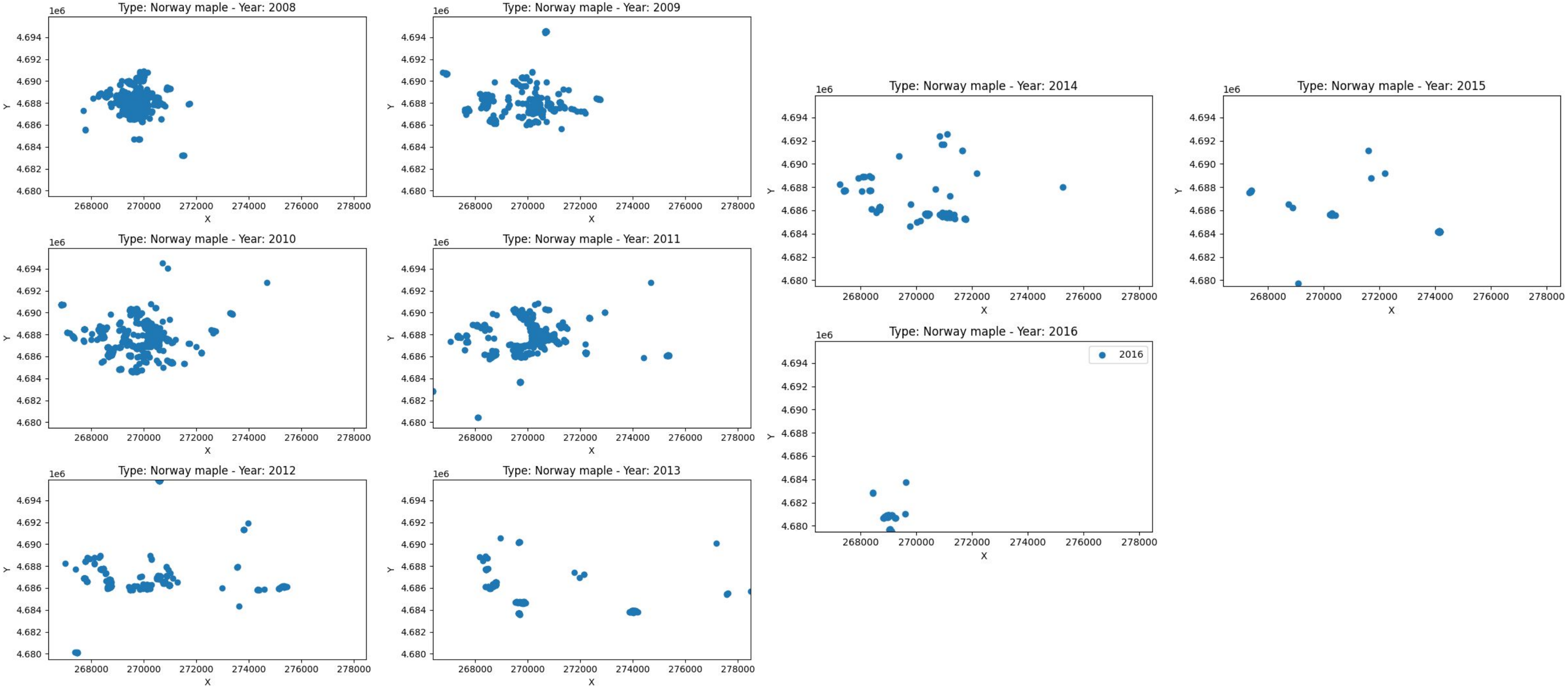
Data analysis methods:

- KDE (Kernel Density Estimation)

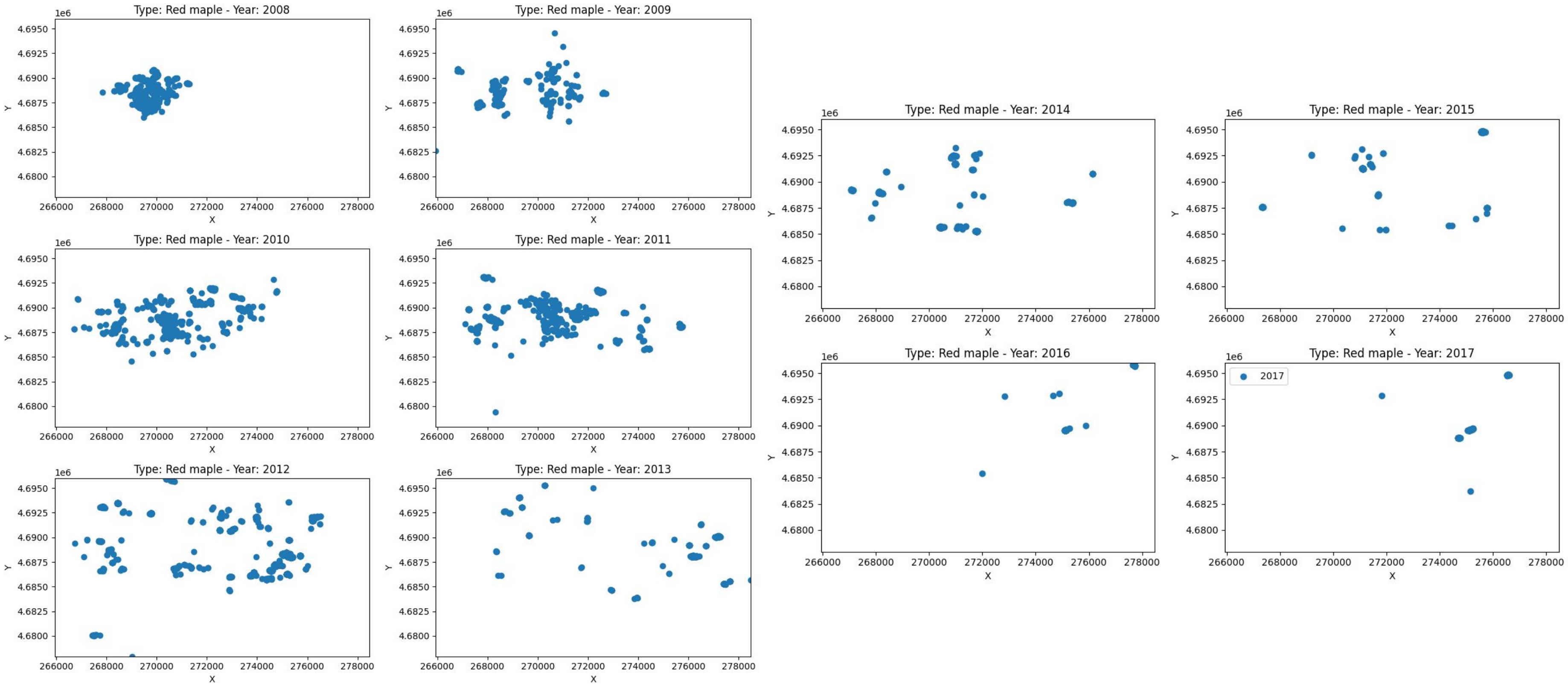
Modeling Methods:

- Bayesian Inference
- Maximum Likelihood Approach (MLE)

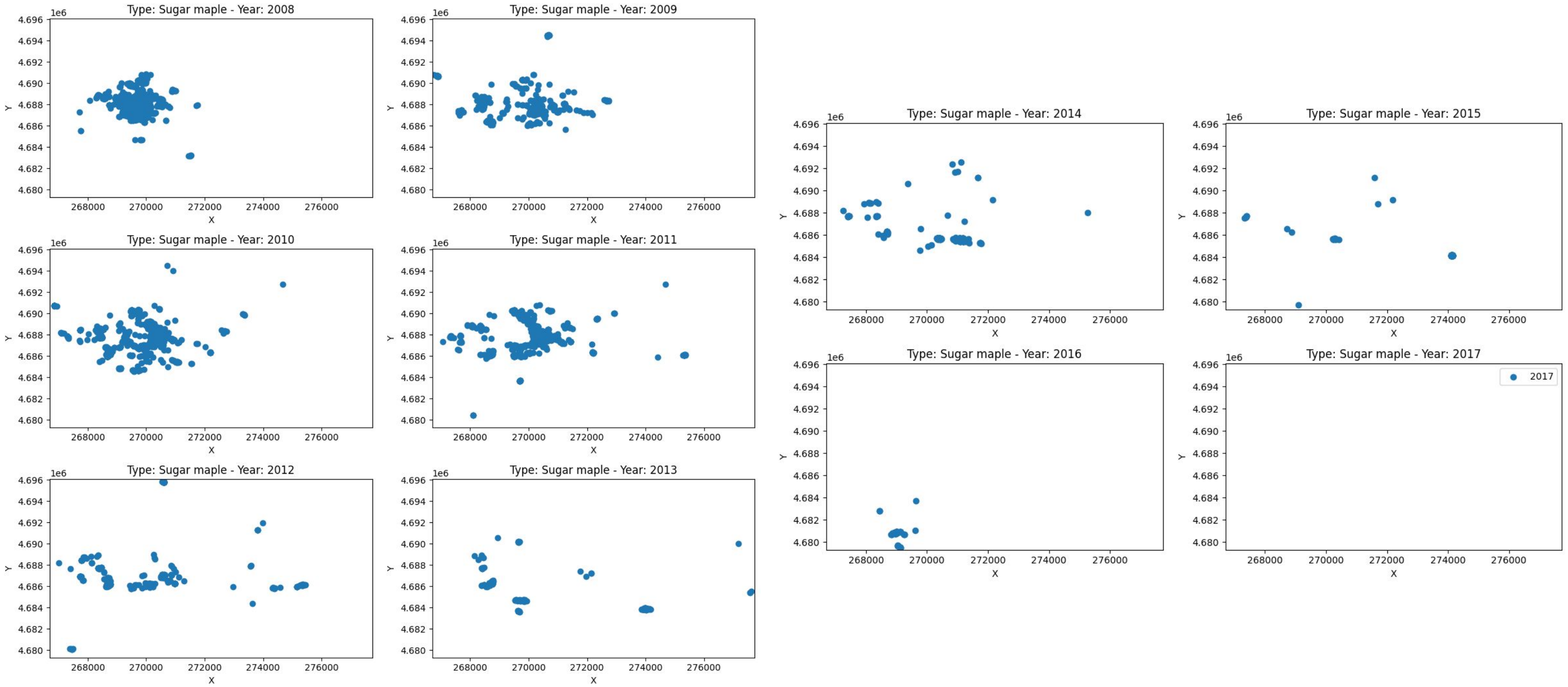
Norway Maple Species by Year



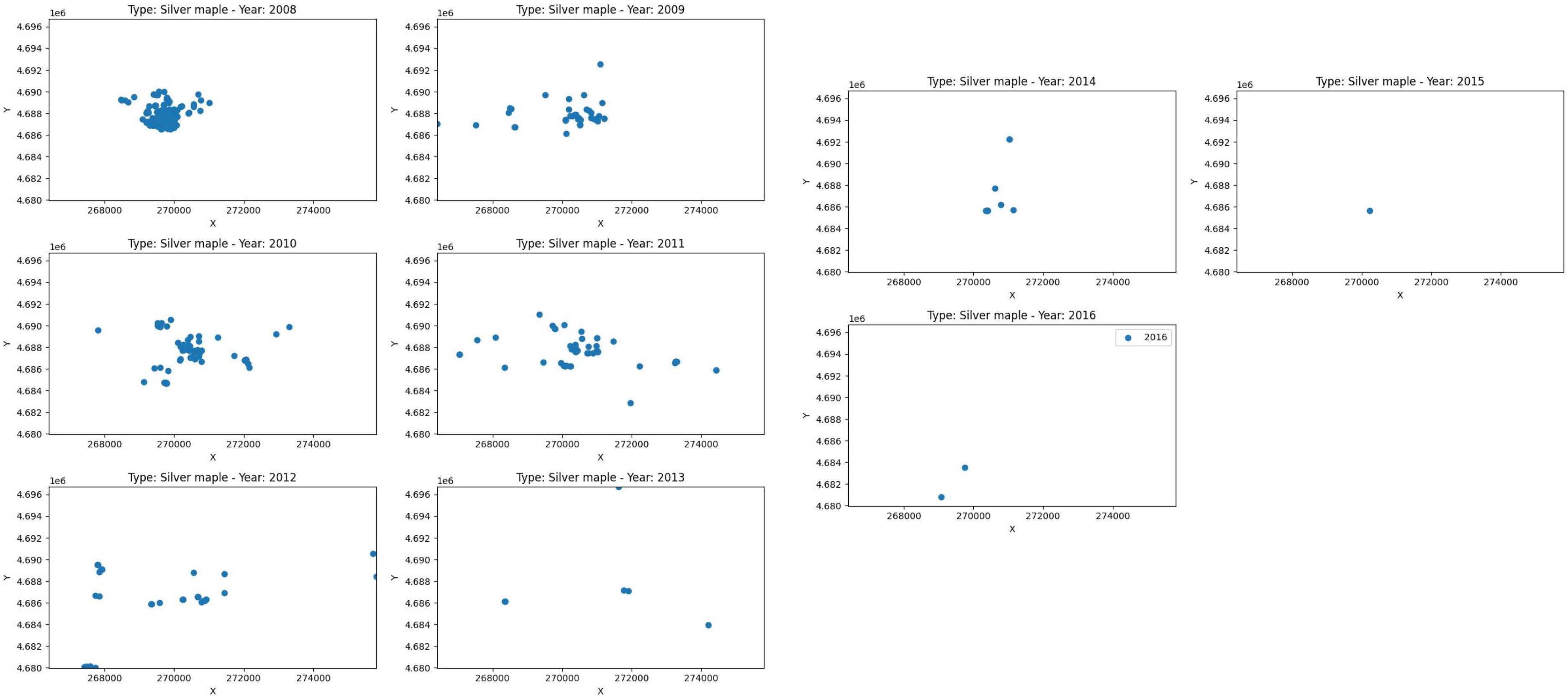
Red Maple Species by Year



Sugar Maple Species by Year



Silver Maple Species by Year



Progress - Data Analysis

Circle data (Norway maple : year 2008 & 2009)

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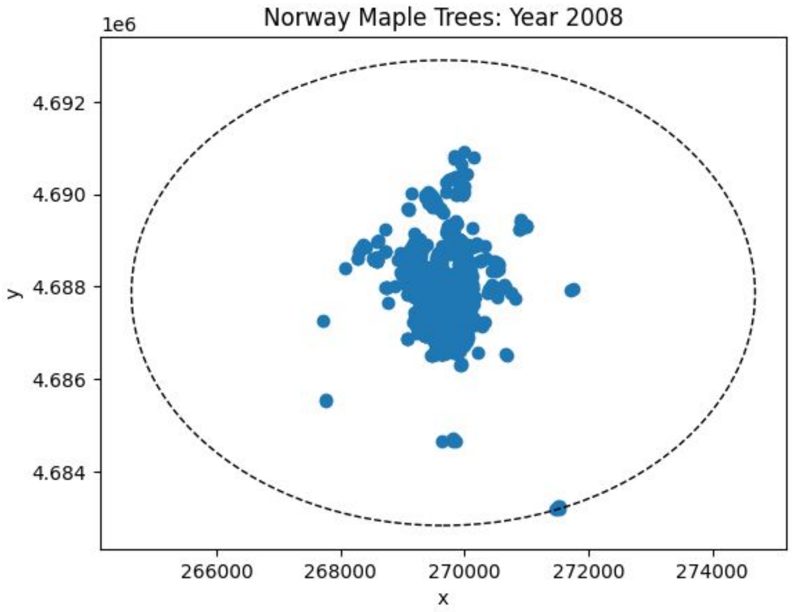

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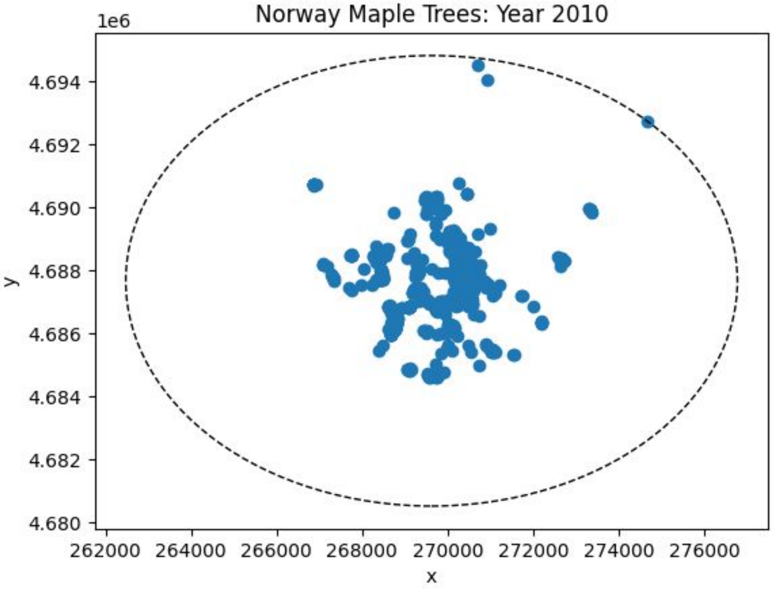

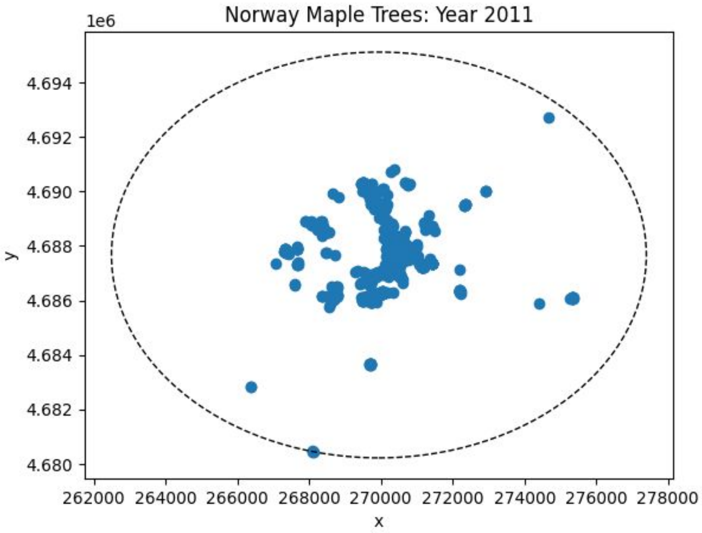

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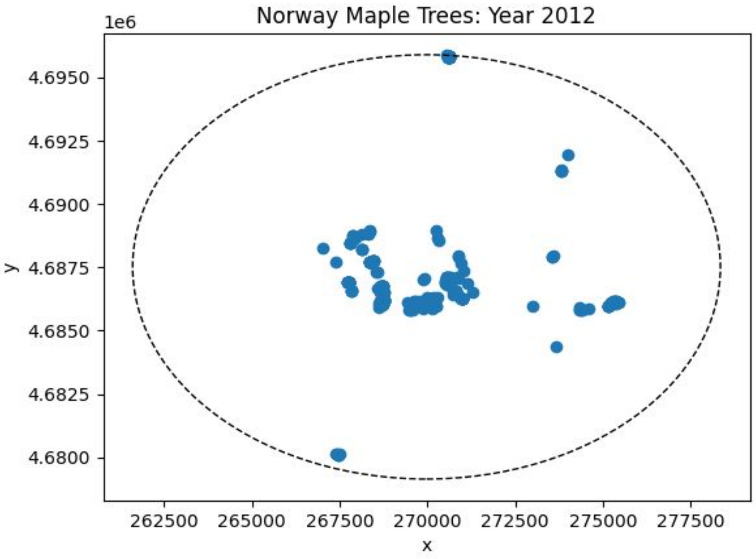

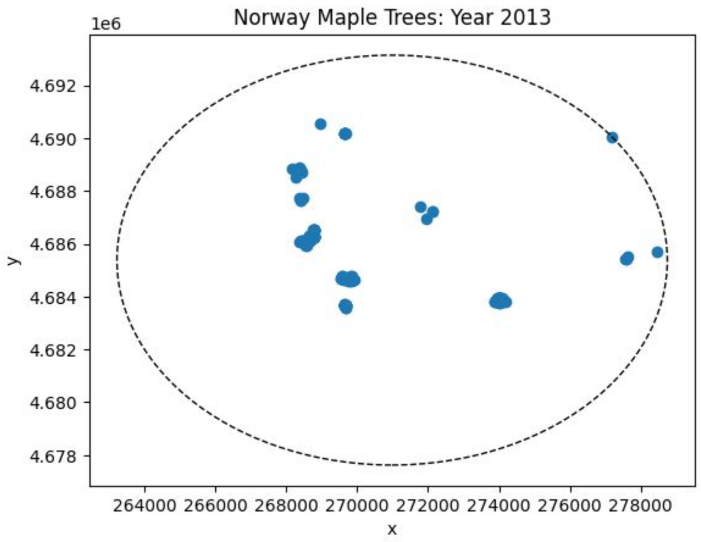

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Circle data (Norway maple: year 2014 & 2015)

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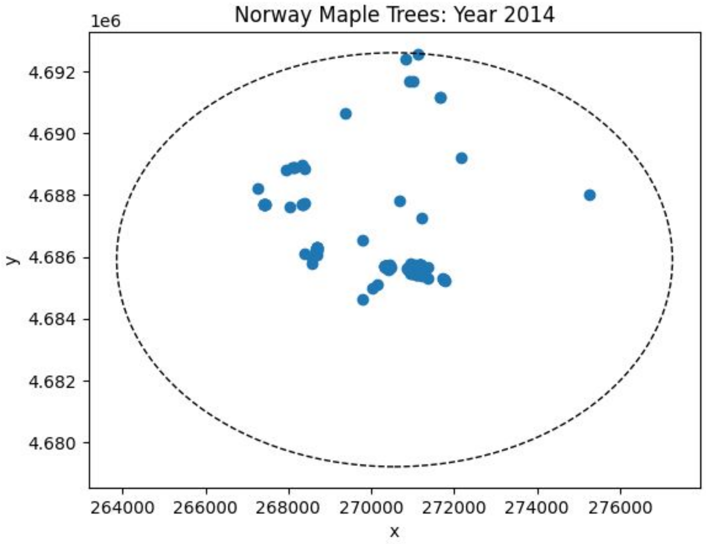

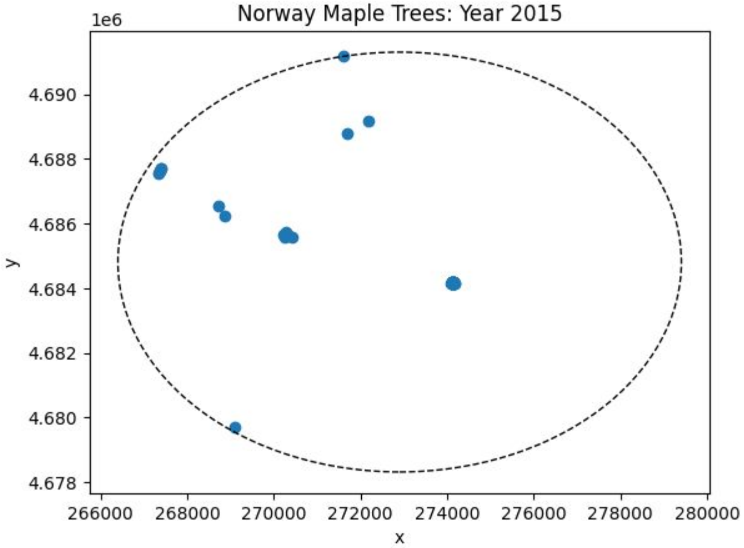

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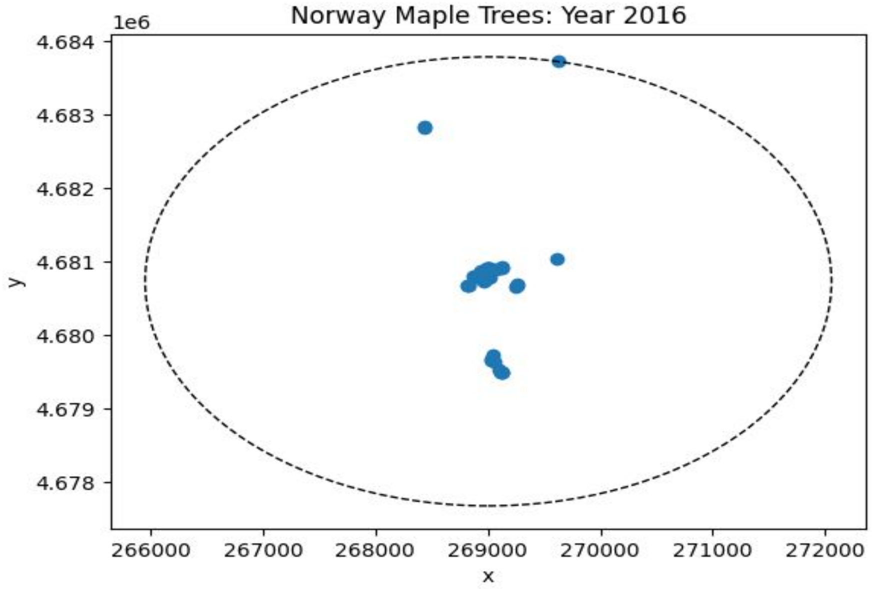
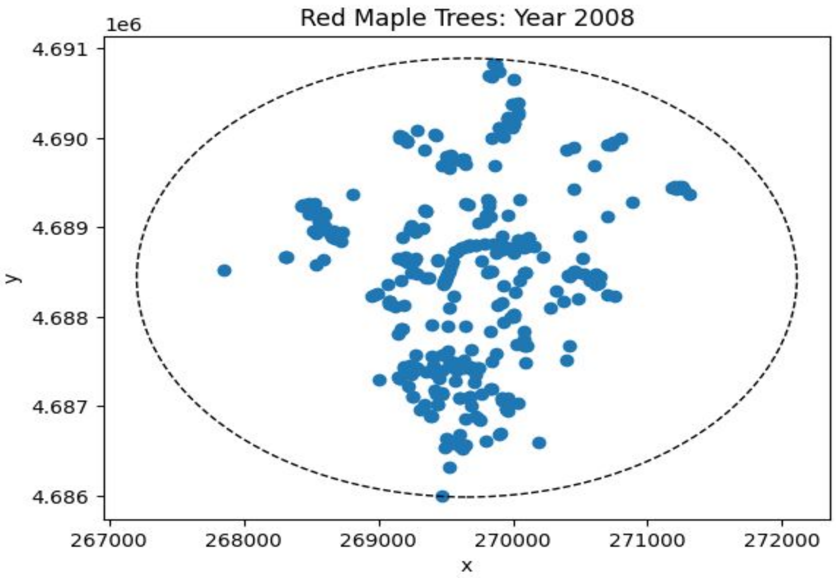
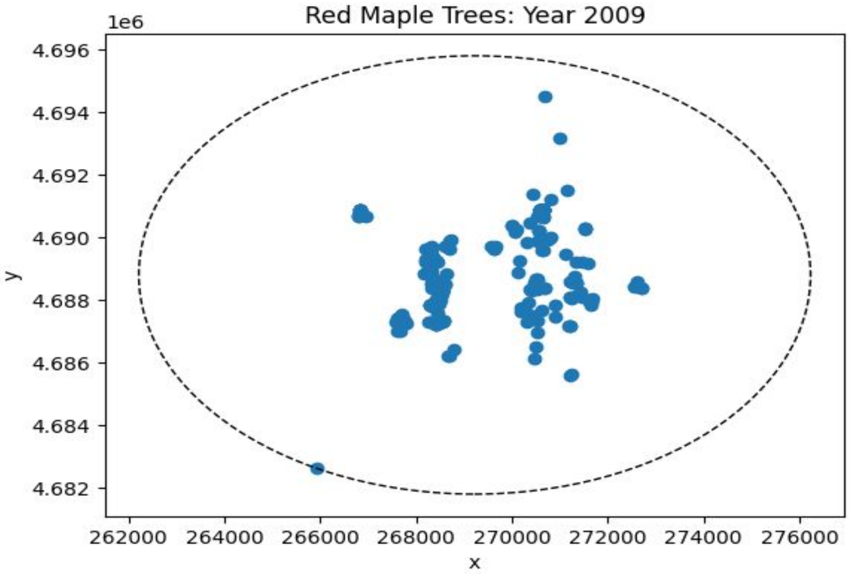


Red Maple Species by Year

Spatial Poisson point process of infected tree species by year





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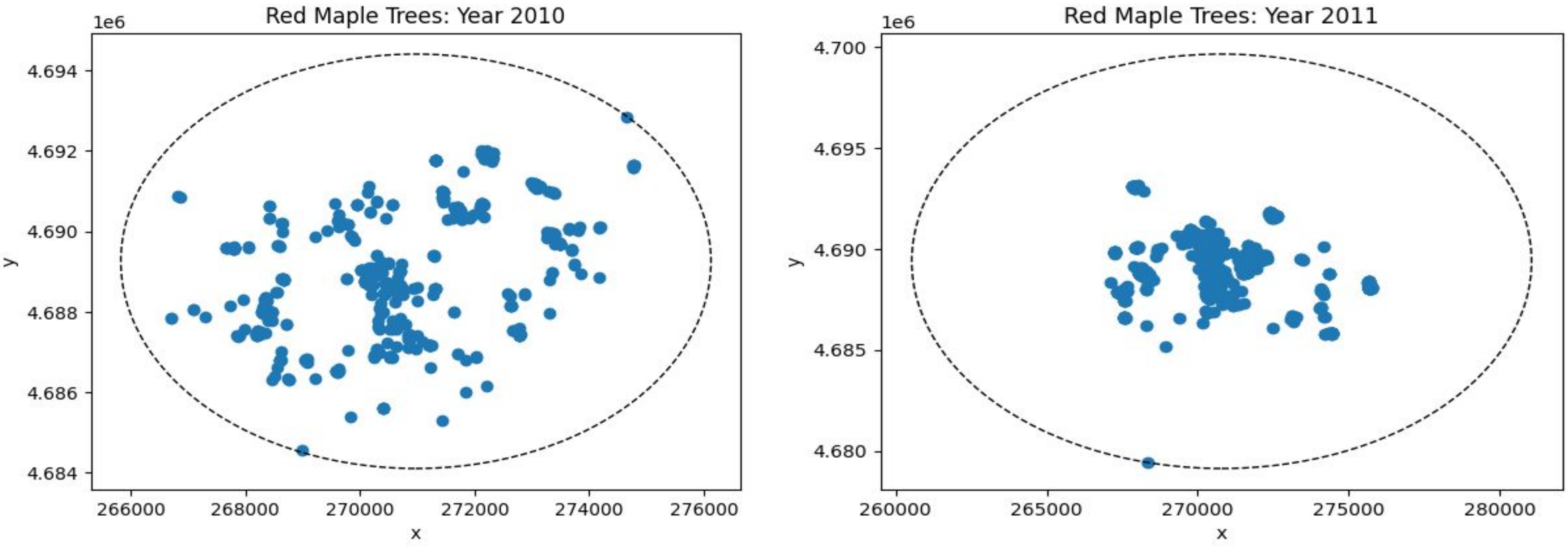

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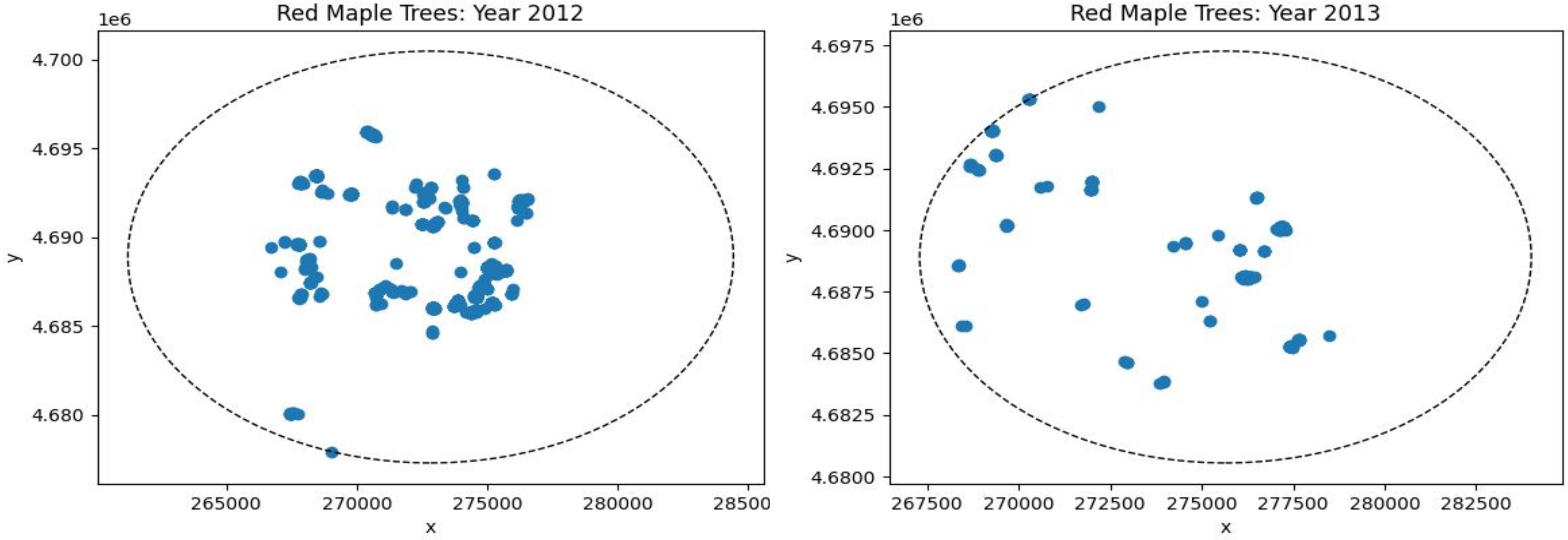

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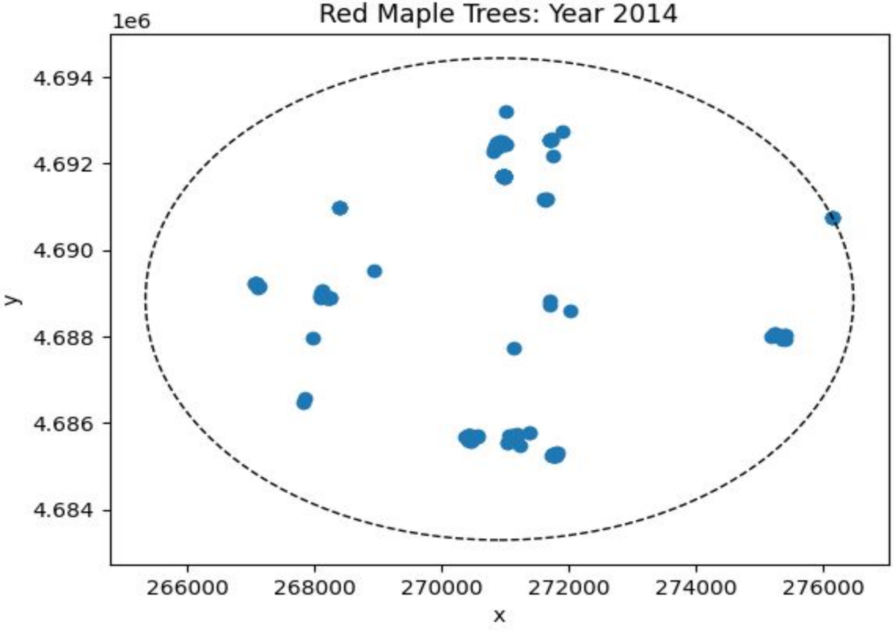

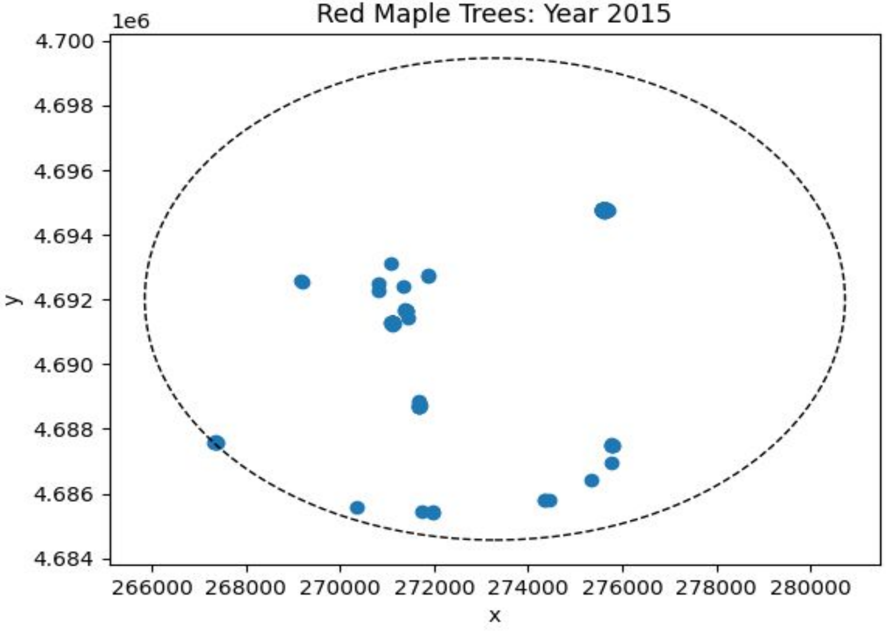

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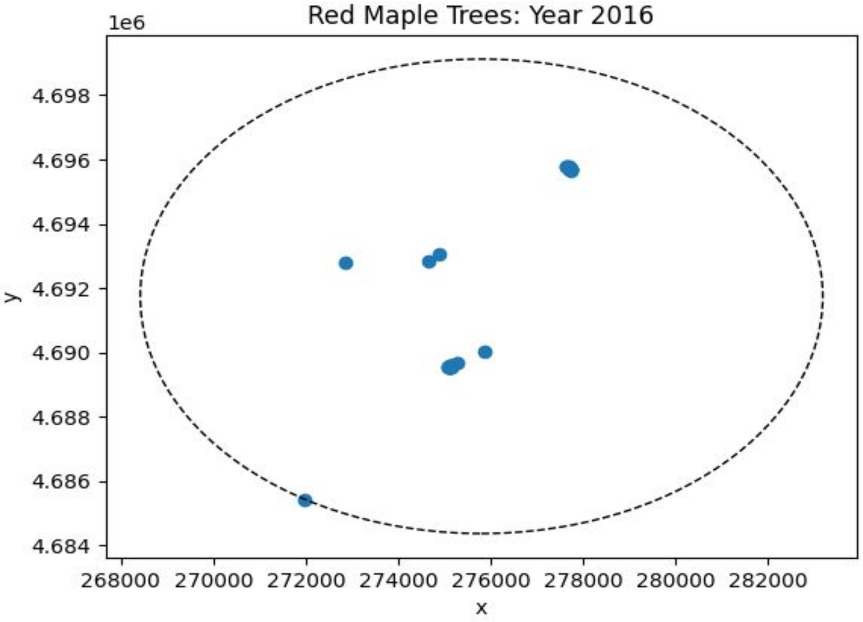

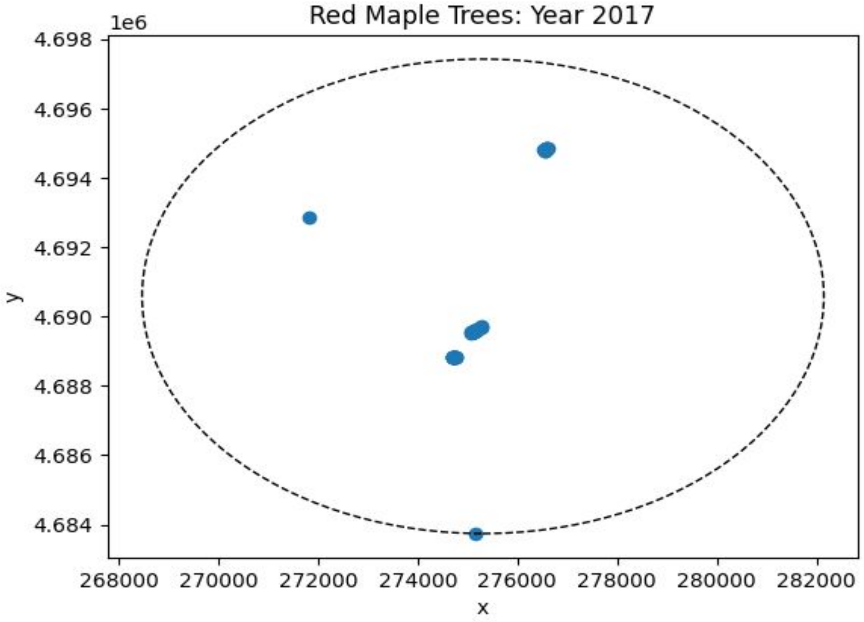

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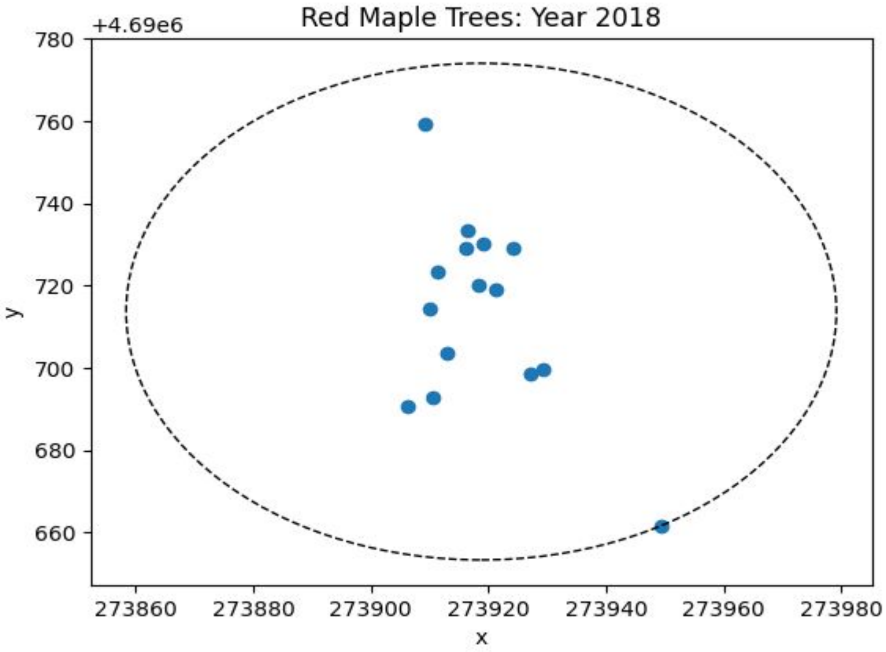

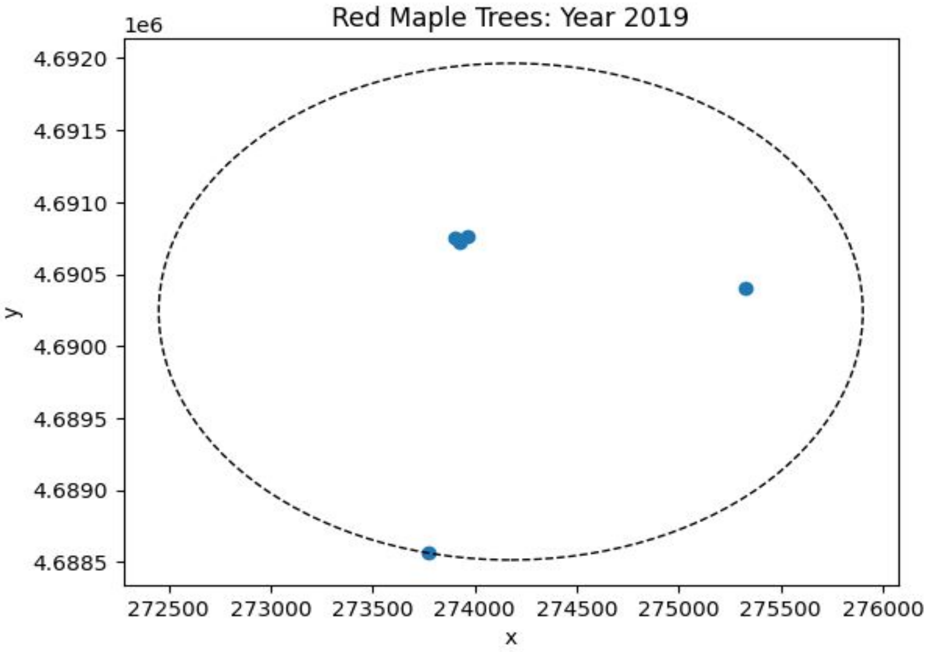

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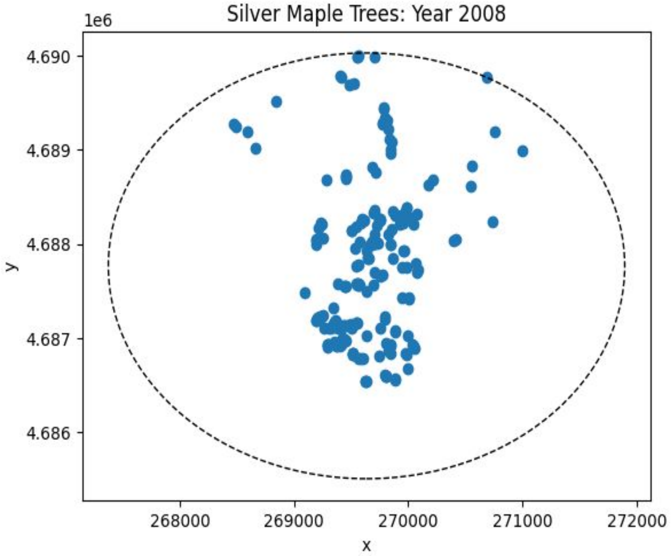
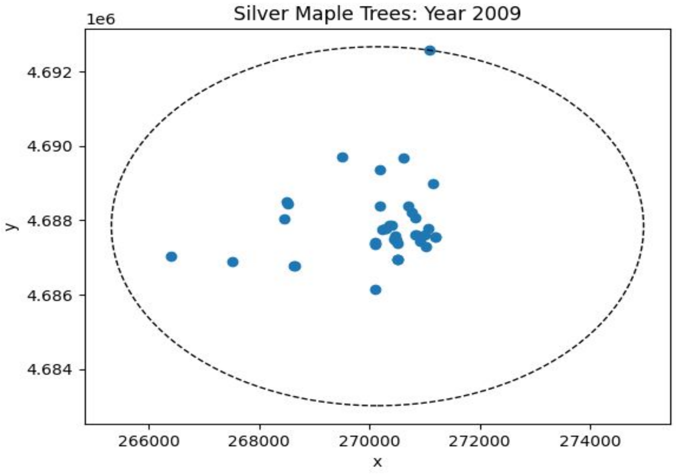
Circle data (Silver maple: year 2008 & 2009)





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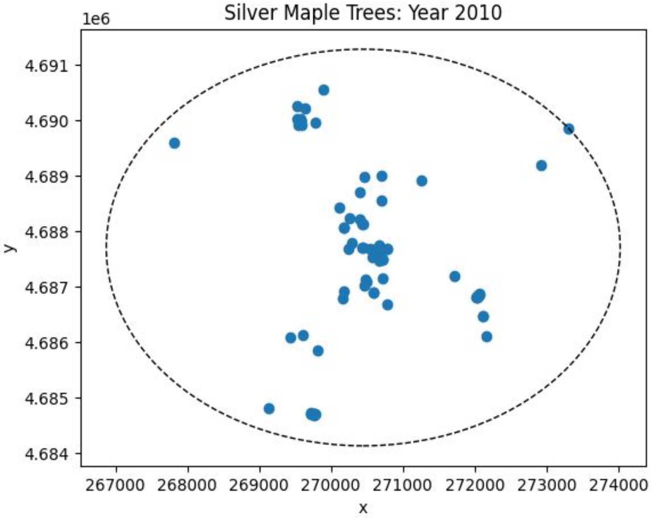

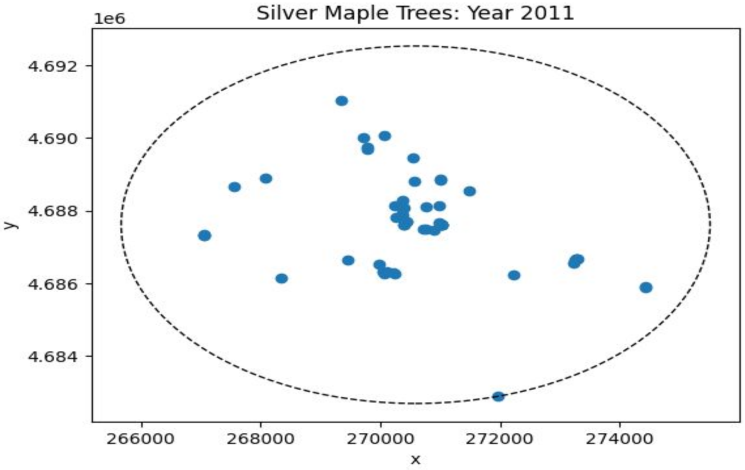

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Circle data (Silver maple: year 2012 & 2013)

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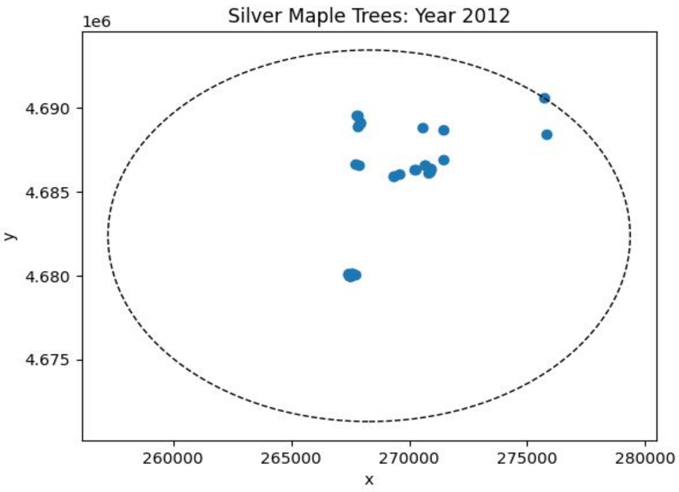

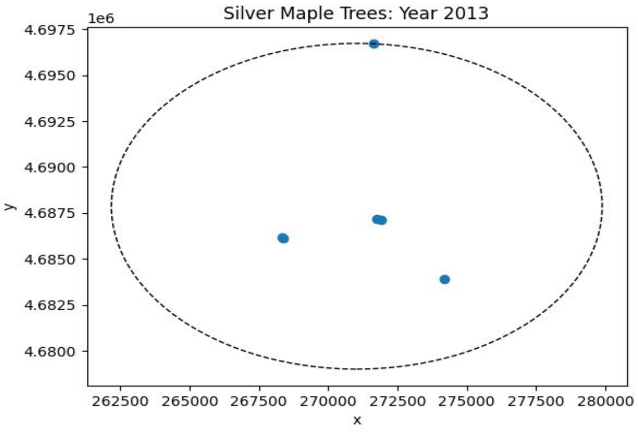

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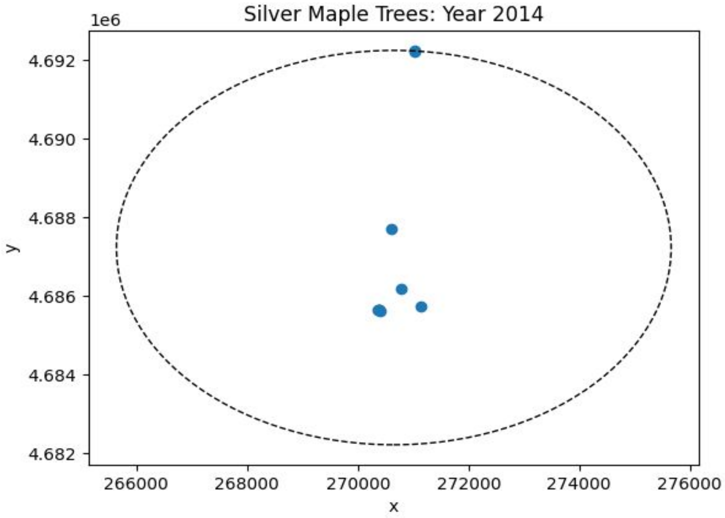

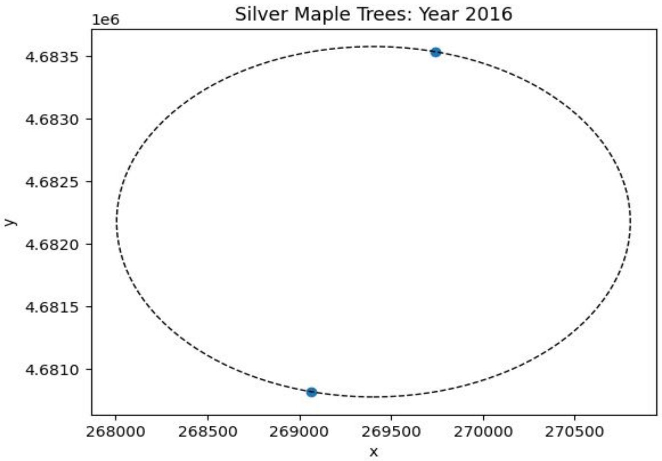

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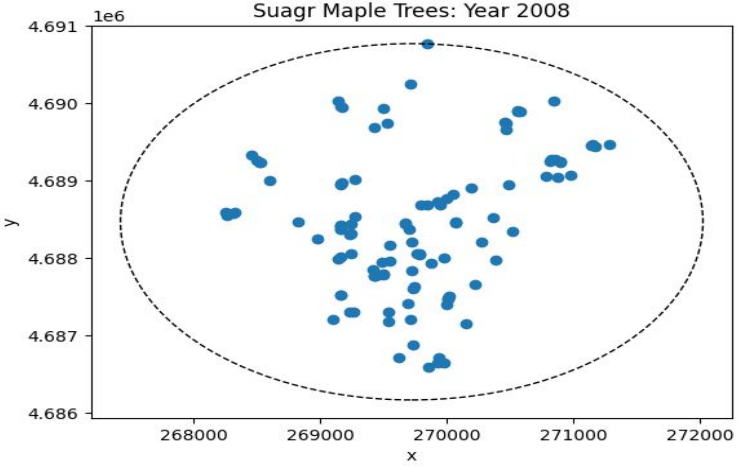

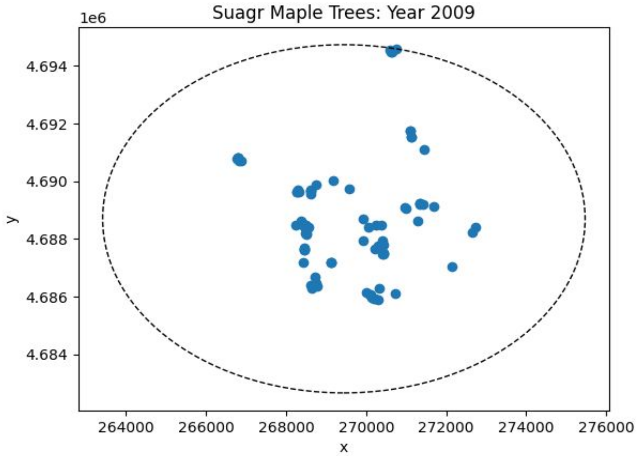

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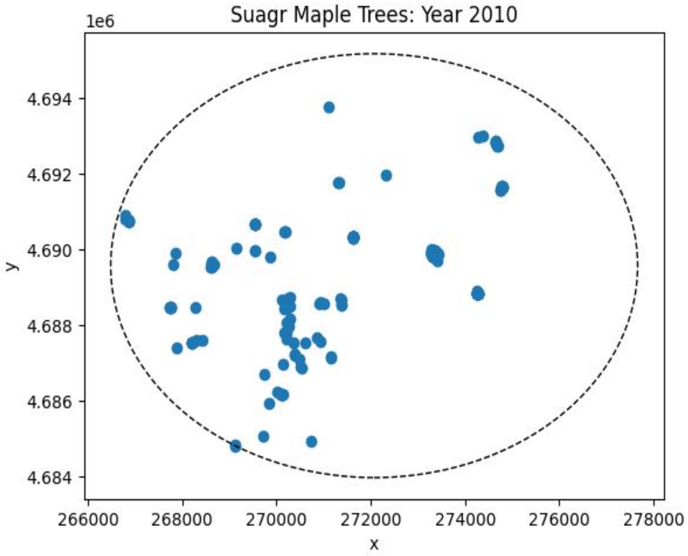

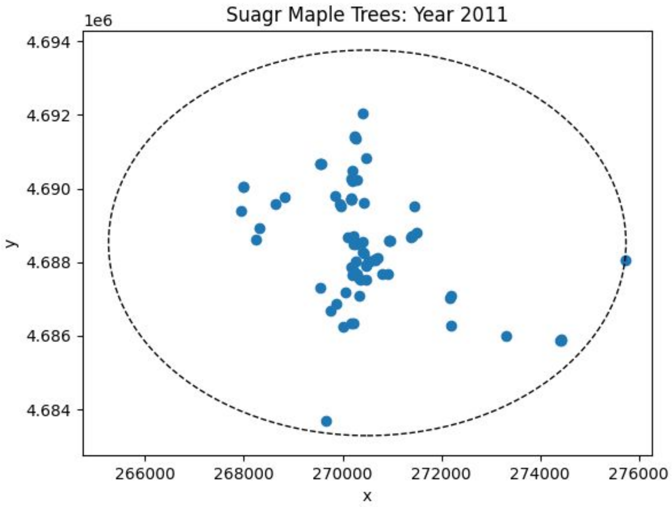

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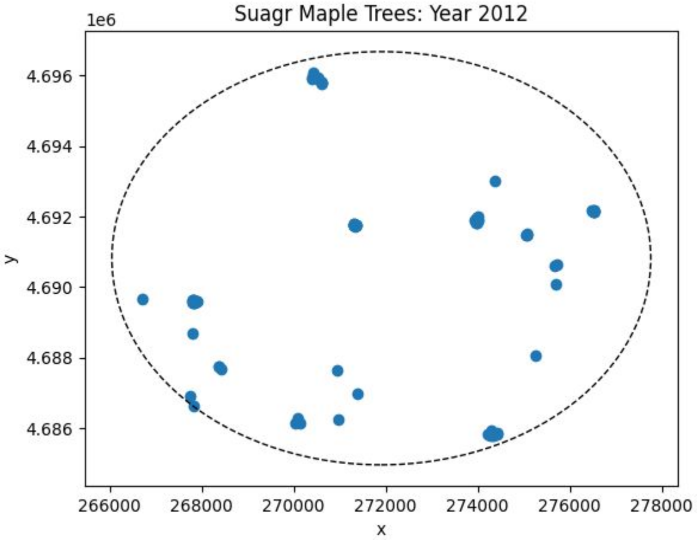

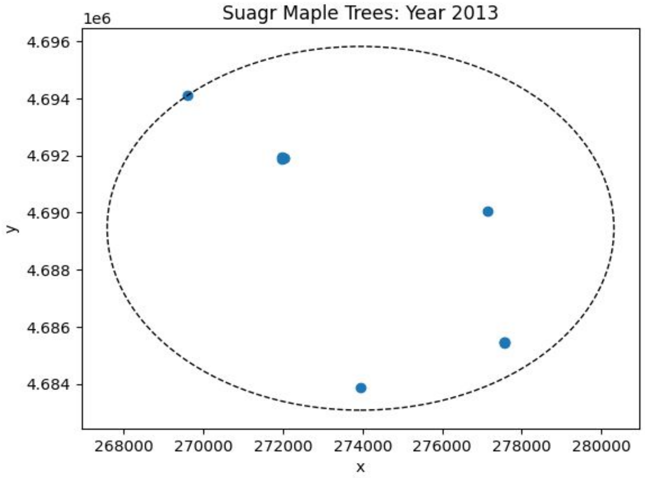

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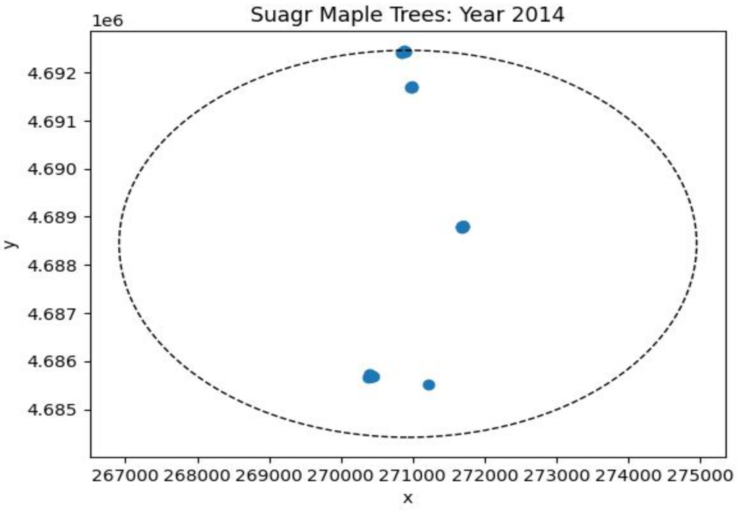
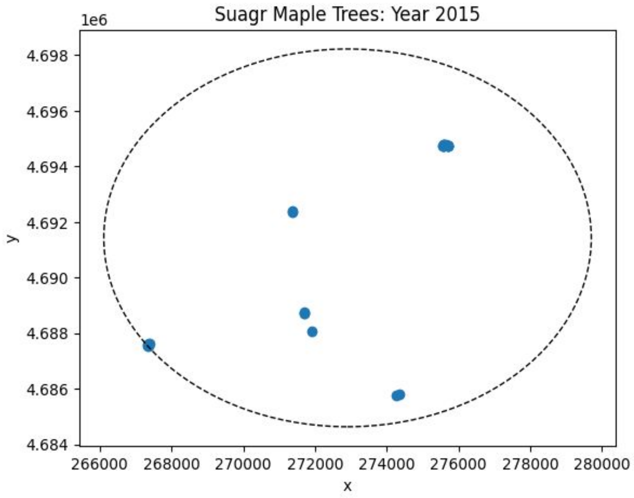
Circle data (Sugar maple: year 2014 & 2015)





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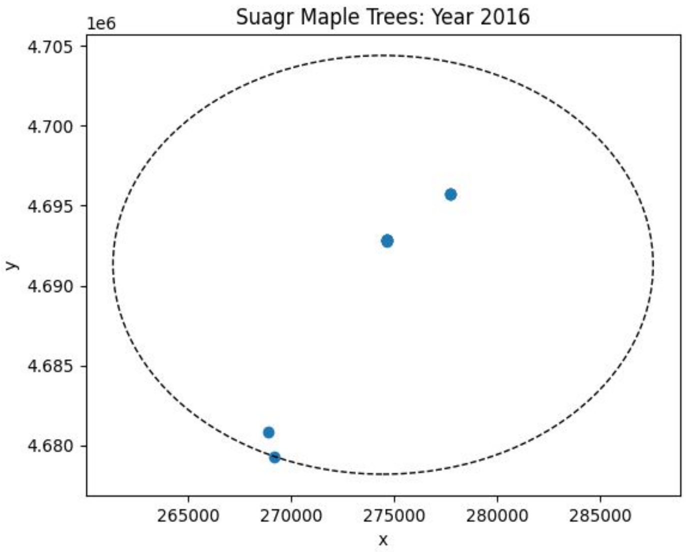

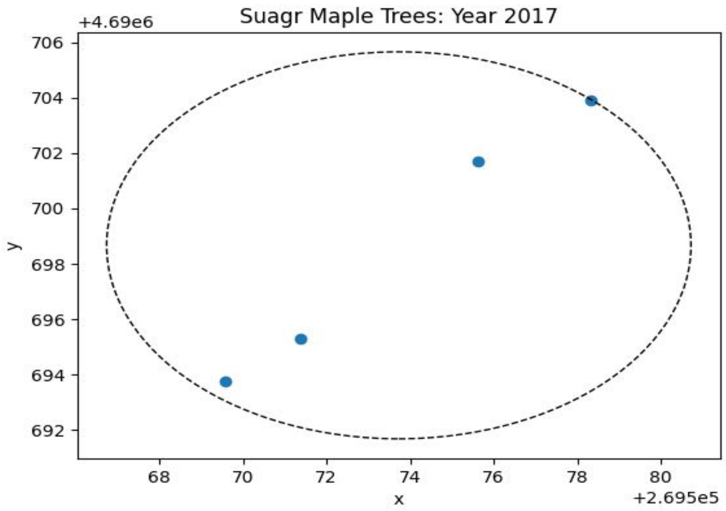

How is the data distributed?

Kernel Density Estimations (KDE)

- We can approximate a 1-D or 2-D distribution as a Gaussian distribution.
- To do this, take a histogram of the data and fit a normal curve to each bin.
- Add all curves together to get our approximate (Gaussian) distribution.

How is the data distributed?

Kernel Density Estimations (KDE)

- If $(x_1, \ldots, x_n)$ are i.i.d. Samples from our distribution, we approximate the probability density function (PDF) via

$$\hat{p}_h(x) = \frac{1}{nh} \sum_{i=1}^{n} K\left(\frac{x - x_i}{h}\right)$$

- The parameter $h > 0$ is the *bandwidth* of the estimation.
- The function *K()* is the *kernel function*. We use the Gaussian probability density function Normal(0, 1).

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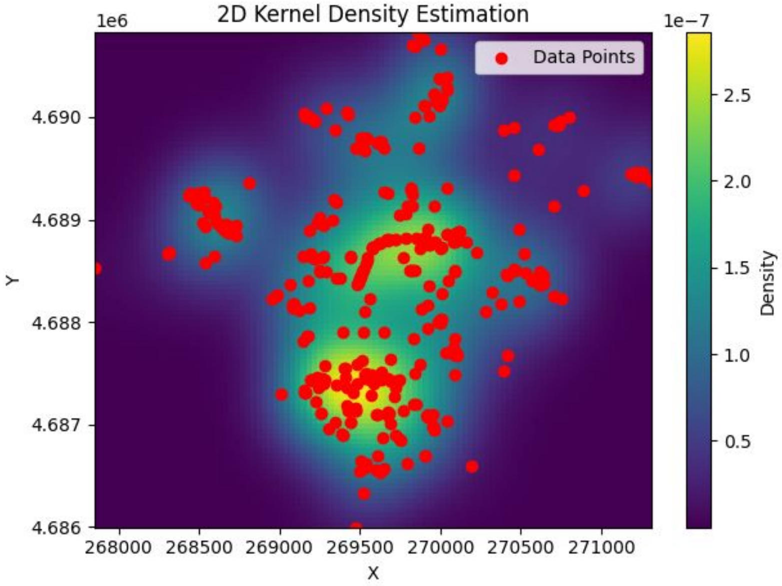
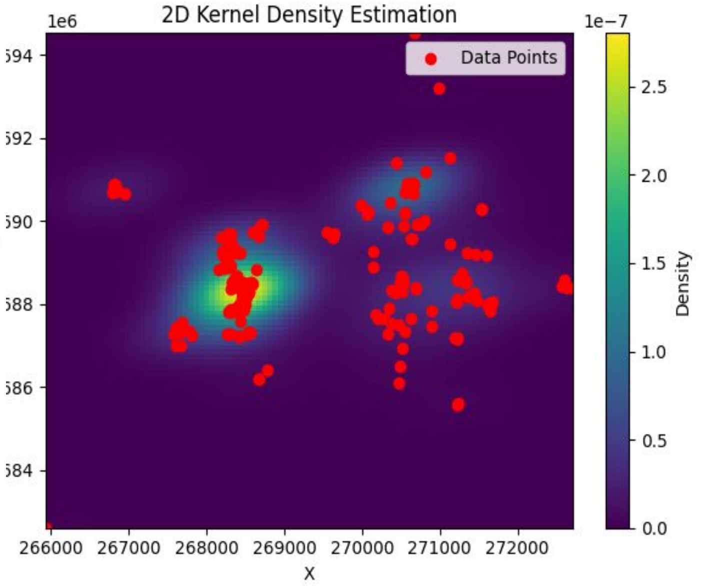
2D Kernel Density Estimation

Red Maple Species: 2008 & 2009





2D Kernel Density Estimation

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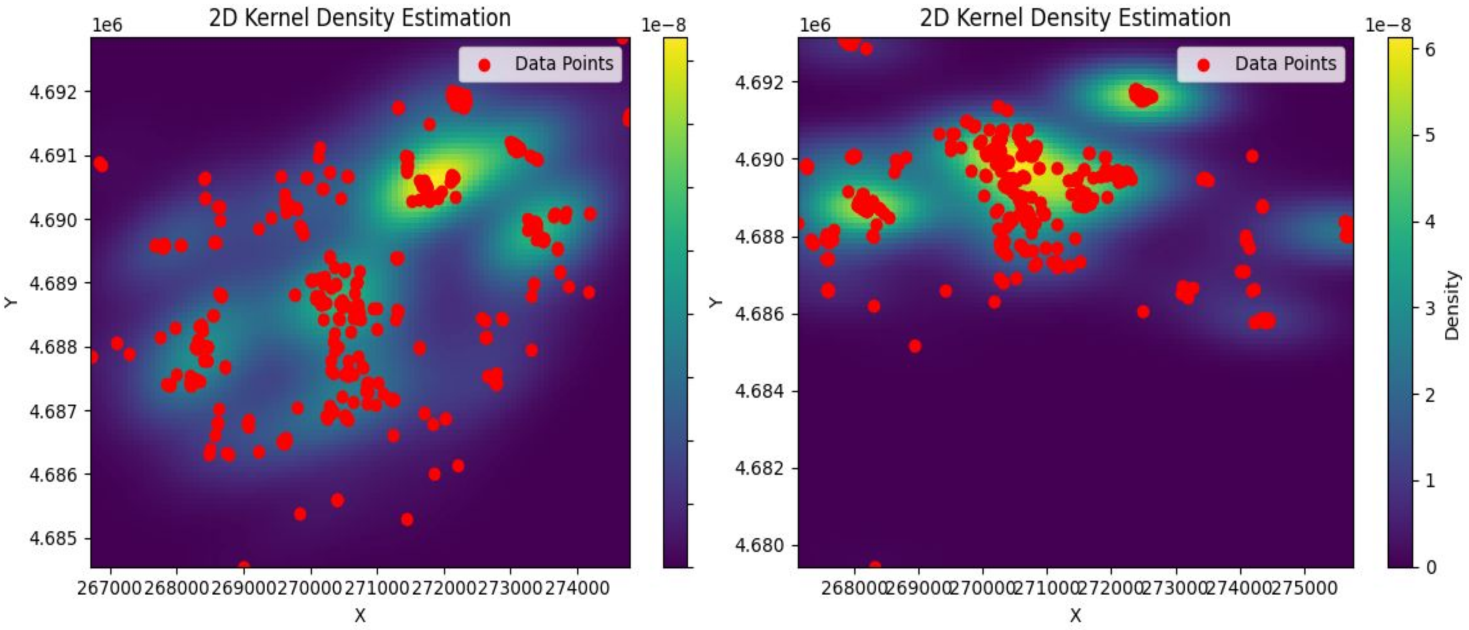
Red Maple Species: 2010 & 2011



2D Kernel Density Estimation

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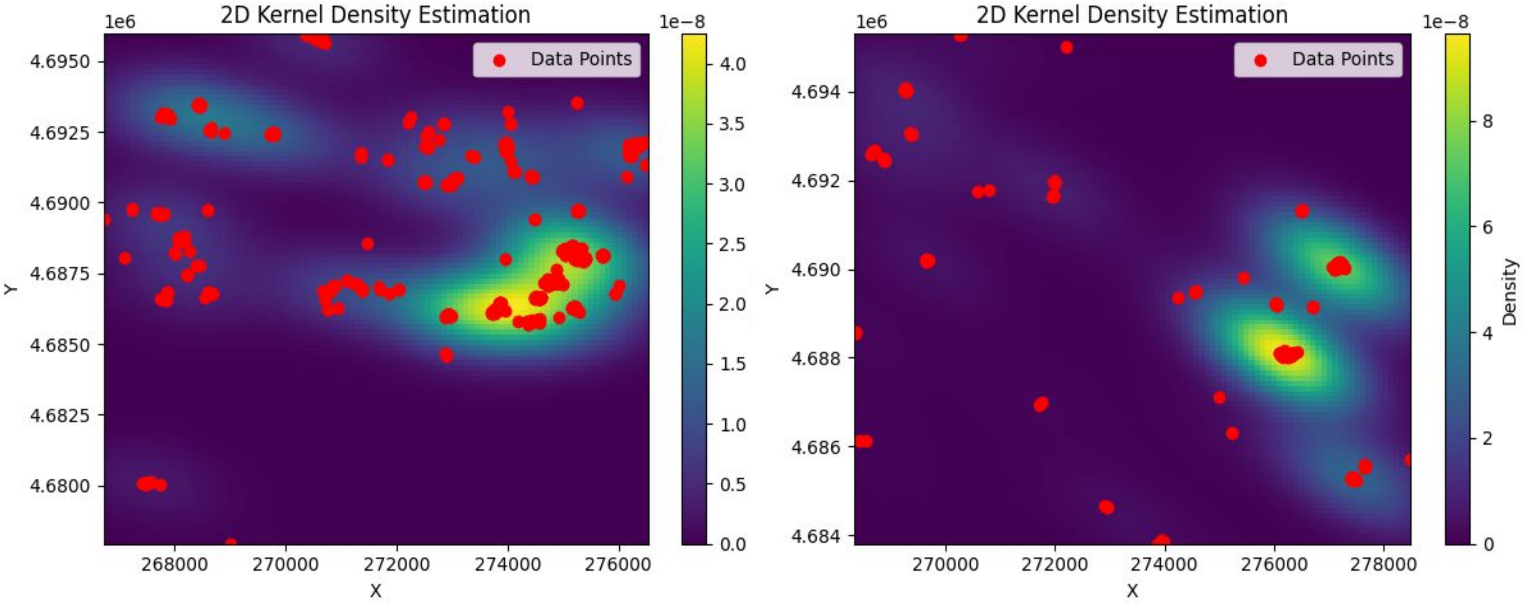
Red Maple Species: 2012 & 2013



2D Kernel Density Estimation

Red Maple Species: 2014 & 2015

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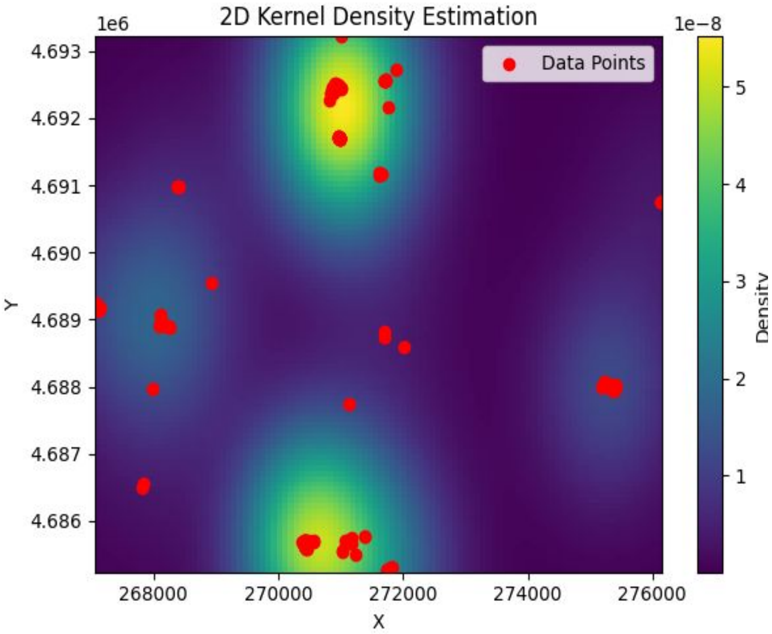

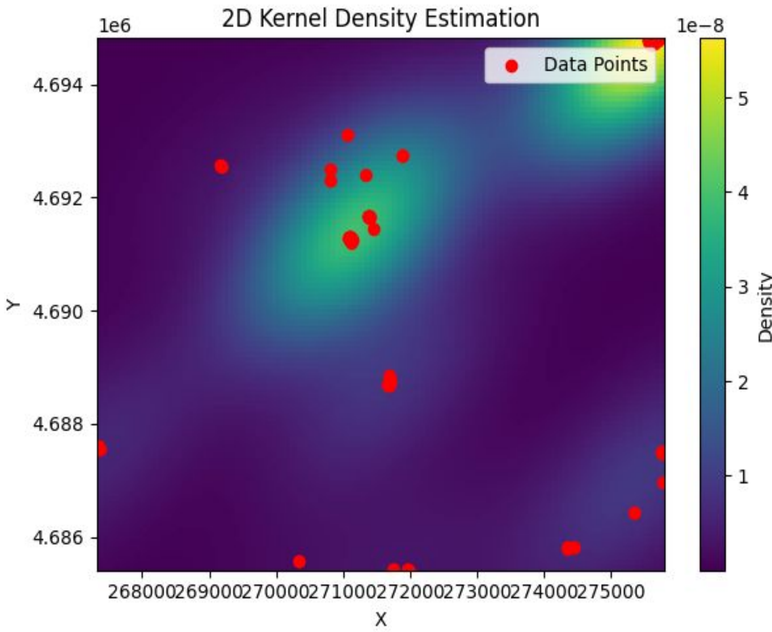

2D Kernel Density Estimation

Red Maple Species: 2016 & 2017

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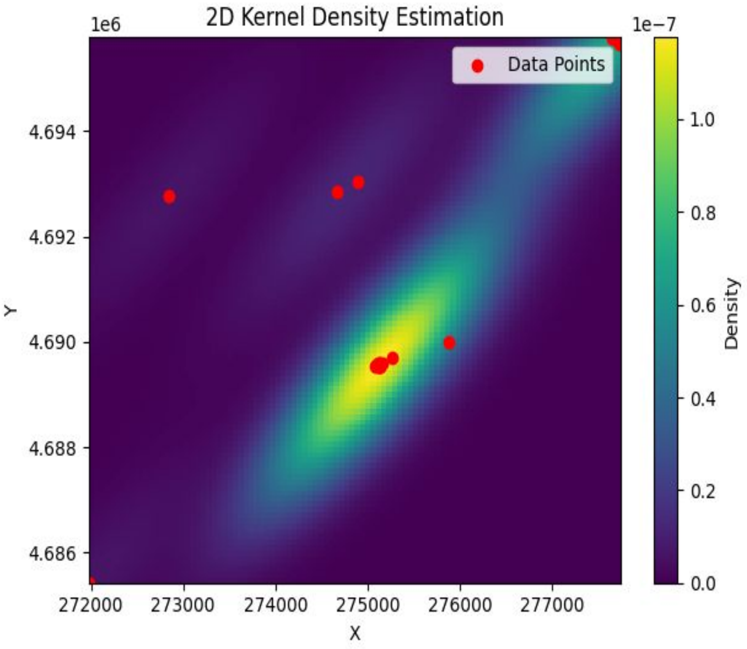

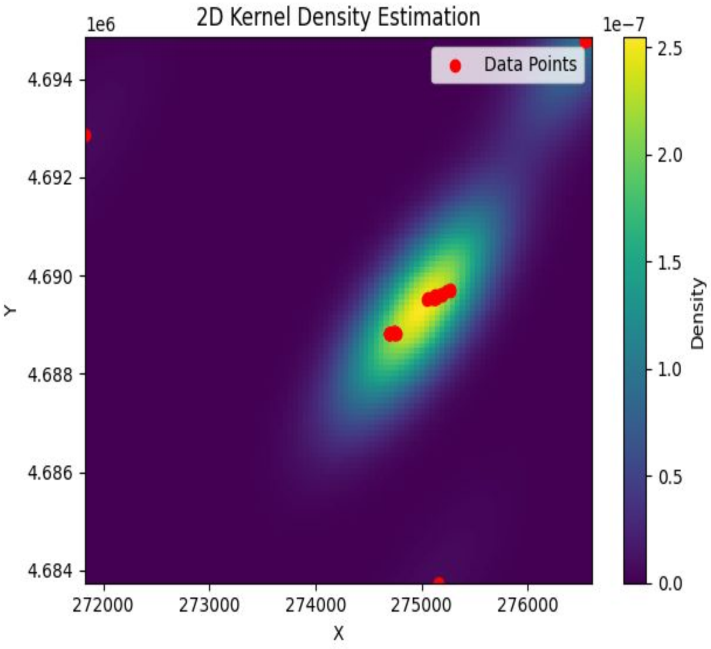

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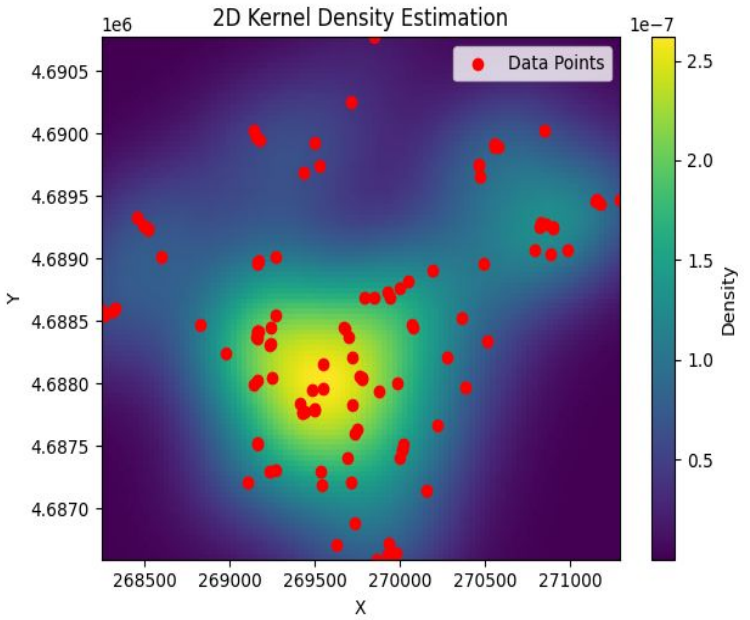
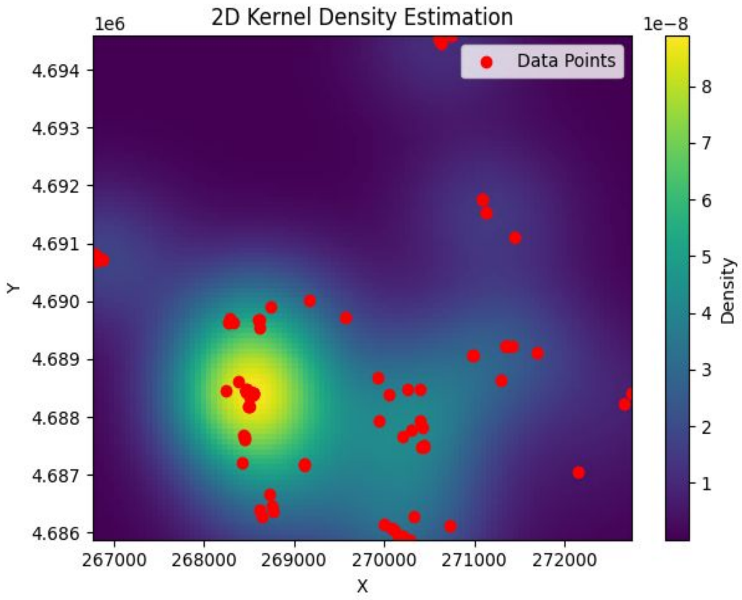
2D KDE (Sugar maple: 2008 & 2009 data)





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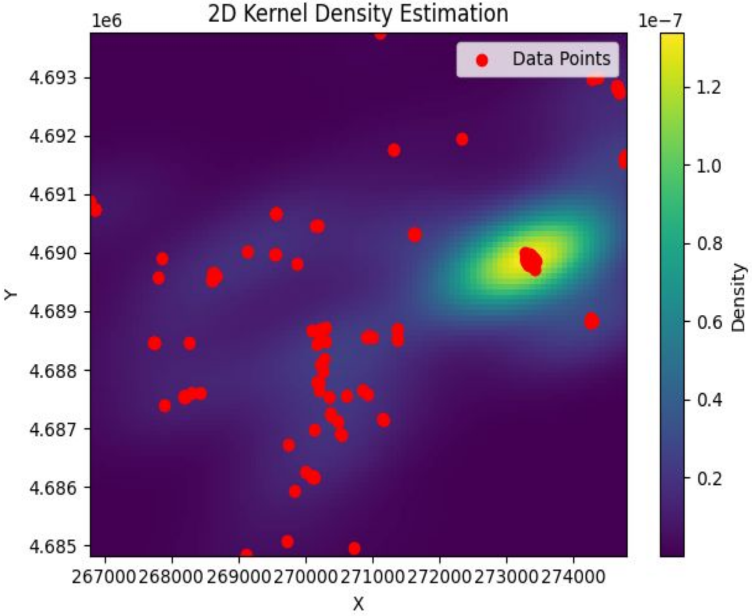

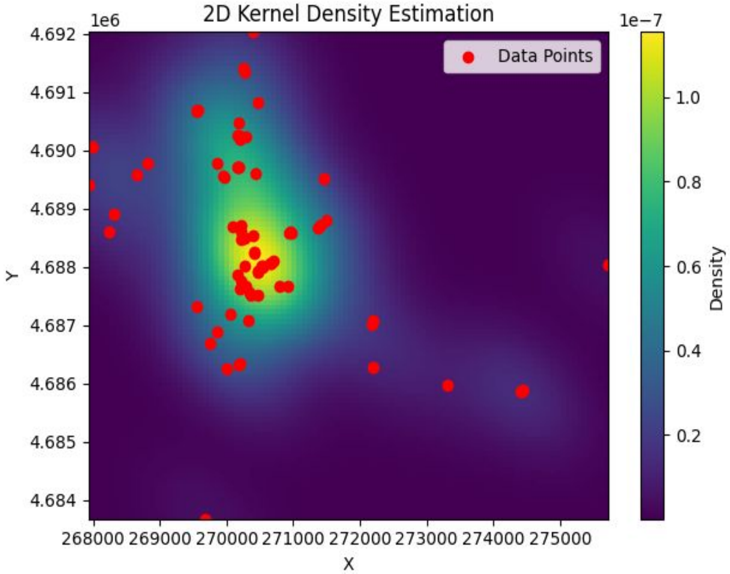

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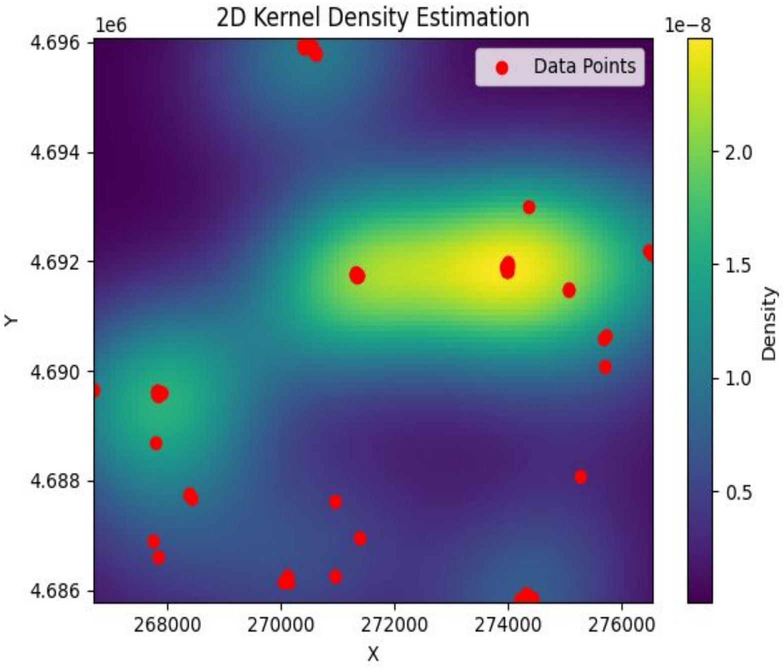

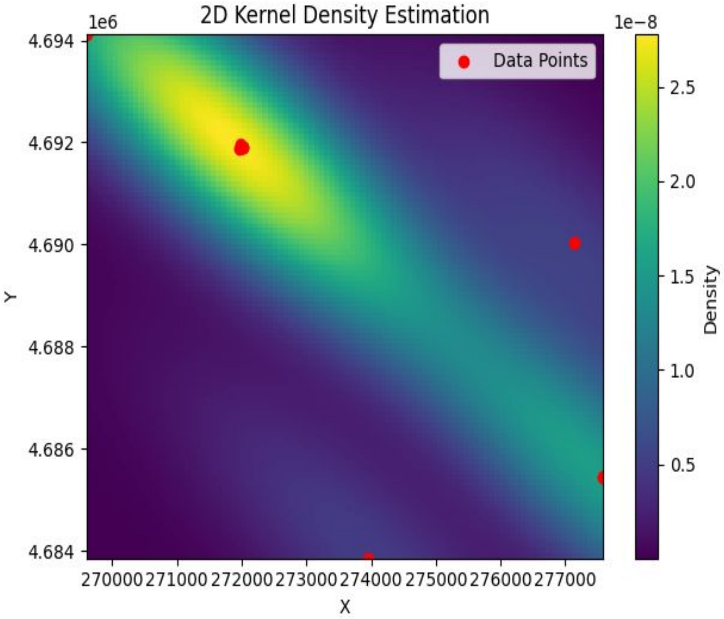

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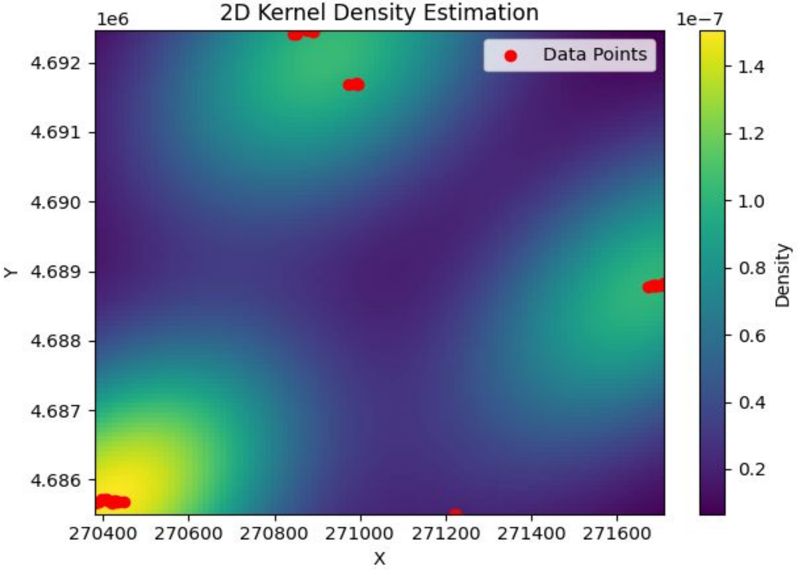
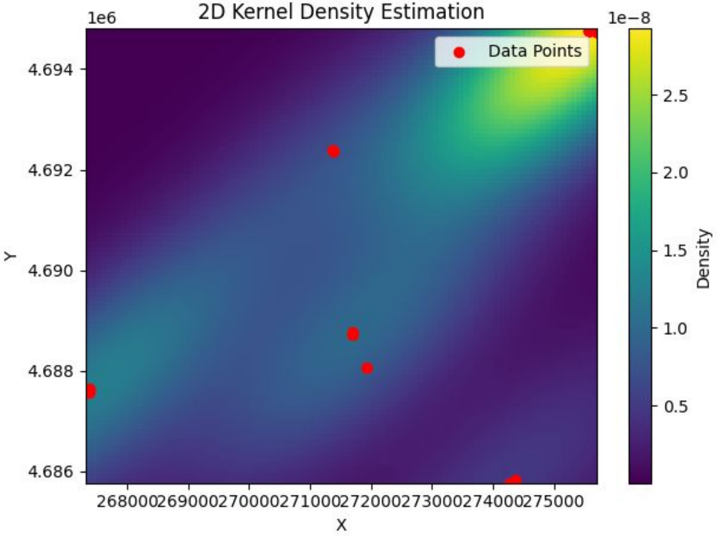




Progress - Data Analysis

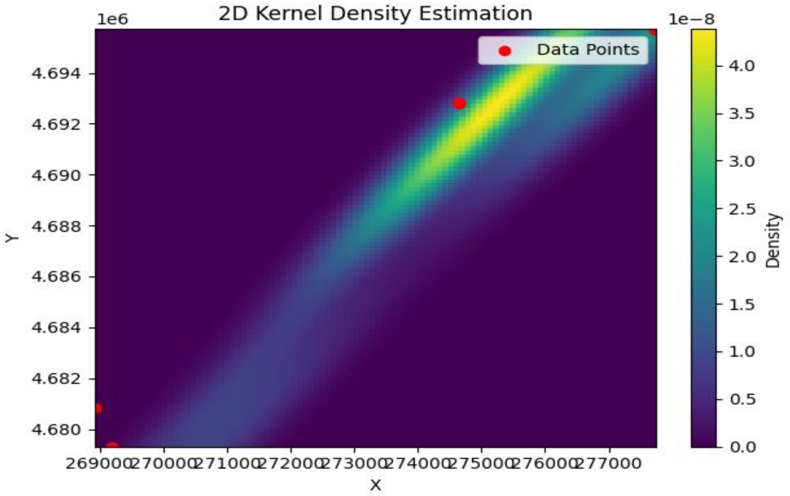
Overview

Motivation

Progress
- **Data Analysis**
- Model Construction

Next Steps

2D KDE (Sugar maple: 2016 data)



Progress - Data Analysis

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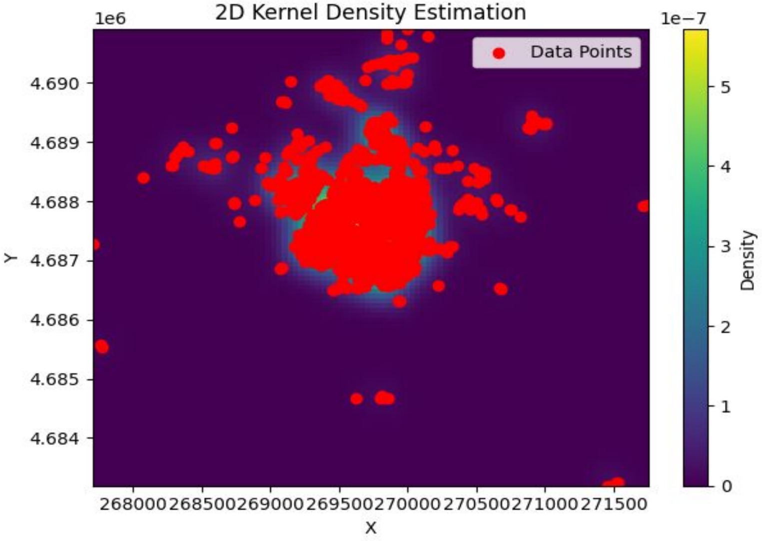
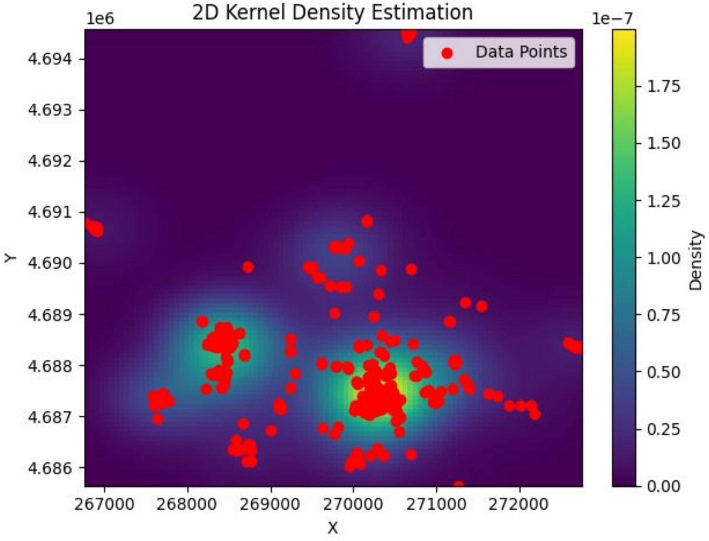
2D KDE (Norway maple: 2008 & 2009 data)





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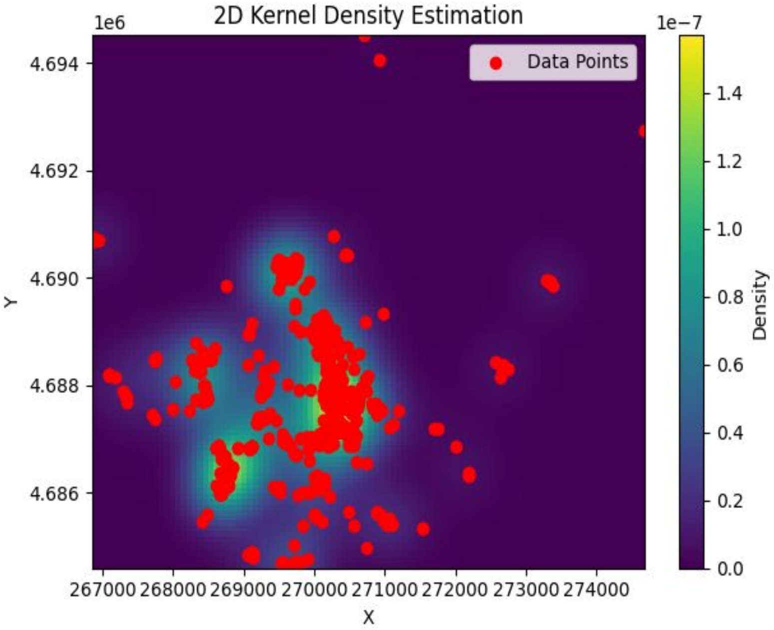
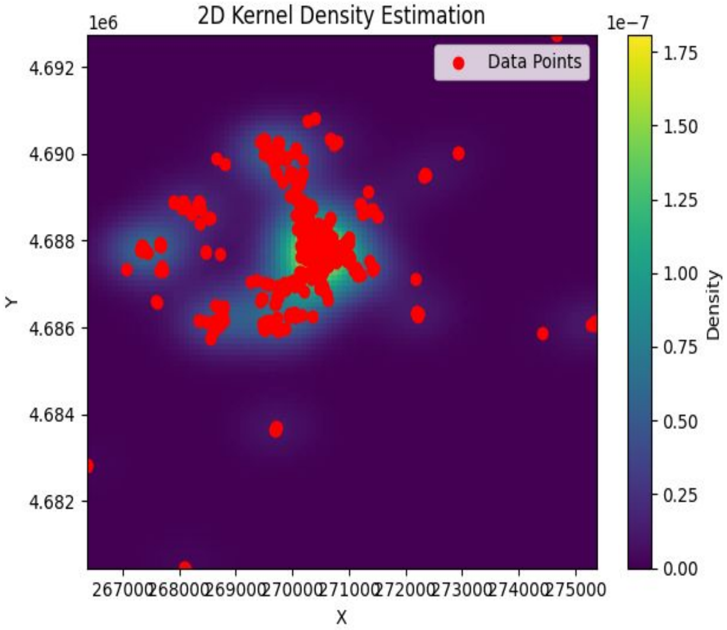
2D KDE (Norway maple: 2010 & 2011 data)





Progress - Data Analysis

2D KDE (Norway maple: 2012 & 2013 data)

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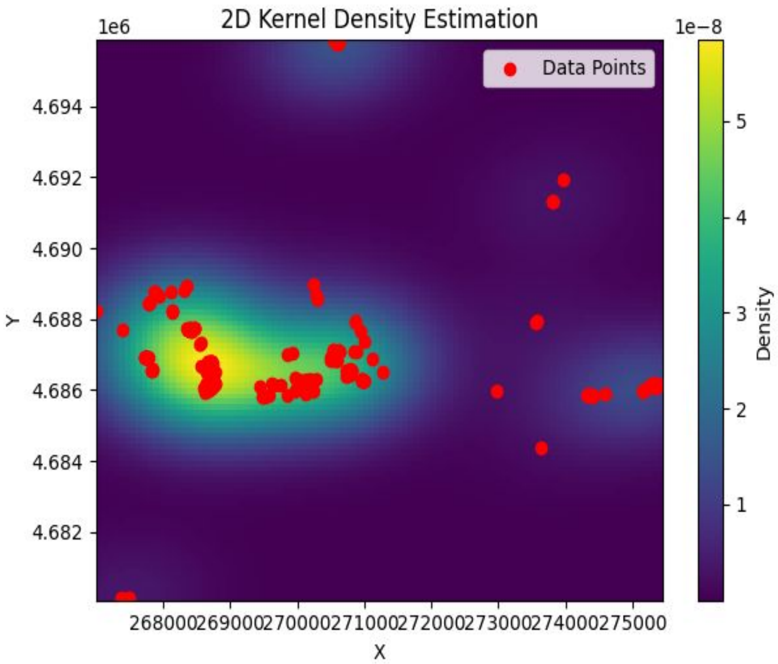

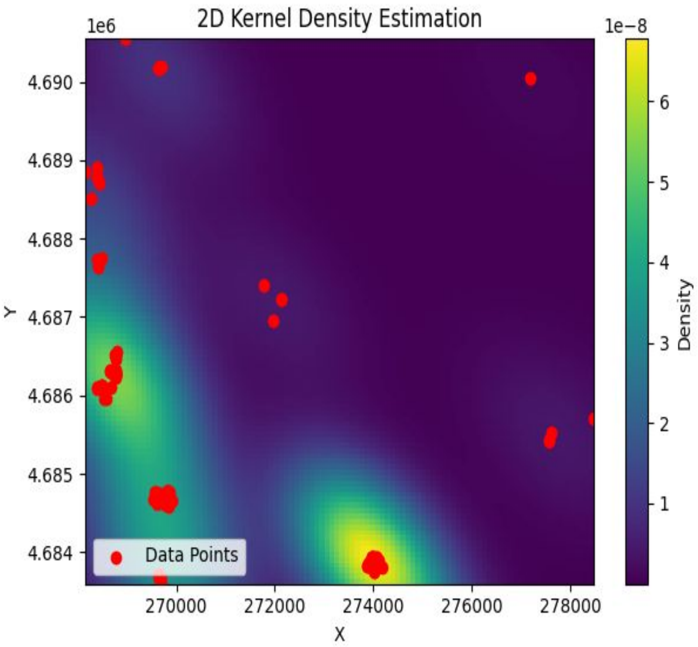

Progress - Data Analysis

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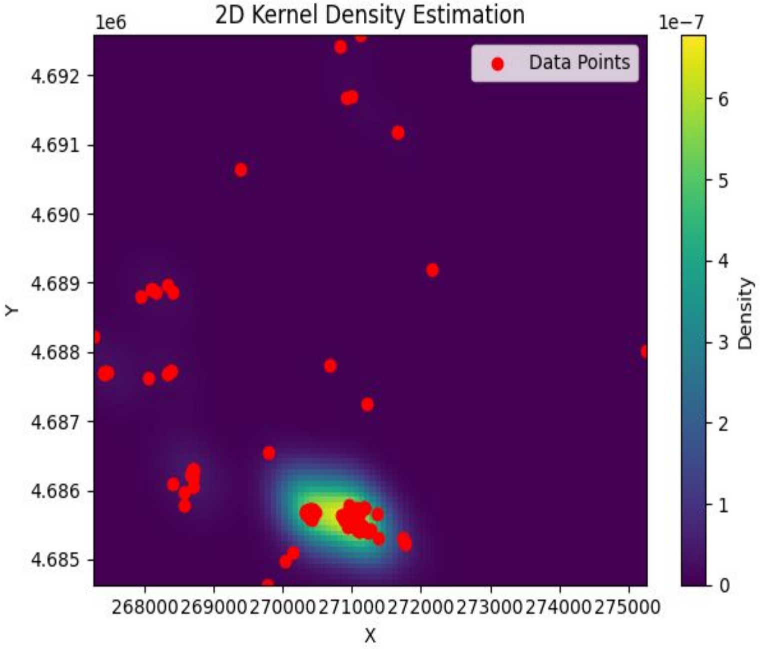
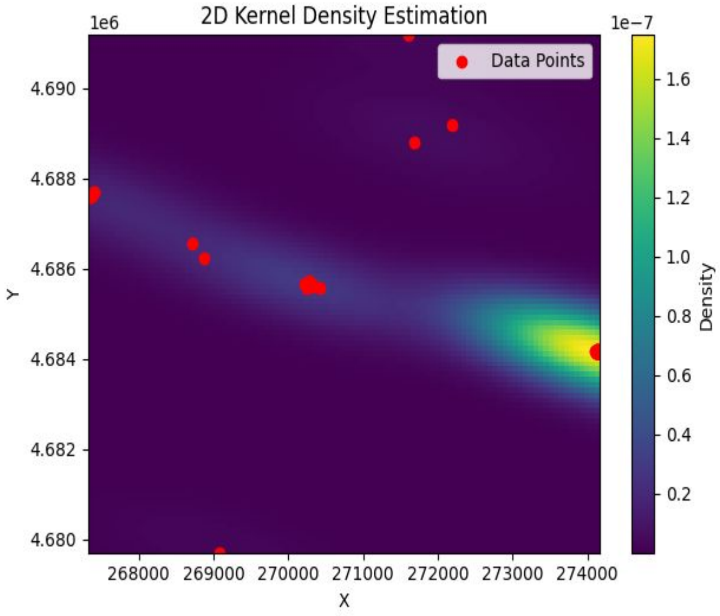
2D KDE (Norway maple: 2014 & 2015 data)





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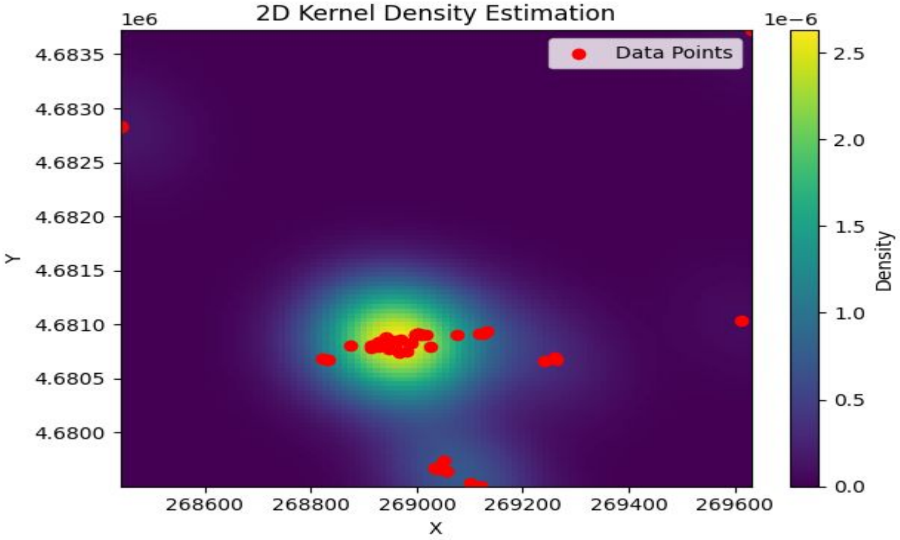
Progress - Data Analysis

2D KDE (Norway maple: 2016)



Progress - Data Analysis

2D KDE (Silver maple: 2008 & 2009 data)

Overview

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- **Data Analysis**
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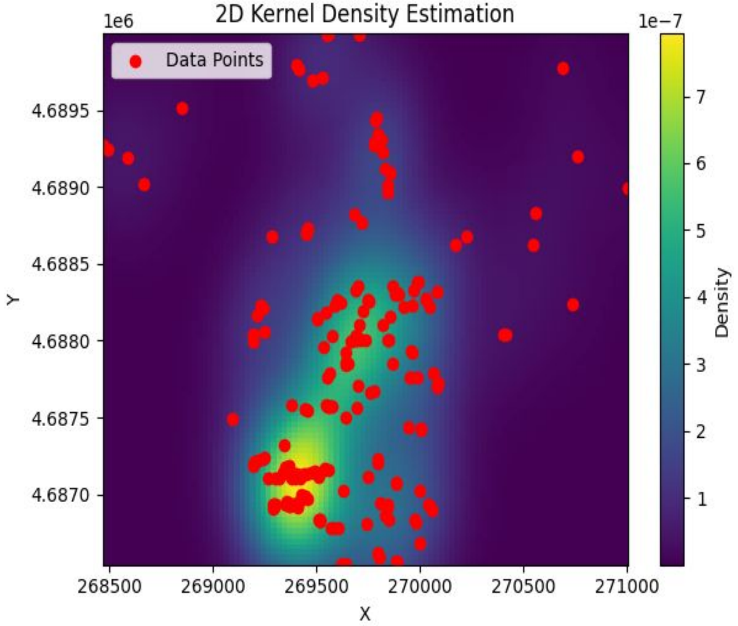

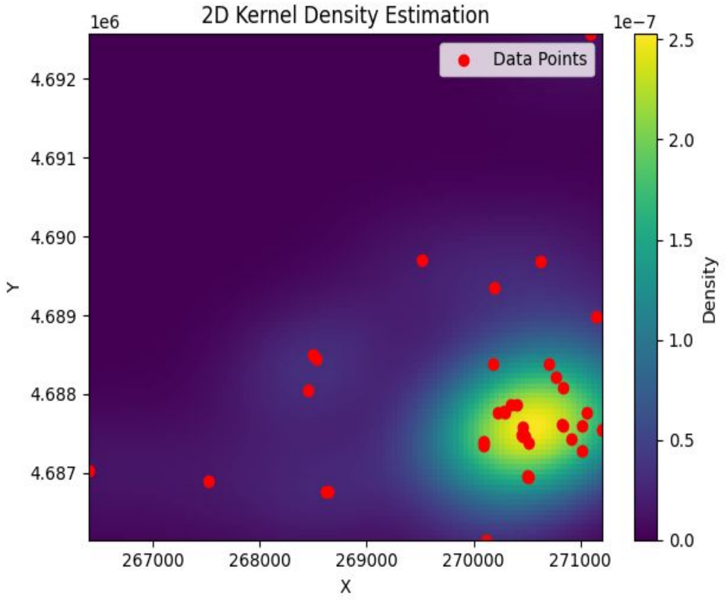

Progress - Data Analysis

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- **Data Analysis**
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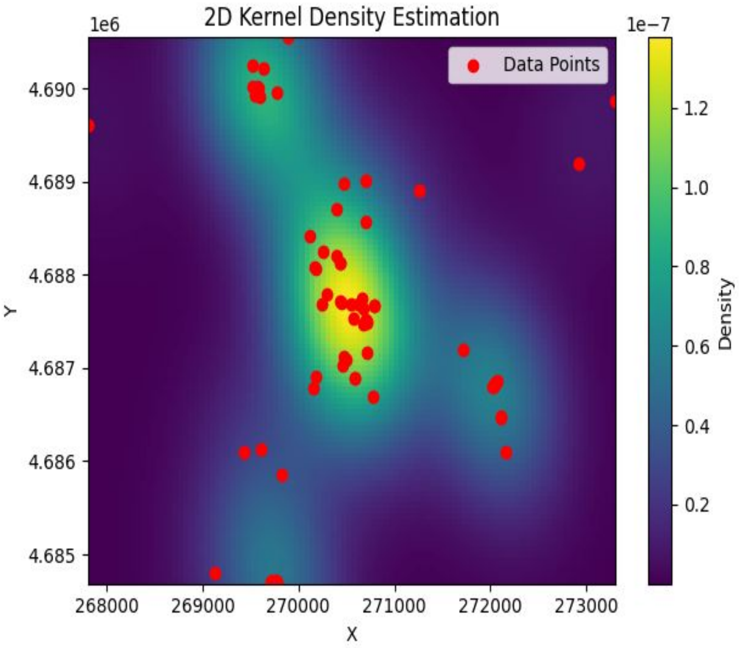
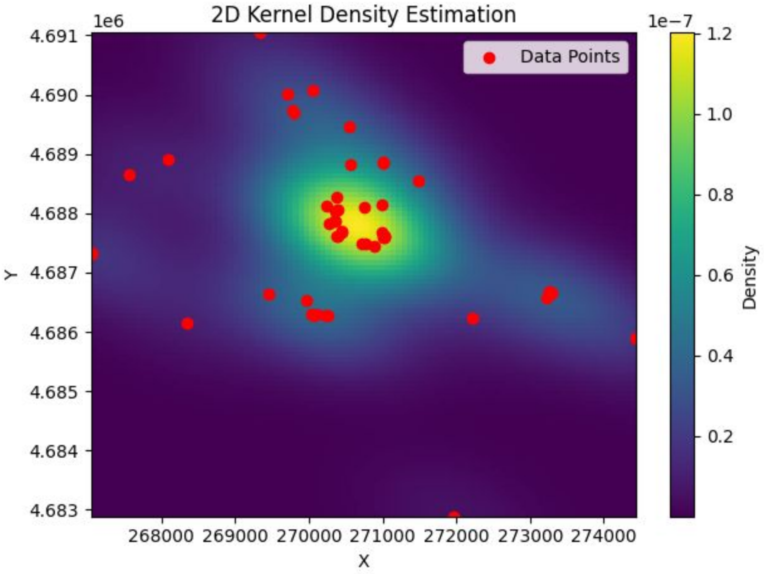
2D KDE (Silver maple: 2010 & 2011 data)





Progress - Data Analysis

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- Model Construction

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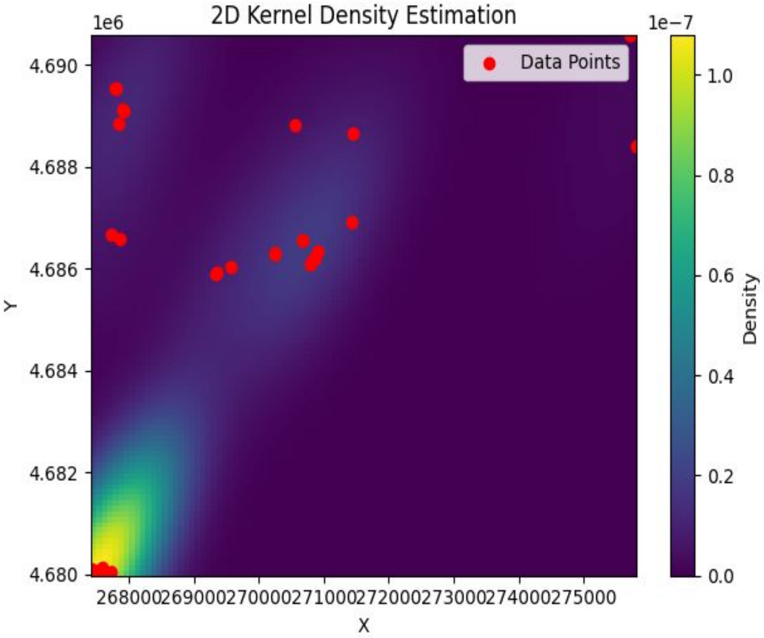

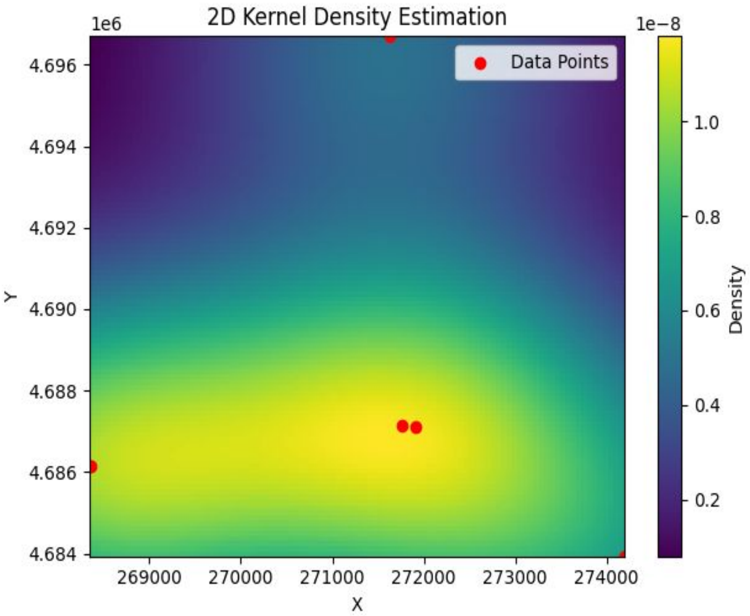

Progress - Data Analysis

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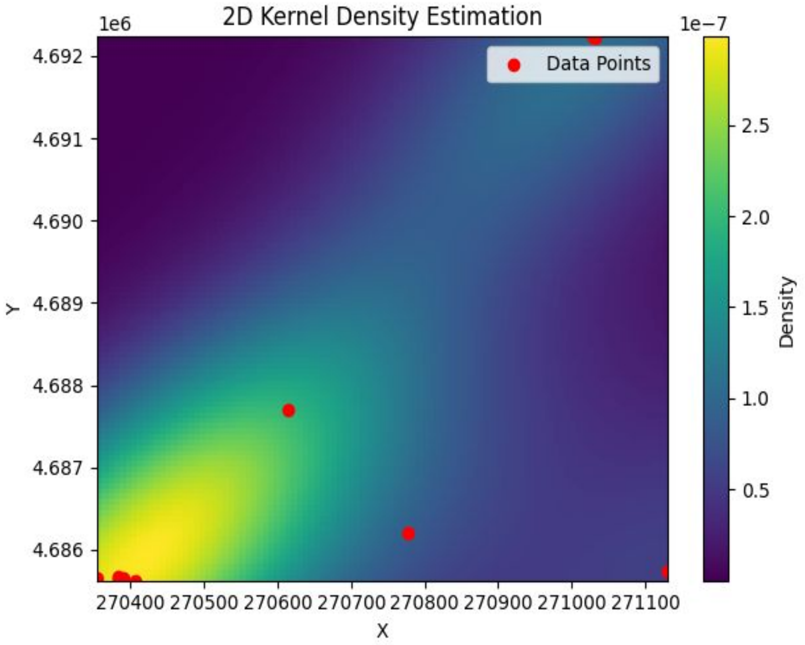

Overview

Motivation

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- **Data Analysis**
- Model Construction

Next Steps

Application of the KDE

Kernel Density Estimations (KDE)

- We can approximate a 1-D or 2-D distribution as a Gaussian distribution.
- Take a histogram of the data and fit a normal curve to each bin. Then, adding all curves together will give us our approximate underlying distribution.
- **The peak of the summed curve should indicate the peak of densities, corresponding to potential clusters.**

Detecting Clusters

Overview

Motivation

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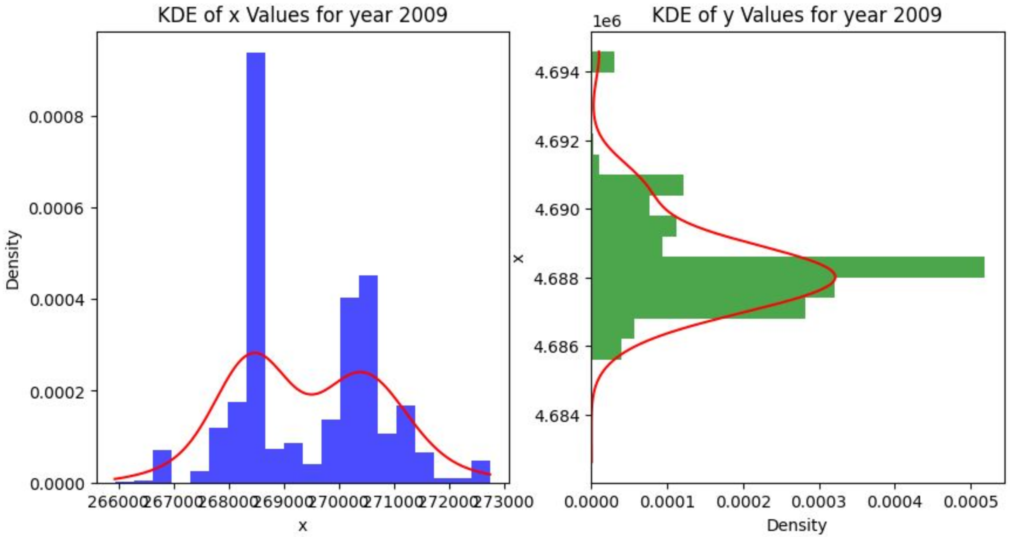
- **Data Analysis**
- Model Construction

Next Steps

One-Dimensional KDE: 2009 Data



Detecting Clusters

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- **Data Analysis**
- Model Construction

Next Steps

In theory...

- Use 1-D KDE to pinpoint the 2-D clusters

In reality...?

Detecting Clusters

Overview

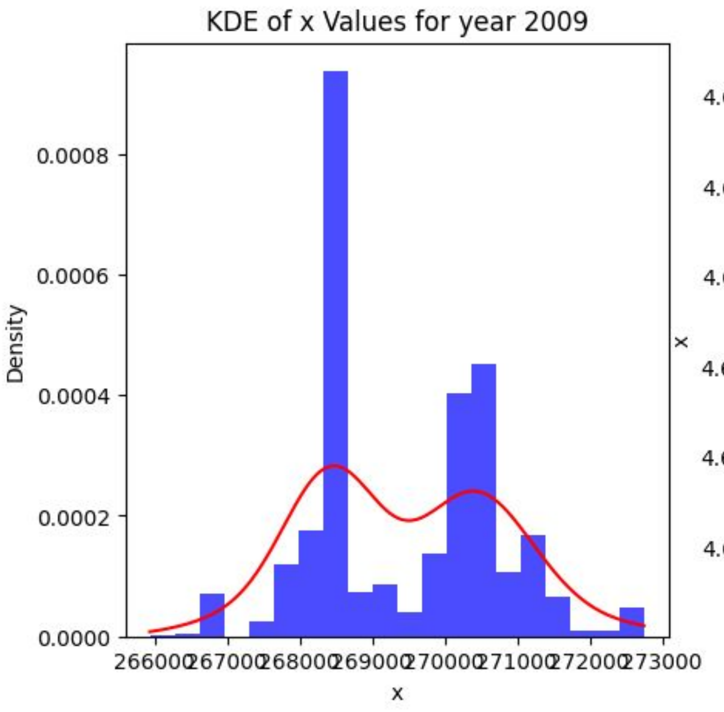
Motivation

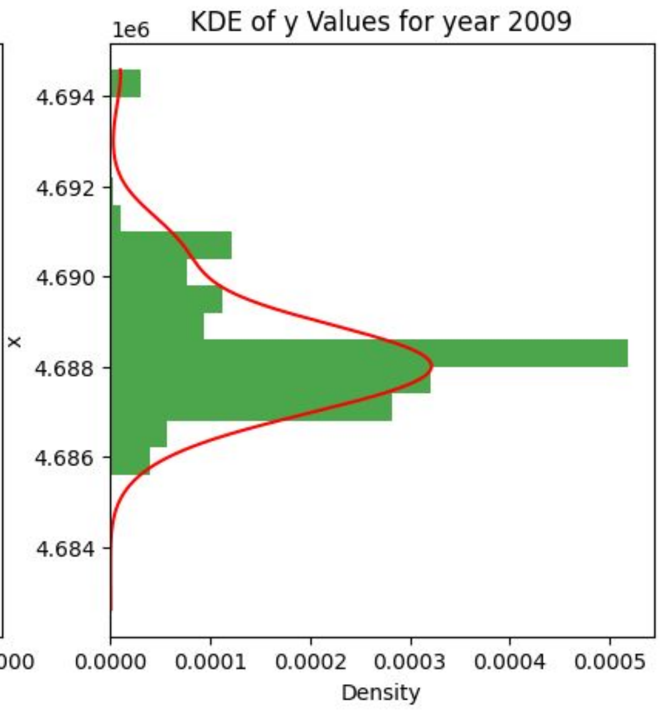
Progress

- **Data Analysis**
- Model Construction

Next Steps

One-Dimensional KDE: 2009 Data





Detecting Clusters

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- **Data Analysis**
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Next Steps

In theory...

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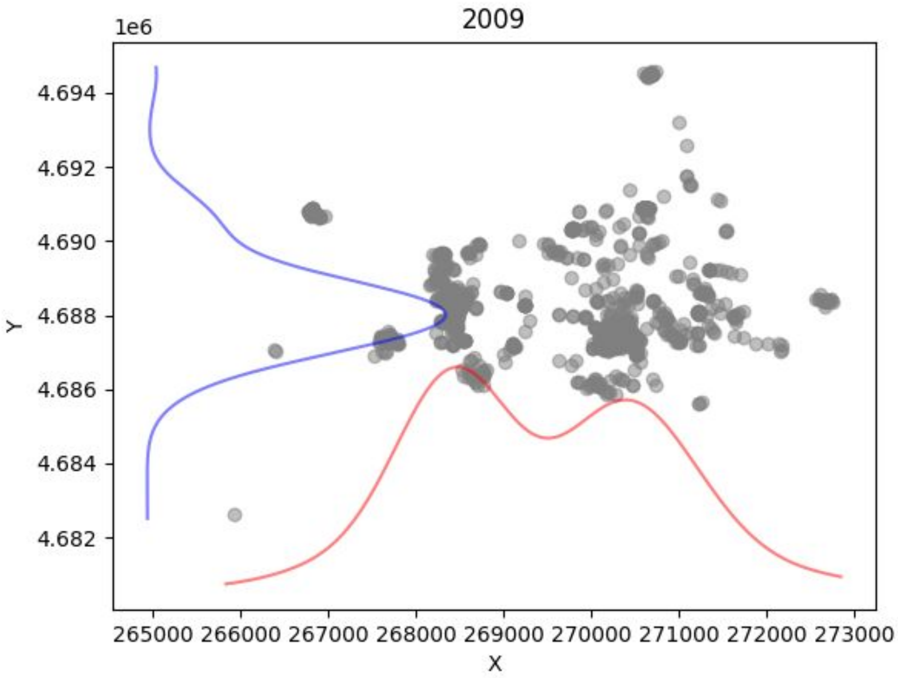

Detecting Clusters

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- **Data Analysis**
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Next Steps

In theory...

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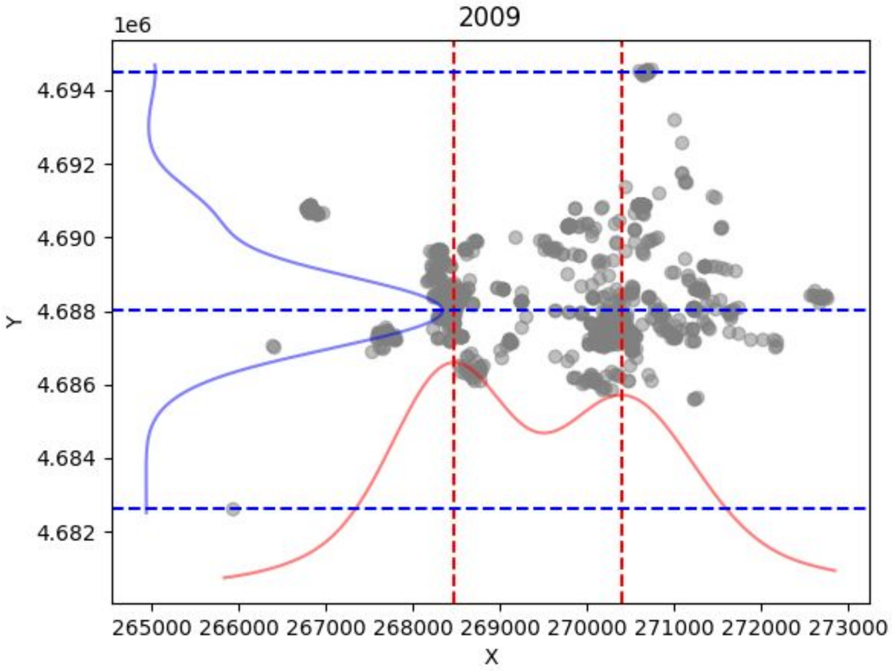


Detecting Clusters

Overview

Motivation

Progress

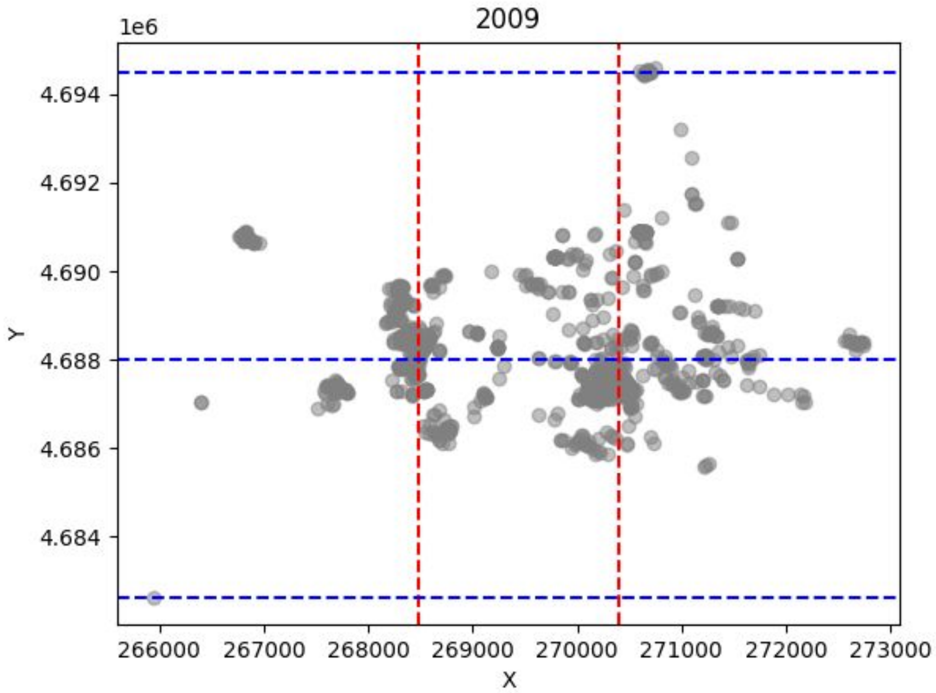
- **Data Analysis**
- Model Construction

Next Steps

In practice...

- Found Cluster!

Or did we?



Detecting Clusters

What about the other years?

Detecting Clusters

Overview

Motivation

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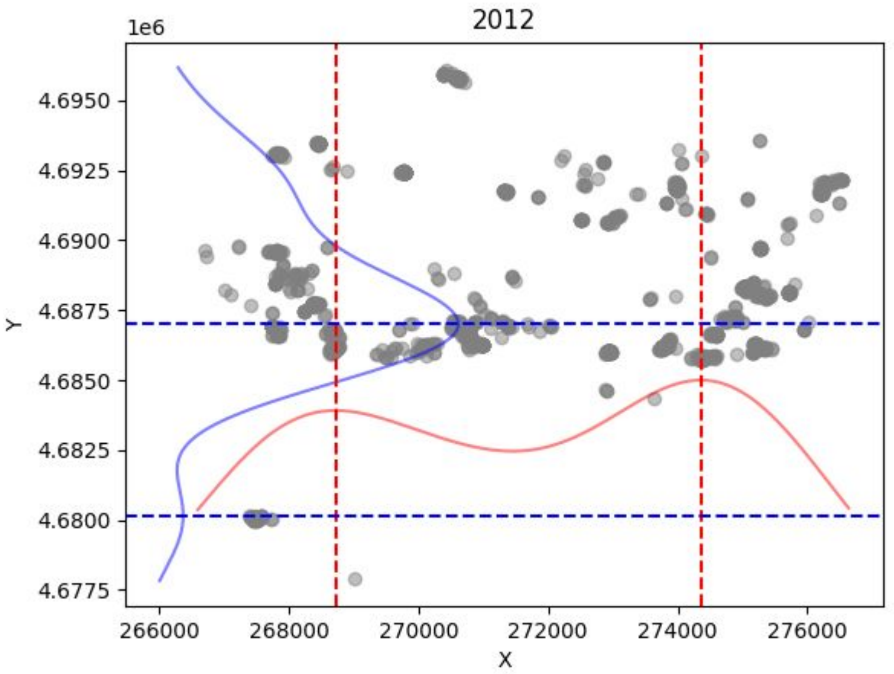
- **Data Analysis**
- Model Construction

Next Steps

In theory...

- Use 1-D to pinpoint the 2-D clusters



Detecting Clusters

Overview

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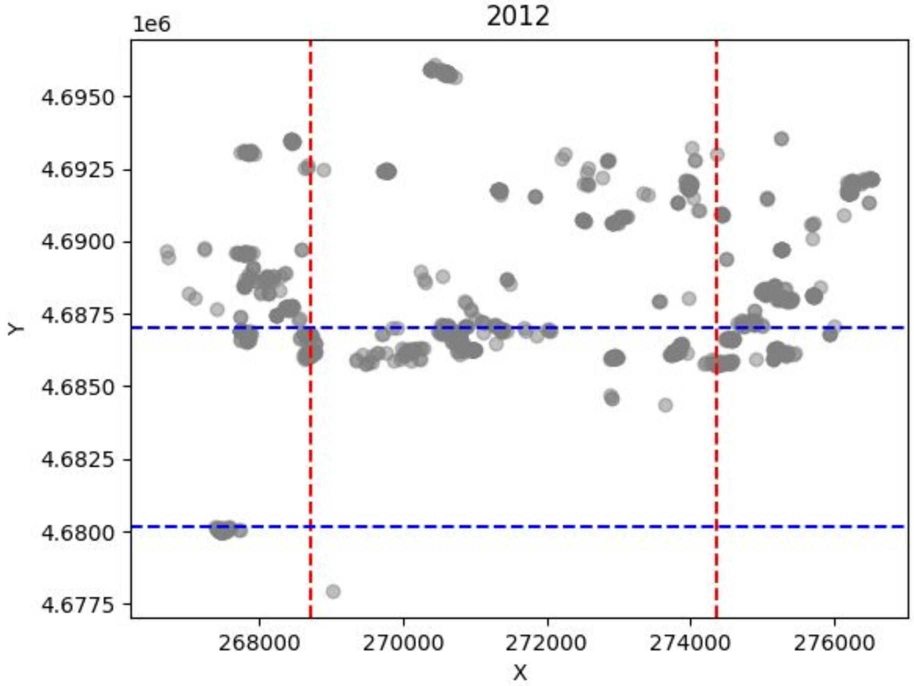
- **Data Analysis**
- Model Construction

Next Steps

In reality...

- Not so nice anymore :(



Detecting Clusters

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- **Data Analysis**
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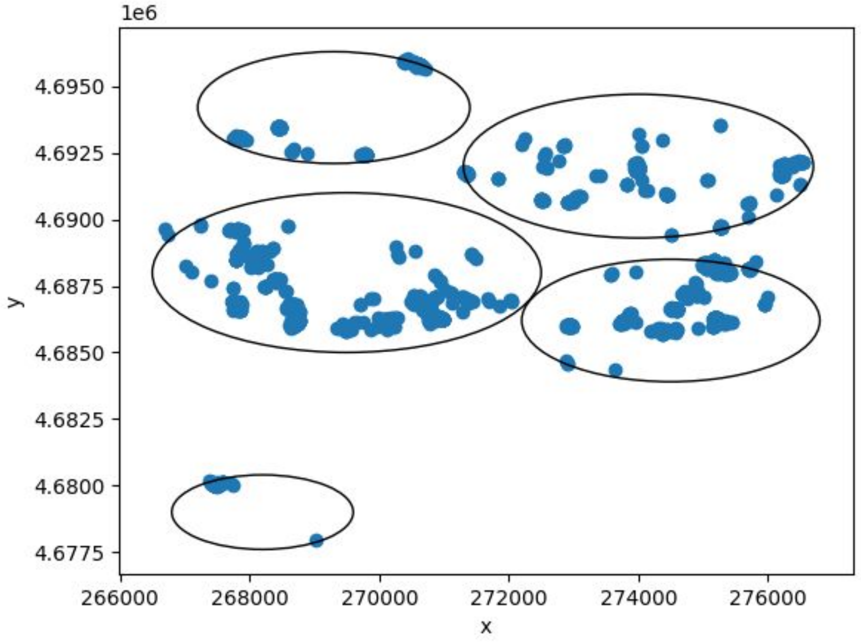
Next Steps

In reality...

- Structures cannot be determined as good as eyeballing the data directly.



Overview

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- **Data Analysis**
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Next Steps

Detecting Clusters

In reality...

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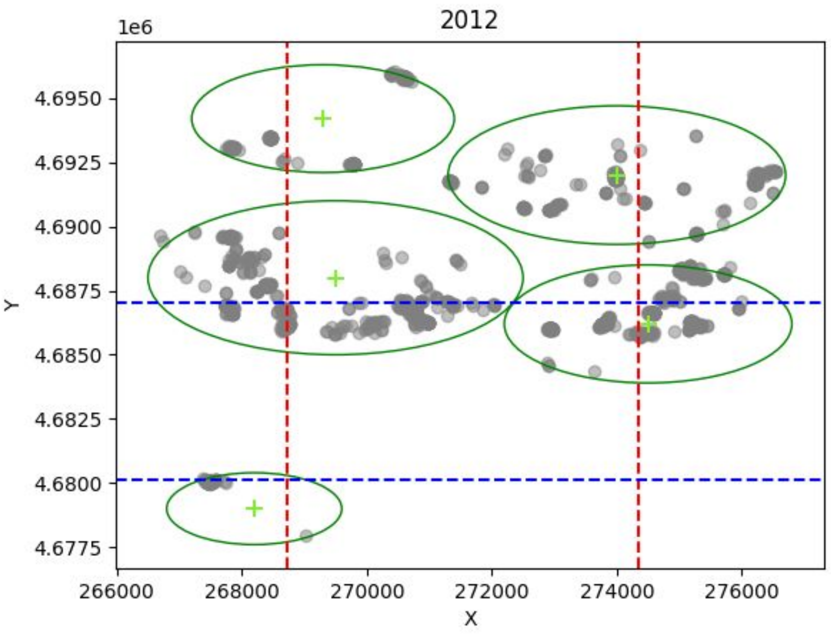

Here the green circles/centers are results of eyeballing with help from 2D KDEs.

Detecting Clusters

Overview

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- **Data Analysis**
- Model Construction

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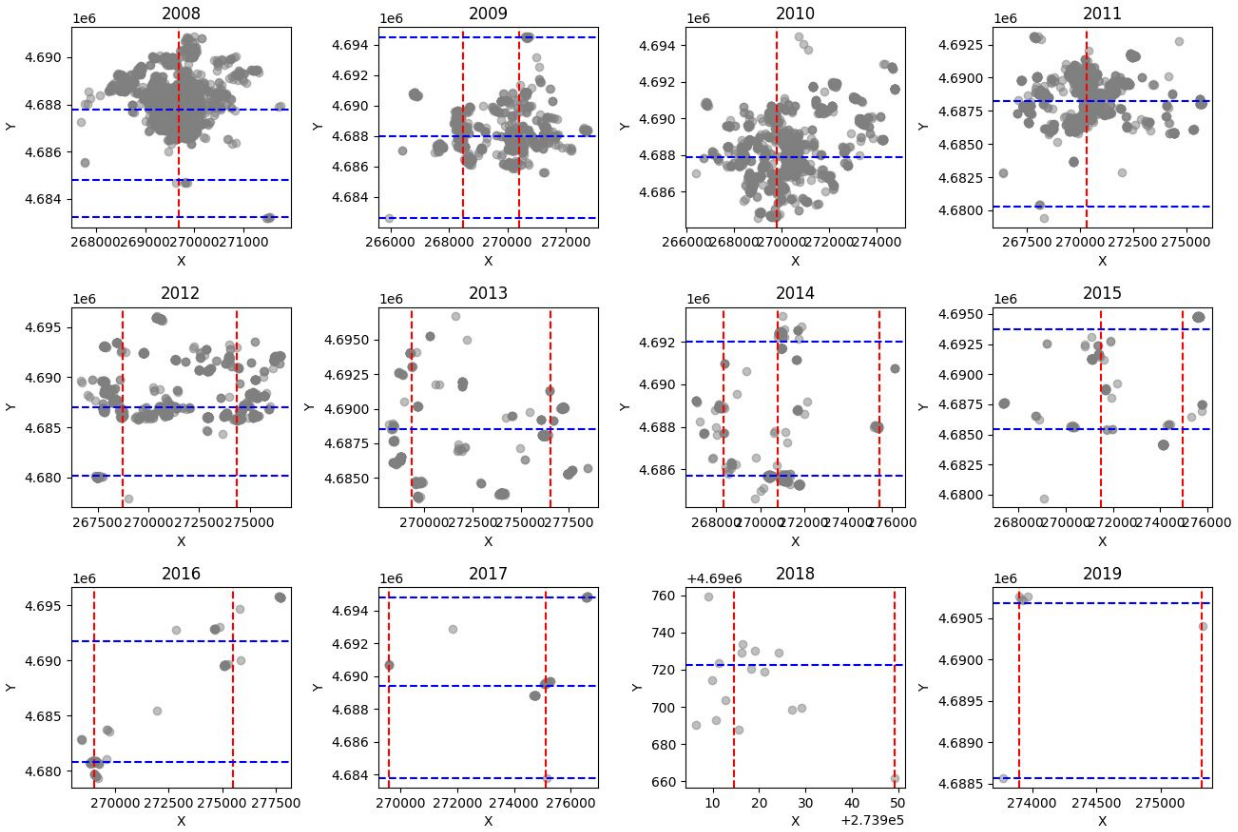


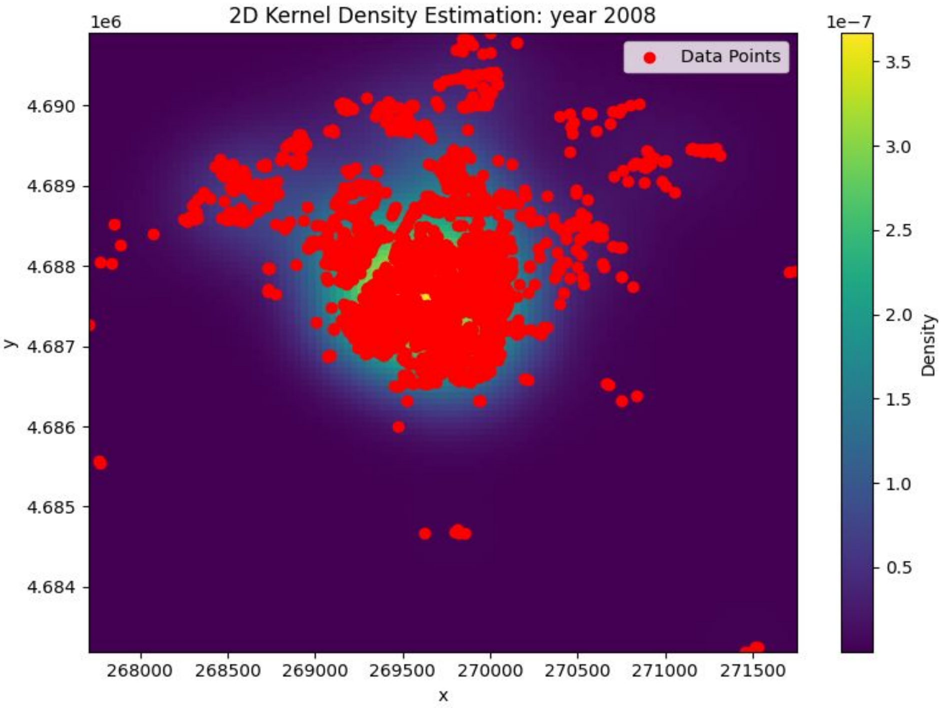
Detecting Clusters

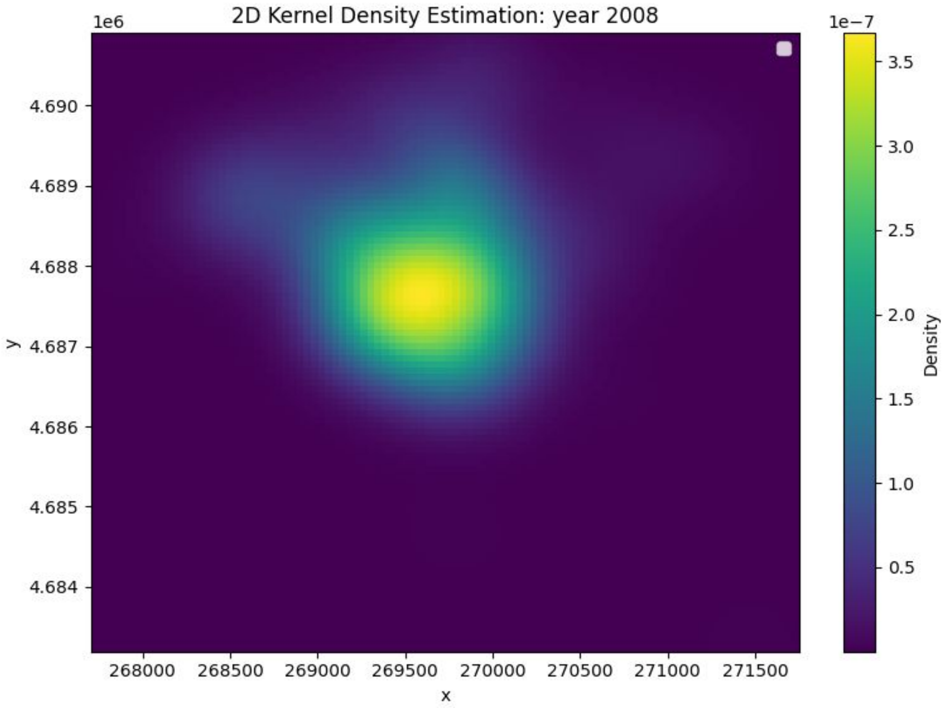
In reality...

- Structures cannot be determined as good as eyeballing the data directly.
  - Resolving to determining clusters by eye

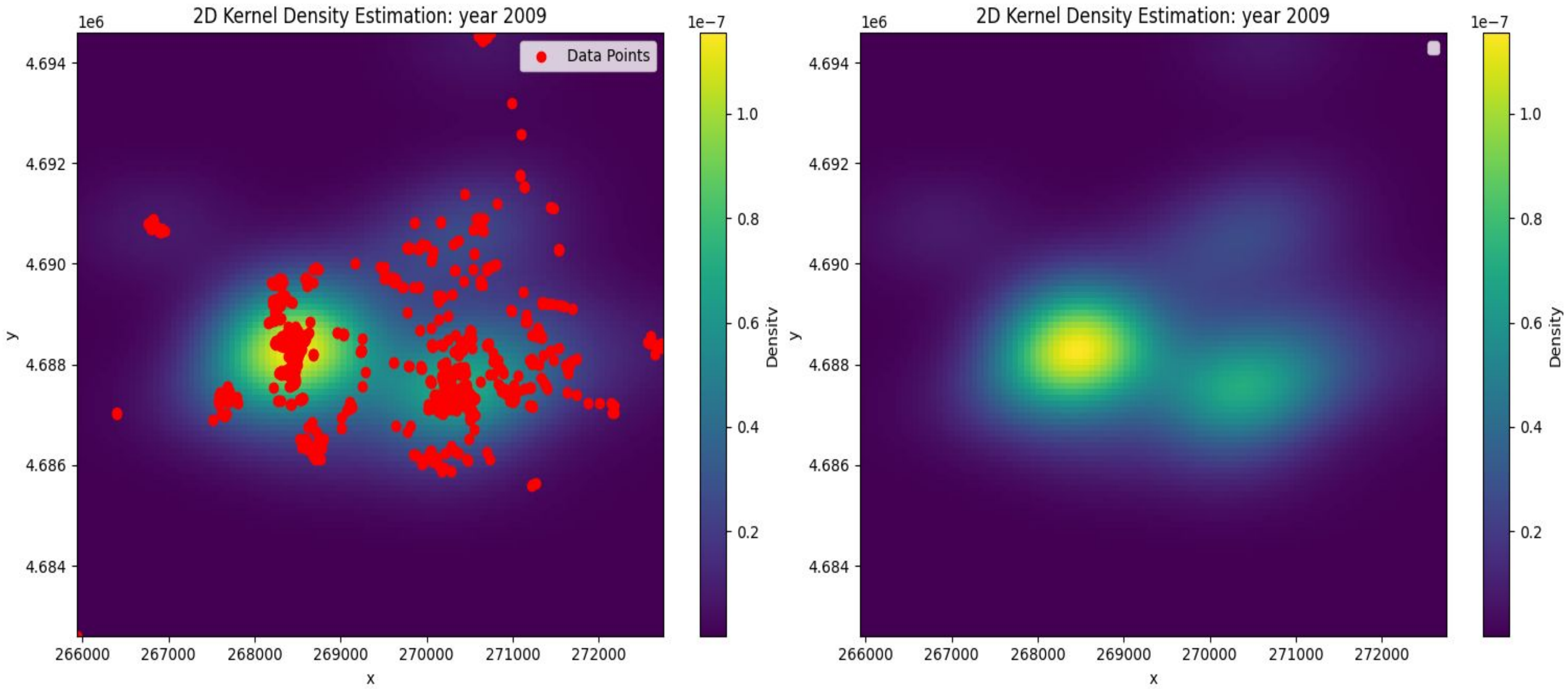
- Currently an open question!

2D KDE for Year 2008





2D KDE for Year 2009



Temporal Shift by Year - 2008

Overview

Motivation

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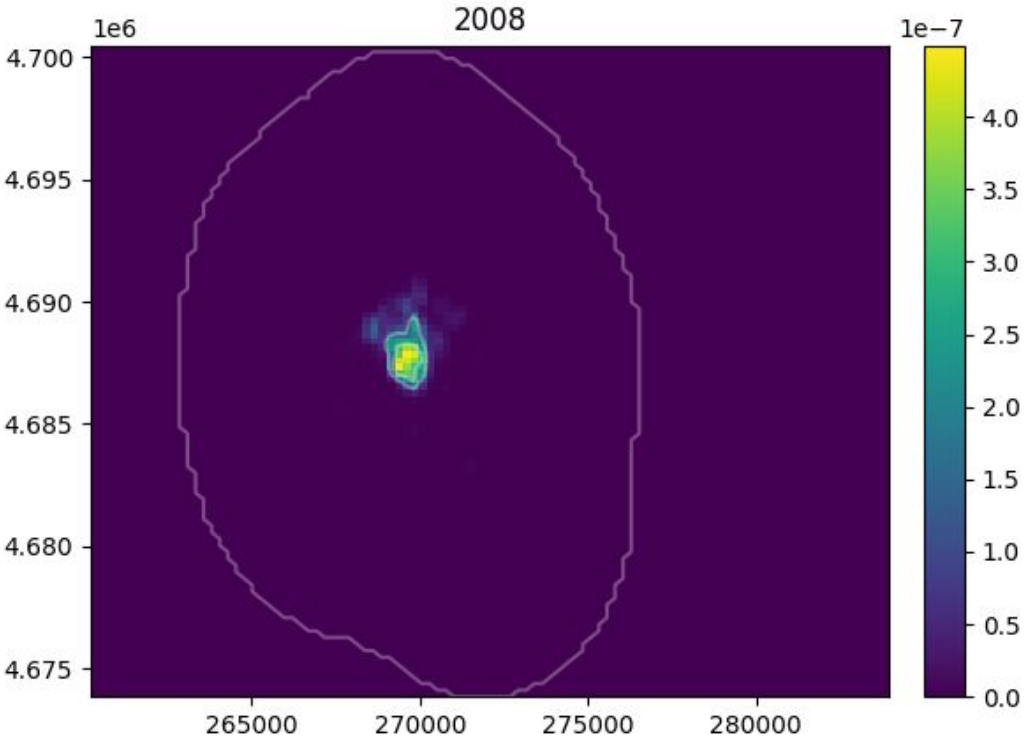
- **Data Analysis**
- Model Construction

Next Steps



Temporal Shift by Year - 2009

Overview

Motivation

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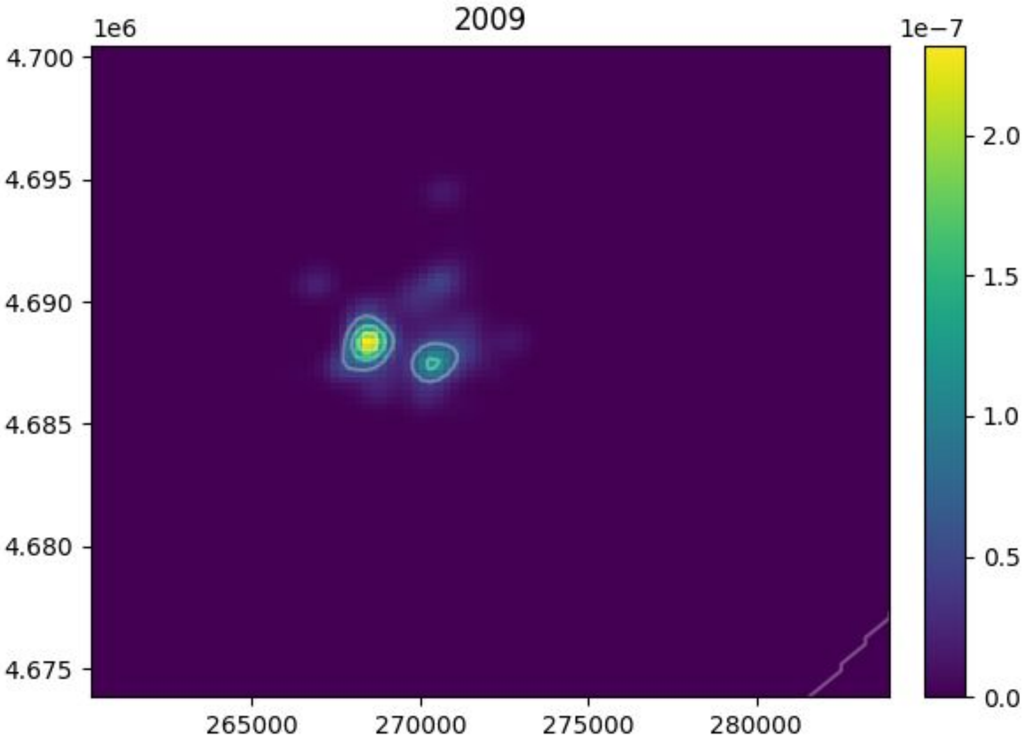
- **Data Analysis**
- Model Construction

Next Steps



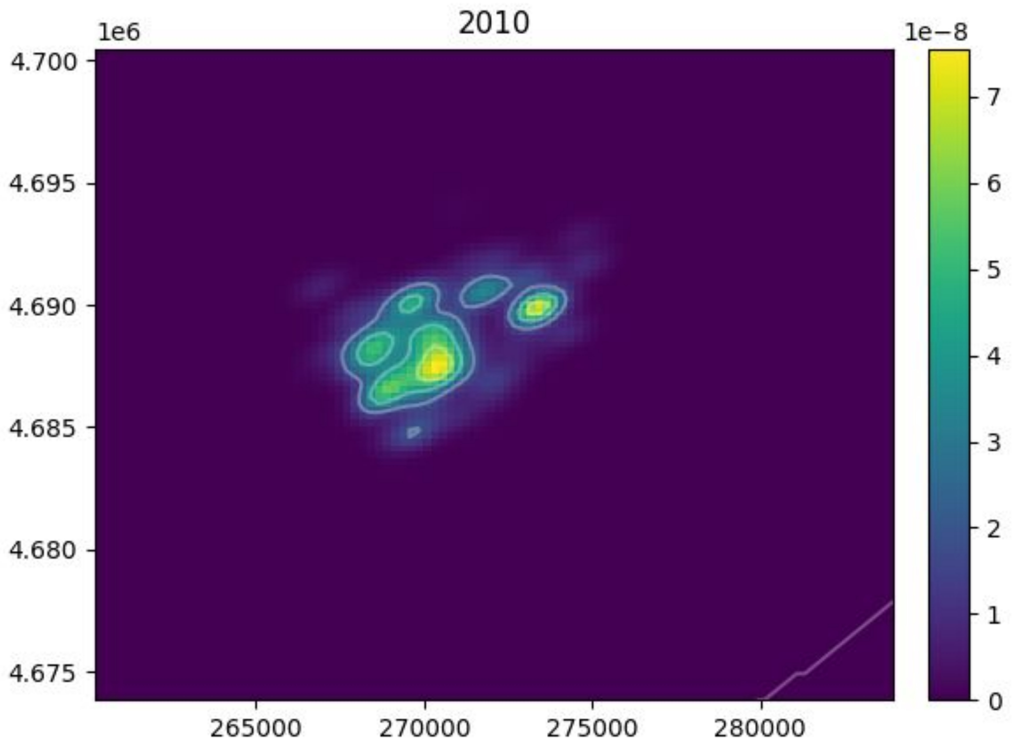
Temporal Shift by Year - 2010

Overview

Motivation

Progress
- **Data Analysis**
- Model Construction

Next Steps



Temporal Shift by Year - 2011

Overview

Motivation

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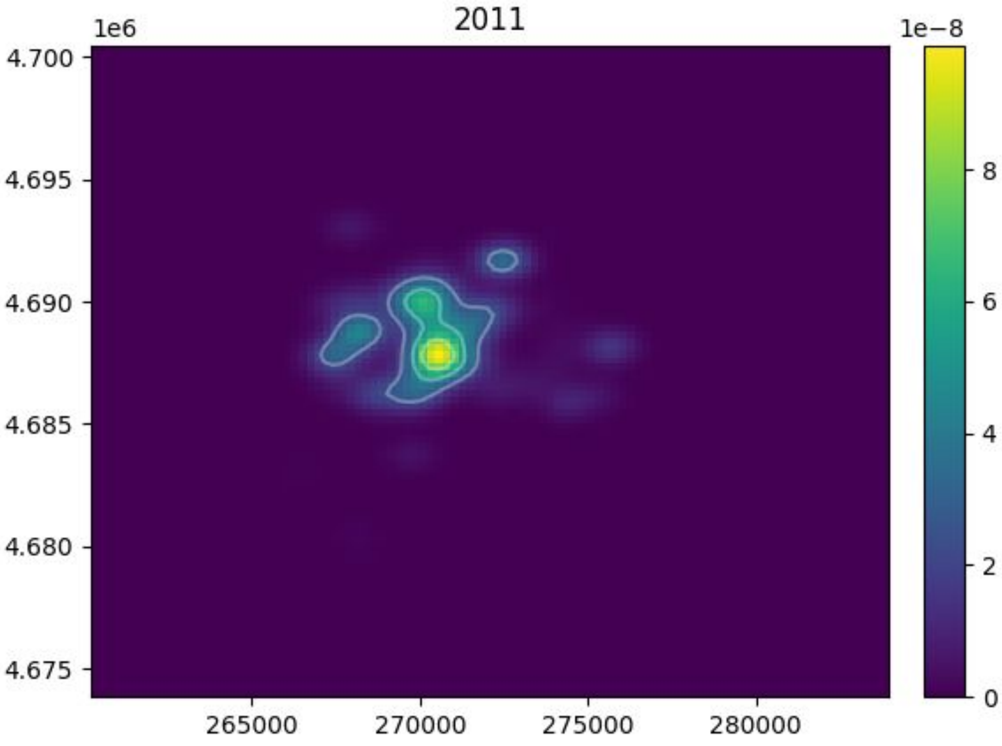
- **Data Analysis**
- Model Construction

Next Steps



Temporal Shift by Year - 2012

Overview

Motivation

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- **Data Analysis**
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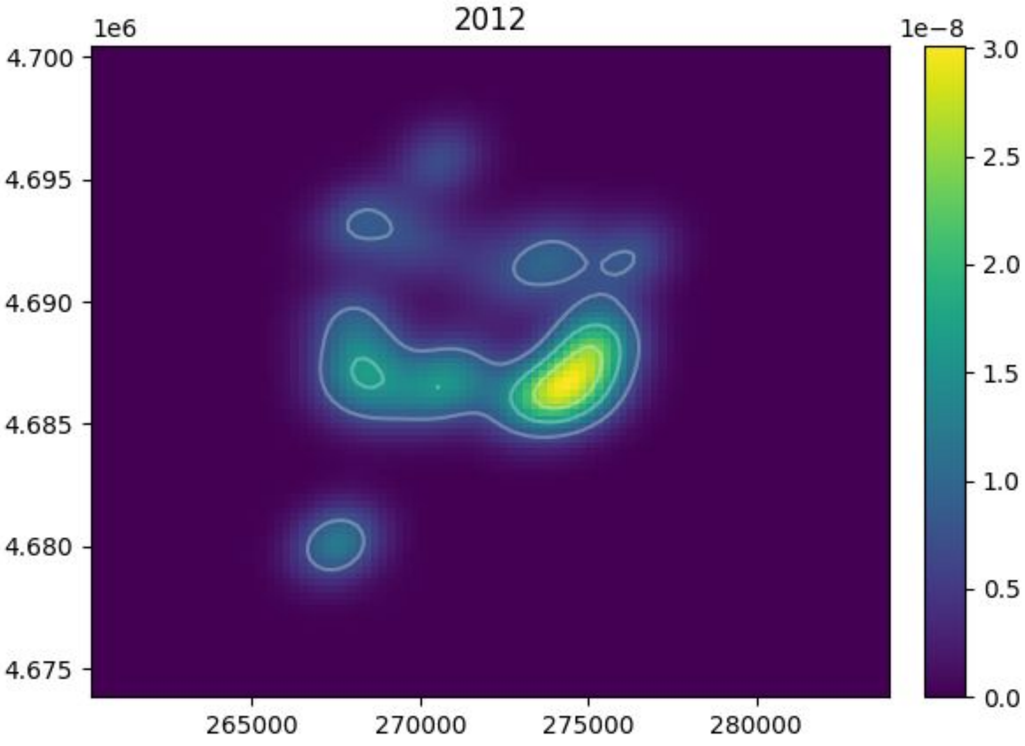
Next Steps



Temporal Shift by Year - 2013

Overview

Motivation

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- **Data Analysis**
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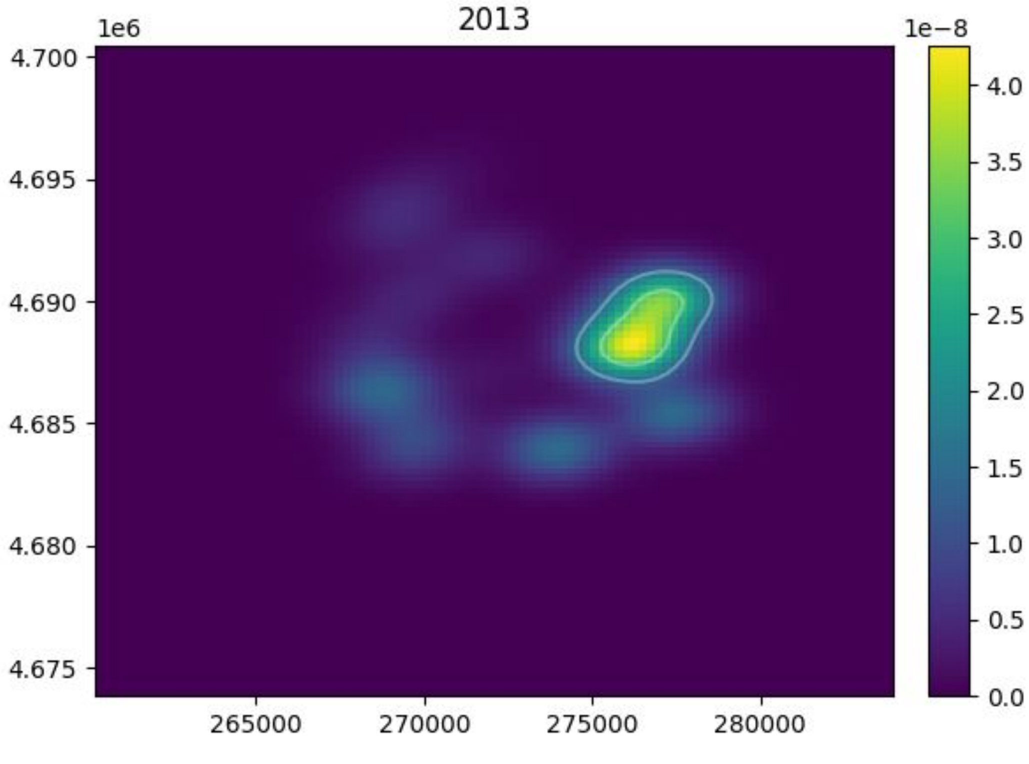
Next Steps



Temporal Shift by Year - 2014

Overview

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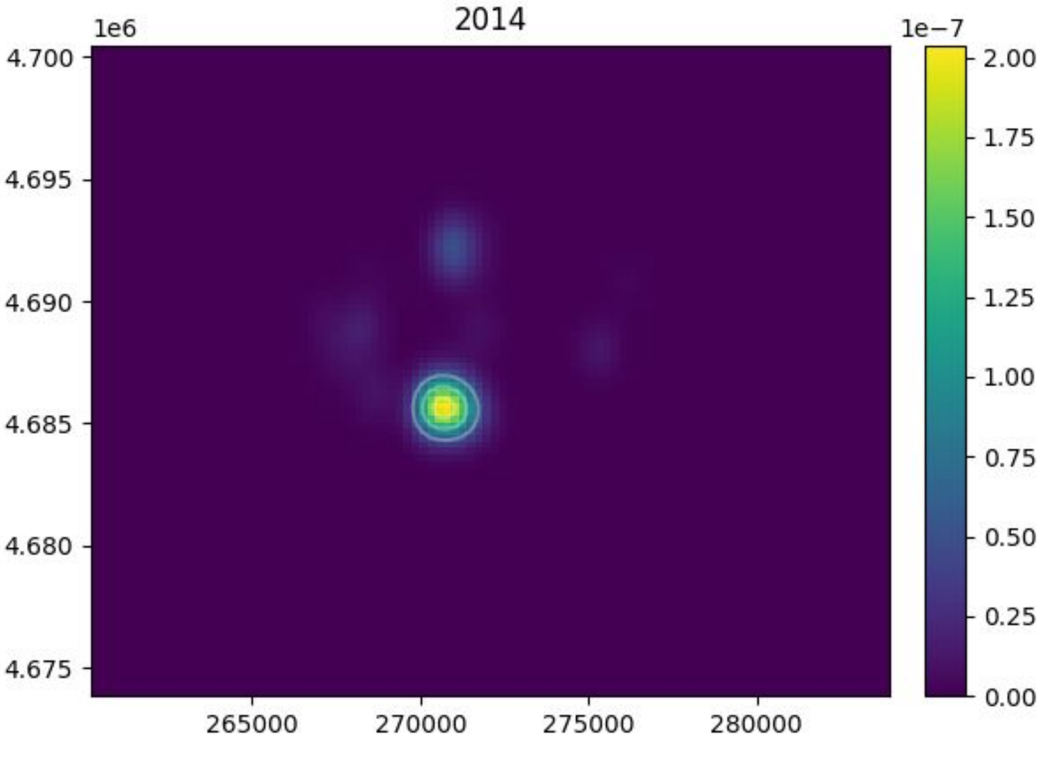
- **Data Analysis**
- Model Construction

Next Steps



Temporal Shift by Year - 2015

Overview

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- **Data Analysis**
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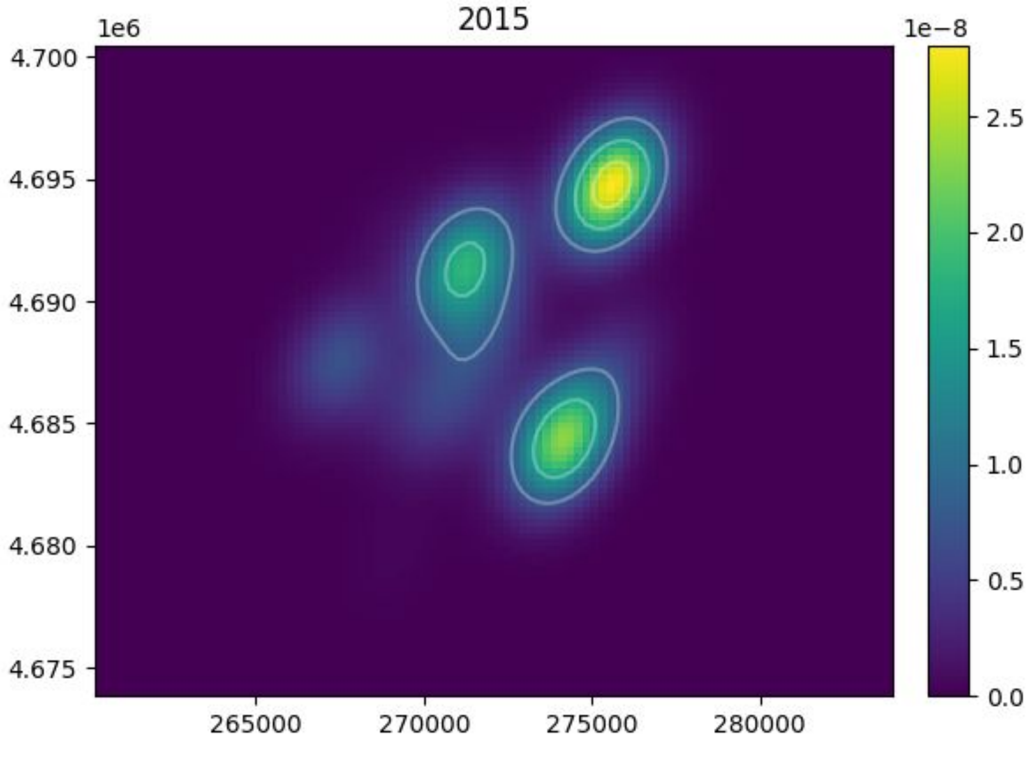

Temporal Shift by Year - 2016

Overview

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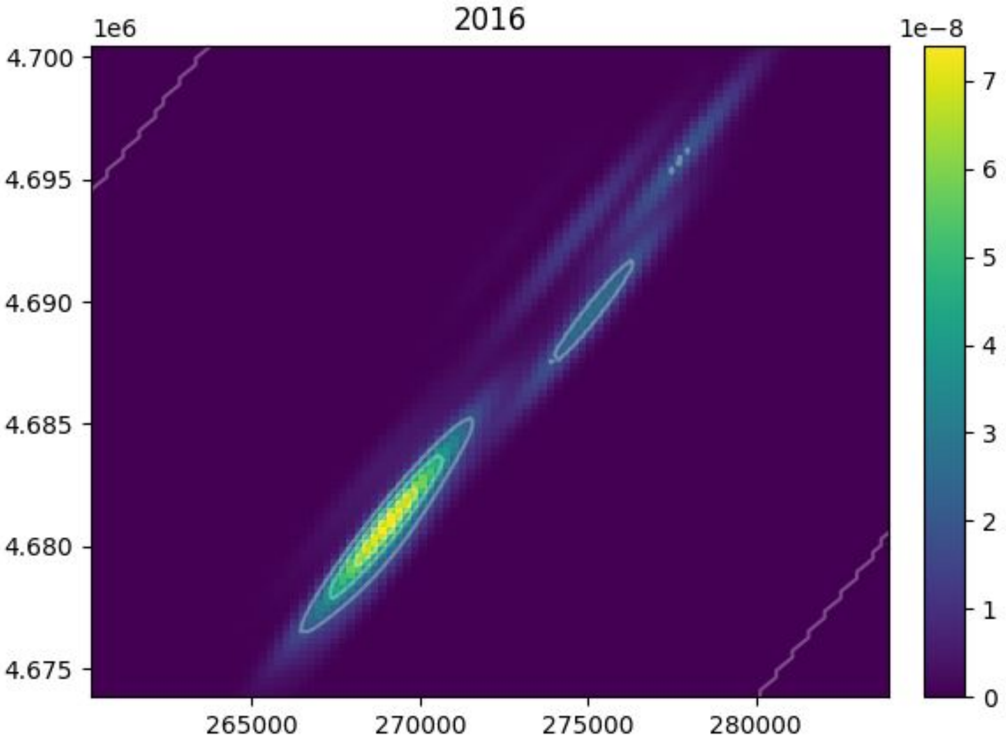
- **Data Analysis**
- Model Construction

Next Steps



Temporal Shift by Year - 2017

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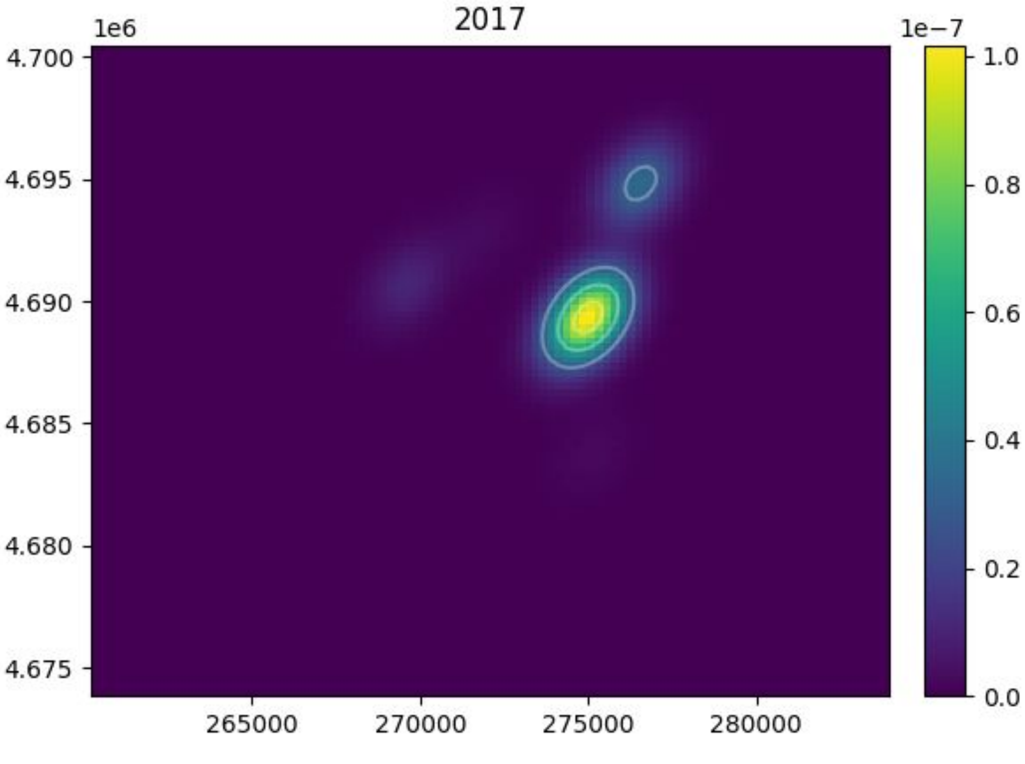
- **Data Analysis**
- Model Construction

Next Steps



Temporal Shift by Year - 2018

Overview

Motivation

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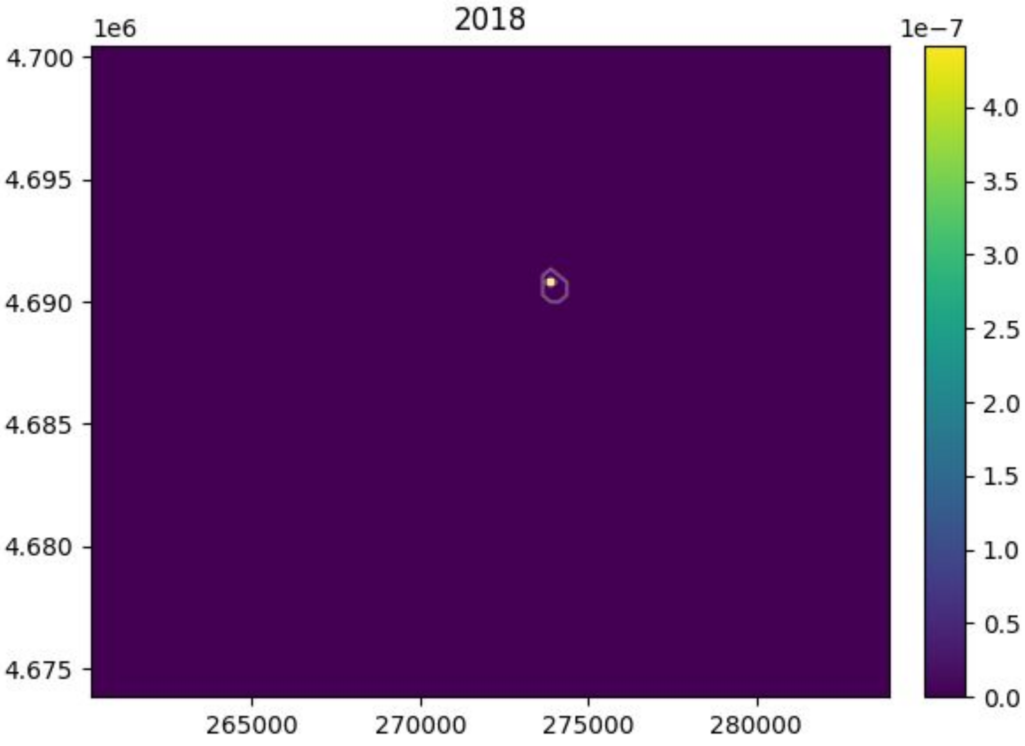
- **Data Analysis**
- Model Construction

Next Steps



Temporal Shift by Year - 2019

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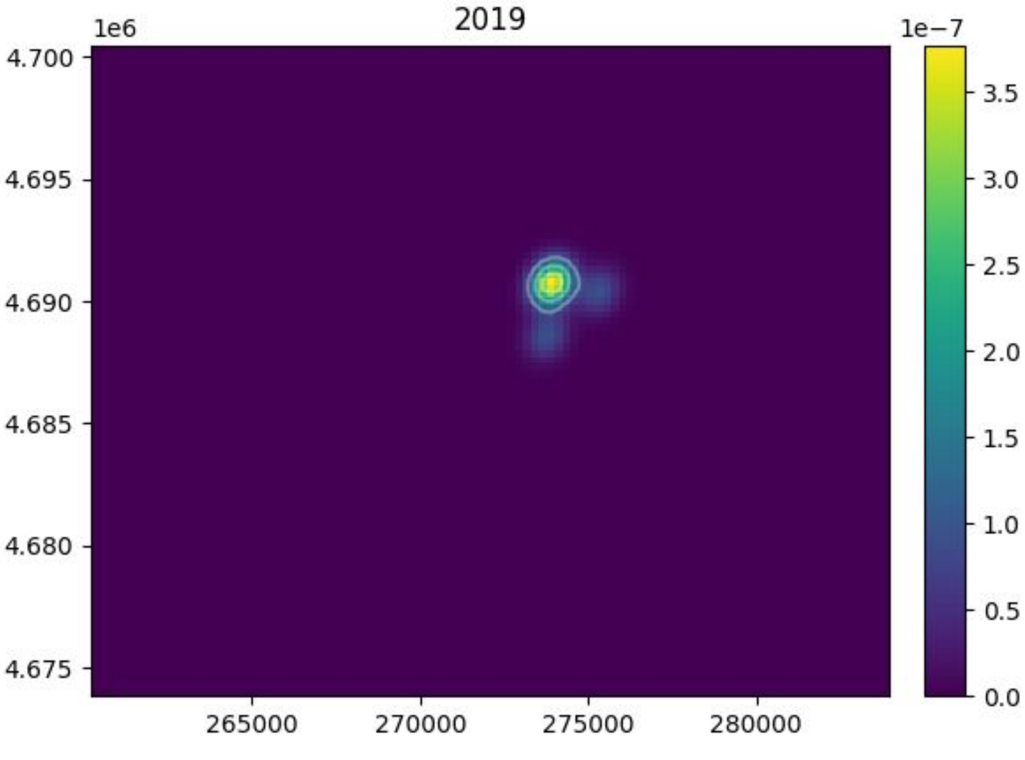
- **Data Analysis**
- Model Construction

Next Steps



Model Construction

Constructing a Model for Our Data

- Important quantities to model:

Model Construction

Constructing a Model for Our Data

- Important quantities to model:
  - The **number of points** in the grid

Model Construction

Constructing a Model for Our Data

- Important quantities to model:
    - The **number of points** in the grid
    - The **spread of the points** within the grid

Model Construction

Constructing a Model for Our Data

- Important quantities to model:
    - The **number of points** in the grid
    - The **spread of the points** within the grid

- We must create a model that **fits both of these parameters.**

Spatial Poisson Point Process

- We model the **number of points** λ as a *Poisson random variable.*
  - This distribution "counts" the number of events of the process in our grid.

Spatial Poisson Point Process

Overview

Motivation

Progress
- Data Analysis
- **Model Construction**

Next Steps

- We model the **number of points** λ as a *Poisson random variable.*
  - This distribution "counts" the number of events of the process in our grid.
- The probability of k events occurring is given by

$$P\{X = k\} = \frac{\lambda^k e^{-\lambda}}{k!}$$

- The parameter λ is both the **mean** and **variance.**

Non-Homogeneous Point Process

- For a *homogeneous* point process, λ remains constant along the entire grid.

Non-Homogeneous Point Process

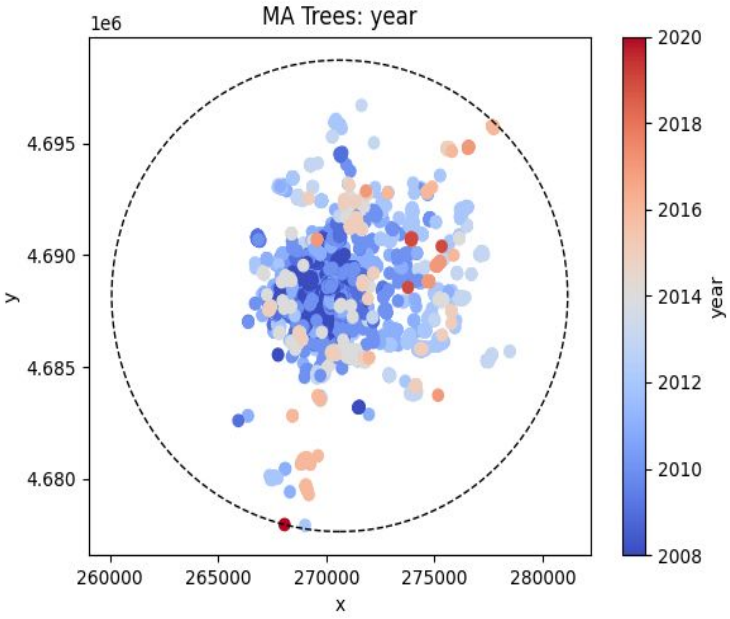
Overview

Motivation

Progress
- Data Analysis
- **Model Construction**

Next Steps

- For a *homogeneous* point process, λ remains constant along the entire grid.
- However, our data is **non-homogeneous:** $\lambda(x)$ depends on the point x in the grid.



Estimating the Values of $\lambda(x)$

Overview

Motivation

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- Data Analysis
- **Model Construction**

Next Steps

- We estimate the values of $\lambda(x)$ by first counting the points in our data set:

Estimating the Values of $\lambda(x)$

- We estimate the values of $\lambda(x)$ by first counting the points in our data set:

  1. Draw circles $C_1, \ldots, C_N$ around point clusters.

Estimating the Values of $\lambda(x)$

Overview

Motivation

Progress
- Data Analysis
- **Model Construction**

Next Steps

- We estimate the values of $\lambda(x)$ by first counting the points in our data set:

1. Draw circles $C_1, \ldots, C_N$ around point clusters.
2. Find the number of data points $\hat{\lambda}(C_i)$ in each circle.

Estimating the Values of $\lambda(x)$

- We estimate the values of $\lambda(x)$ by first counting the points in our data set:

    1. Draw circles $C_1, \ldots, C_N$ around point clusters.
    2. Find the number of data points $\hat{\lambda}(C_i)$ in each circle.
    3. Divide by the area of the circle:

$$\hat{\lambda}_i = \frac{\hat{\lambda}(C_i)}{area(C_i)}$$

Estimating the Values of $\lambda(x)$

- This gives us an estimate of the λ value within each circle C_i.
    - We want a formula for each point $x \in C_i$.

Estimating the Values of $\lambda(x)$

- This gives us an estimate of the λ value within each circle C_i.
  - We want a formula for each point $x \in C_i$.
- To do this, we use a formula that decreases with distance:

$$\hat{\lambda}_i(x) = \hat{\lambda}_i e^{-dr^2(x)}$$

  - $r(x)$ is the distance from the circle's center
  - *d > 0* is the decay rate (Default: 0.01)

Bayesian Inference

- Inside each circle C_i, we have some M_i data points

$$X_i = \{x_1, \ldots, x_{M_i}\}$$

which we interpret as random samples.

Bayesian Inference

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which we interpret as random samples.

- For each circle, we aim to calculate the *posterior distribution*

$$p_i(\lambda_i | x)$$

Bayesian Inference

- If we make an initial assumption about the distribution of the (random) values $\lambda = (\lambda_1, \ldots, \lambda_N) \in \mathbb{R}^N$, then we can use *Bayesian Inference* to approximate the true distribution:

$$p_i(\lambda_i|x) = p_i(x|\lambda_i) f_i(\lambda_i)$$

Bayesian Inference

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$$p_i(\lambda_i|x) = p_i(x|\lambda_i)f_i(\lambda_i)$$

- We assume a *prior distribution* $f_i(\lambda_i)$ for these parameters.

Overview

Motivation

Progress
- Data Analysis
- **Model Construction**

Next Steps

Bayesian Inference

- We use a **Gamma distribution** as our prior:

$$\lambda_i \sim f_i(\lambda_i) = Gamma(a_{\lambda_i}, b_{\lambda_i})$$

Overview

Motivation

Progress
- Data Analysis
- **Model Construction**

Next Steps

Bayesian Inference

- We use a **Gamma distribution** as our prior:

$$\lambda_i \sim f_i(\lambda_i) = Gamma(a_{\lambda_i}, b_{\lambda_i})$$

- A gamma distribution describes the spread of points with a *shape* and a *rate* a_{λ_i} . b_{λ_i}
  - These can be thought of as **additional model parameters**.

Overview

Motivation

Progress
- Data Analysis
- **Model Construction**

Next Steps

Bayesian Inference

- The *likelihood distribution* can be calculated from our data points:

$$p_i(x|\lambda_i) = \prod_{j=1}^{M} p_i(x_j|\lambda_i(x_j))$$

Bayesian Inference

- The *likelihood distribution* can be calculated from our data points:

$$p_i(x|\lambda_i) = \prod_{j=1}^{M} p_i(x_j|\lambda_i(x_j))$$

- Each of these functions is given by the Poisson distribution.

Overview

Motivation

Progress
- Data Analysis
- **Model Construction**

Next Steps

Bayesian Inference

- The prior distribution can be estimated from our (sampled) points:

$$f_i(\lambda_i) = \prod_{j=1}^{M} f_i(\lambda_i(x_j))$$

Bayesian Inference

- The prior distribution can be estimated from our (sampled) points:

$$f_i(\lambda_i) = \prod_{j=1}^{M} f_i(\lambda_i(x_j))$$

  - Recall that $f_i(\lambda_i)$ is a Gamma distribution.

Sampling from a Posterior

Markov Chain Monte Carlo (MCMC) Model

- From the paper "Bayesian Nonparametric Nonhomogeneous Poisson Process..." (arXiv:1907.03186):

$$
\begin{aligned}
&k \sim p(\cdot), \text{where } p(\cdot) \text{ is a p.m.f on } \{1, 2, \ldots\} \\
&\lambda_r \overset{\text{ind}}{\sim} \text{Gamma}(a, b), \quad r = 1, \ldots, k, \\
&P(z_i = j \mid \pi, k) = \pi_j, \quad j = 1, \ldots, k, \ i = 1, \ldots, n, \\
&\pi \mid k \sim \text{Dirichlet}(\gamma, \ldots, \gamma), \\
&N(A_i) \mid z, \lambda, k \overset{\text{ind}}{\sim} \text{Poisson}(\lambda_{z_i}), \quad i = 1, \ldots, n,
\end{aligned}
$$

Estimating Parameter Space

Maximum Likelihood Estimate

- Our goal is to **maximize the posterior distribution**

$$\tilde{\lambda}_i = \underset{\lambda_i}{\operatorname{argmax}}\{p_i(\lambda_i|x)\}$$

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$$= \underset{\lambda_i}{\operatorname{argmin}}\{-\log p_i(\lambda_i|x)\}$$

Overview

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Progress
- Data Analysis
- **Model Construction**

Next Steps

Estimating Parameter Space

Maximum Likelihood Estimate

- Using Bayesian Inference,

$$\tilde{\lambda}_i = \underset{\lambda_i}{\operatorname{argmin}}\{-\log p_i(x|\lambda_i) - \log f_i(\lambda_i)\}$$

Estimating Parameter Space

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Maximum Likelihood Estimate

- Using Bayesian Inference,

$$\tilde{\lambda}_i = \underset{\lambda_i}{\operatorname{argmin}}\{-\log p_i(x|\lambda_i) - \log f_i(\lambda_i)\}$$

$$= \underset{\lambda_i}{\operatorname{argmin}}\left\{-\sum_{j=1}^{M}\Big(\log p_i(x_j|\lambda_i(x_j)) + \log f_i(\lambda_i(x_j))\Big)\right\}$$

Algorithm

Step 1

- Take the set of data points and draw circles $C_1, \ldots, C_N$ around the **point clusters**.

Overview

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Algorithm

Step 1

- Take the set of data points and draw circles $C_1, \ldots, C_N$ around the **point clusters**.
  - Clusters should be at the center of the circle and the data points become more sparse as distance increases from the center.

Overview

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Algorithm

Step 1

- Take the set of data points and draw circles $C_1, \ldots, C_N$ around the **point clusters**.
    - Clusters should be at the center of the circle and the data points become more sparse as distance increases from the center.
- In our case, this was done by hand.
    - There is no ideal algorithm to capture all clusters.

Algorithm

Overview

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Next Steps

Step 2

- Estimate the $\lambda(x)$ values for our data points inside the circle:

$$\hat{\lambda}_i(x_j) = \hat{\lambda}_i e^{-dr^2(x_j)}$$

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Algorithm

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- Estimate the $\lambda(x)$ values for our data points inside the circle:

$$\hat{\lambda}_i(x_j) = \hat{\lambda}_i e^{-dr^2(x_j)}$$

- Recall that $\hat{\lambda}_i$ is the estimated value for the circle C_i.
- This gives us an initial **expected** number of points around x_j.

Algorithm

Step 3

- Next, use the values $\hat{\lambda}_i(x_j)$ to generate the (initial) **number of observed events** k_j for each point x_j.

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Next Steps

Algorithm

Step 3

- Next, use the values $\hat{\lambda}_i(x_j)$ to generate the (initial) **number of observed events** k_j for each point x_j.
  - This is done by creating a Poisson random variable with mean $\hat{\lambda}_i(x_j)$.

Algorithm

Step 3

- Next, use the values $\hat{\lambda}_i(x_j)$ to generate the (initial) **number of observed events** k_j for each point x_j.
    - This is done by creating a Poisson random variable with mean $\hat{\lambda}_i(x_j)$.
    - It gives an idea of the number of data points we should find around x_j.

Overview

Motivation

Progress
- Data Analysis
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Next Steps

Algorithm

Step 4

- Use our calculations for the MLE to complete the optimization

$$\tilde{\lambda}_i = \underset{\lambda_i}{\operatorname{argmax}}\{p_i(\lambda_i|x)\}$$

Overview

Motivation

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- Data Analysis
- **Model Construction**

Next Steps

Algorithm

Step 4

- Use our calculations for the MLE to complete the optimization

$$\tilde{\lambda}_i = \underset{\lambda_i}{\operatorname{argmax}}\{p_i(\lambda_i|x)\}$$

  - This also gives us the values $a_{\tilde{\lambda}_i}, b_{\tilde{\lambda}_i}$ for the prior (Gamma) distribution.

Overview

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Next Steps

Algorithm

Step 5

- To create simulations, start with a **uniformly distributed** set of data points $Y = \{y_1, \ldots, y_S\}$ inside our circle.

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Algorithm

Step 5

- To create simulations, start with a **uniformly distributed** set of data points $Y = \{y_1, \ldots, y_S\}$ inside our circle.
    - The number of points should be **proportional to the optimal value** $\tilde{\lambda}_i$.
- Use the optimal values $a_{\tilde{\lambda}_i}, b_{\tilde{\lambda}_i}$ for the Gamma distribution to generate samples from our prior distribution

$$\lambda_k \sim Gamma(a_{\tilde{\lambda}_i}, b_{\tilde{\lambda}_i})$$

Algorithm

Step 5

- These new λ_k samples are used to calculate the distance-adjusted $\lambda_k(y_k)$ values for our new points.

Algorithm

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Algorithm

Step 5

- These new λ_k samples are used to calculate the distance-adjusted $\lambda_k(y_k)$ values for our new points.
    - **Remove** the new points y_k whose lambda value $\lambda_k(y_k)$ is below some tolerance (default: 0.01).
- The remaining points will (approximately) live in the distribution of our data.

Applying the model to the data

Overview

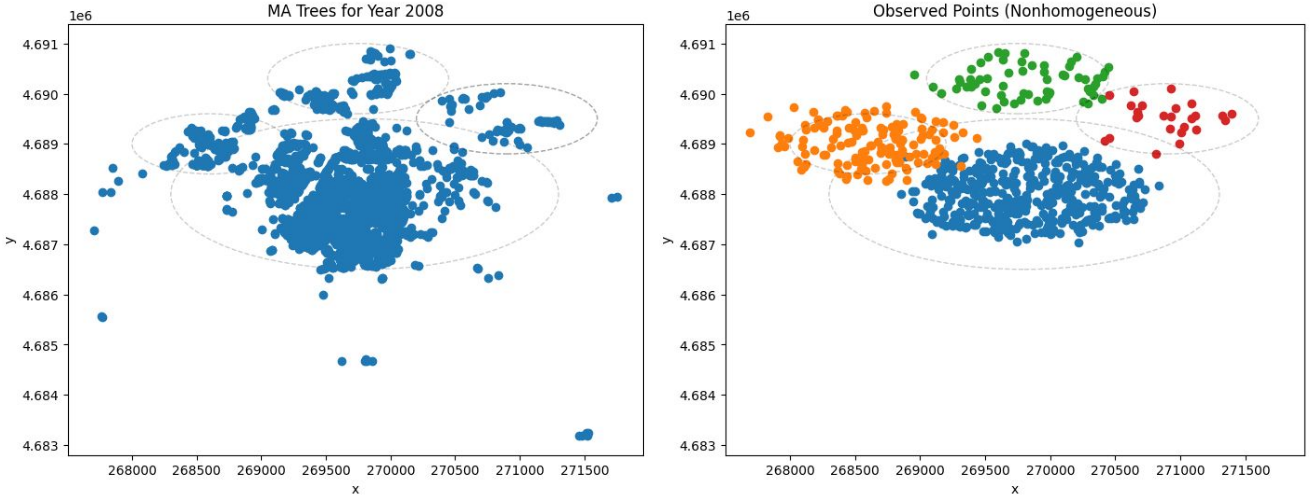
Motivation

Progress

- Data Analysis
- **Model Construction**

Next Steps

Results: 2008 Trees (*decay rate = 1e-5*)



Applying the model to the data

Overview

Motivation

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- Data Analysis
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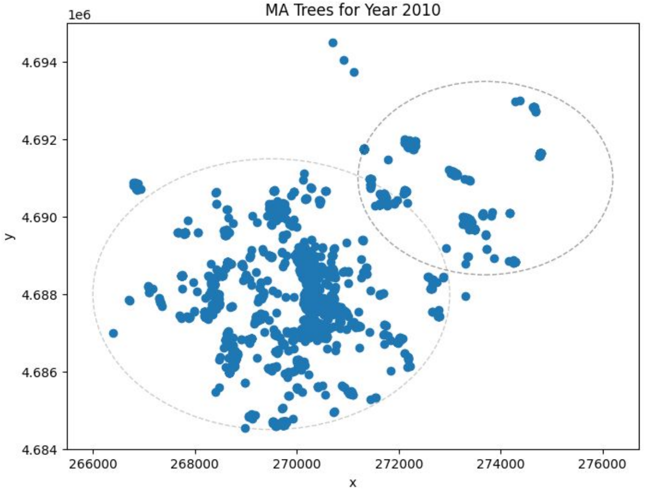
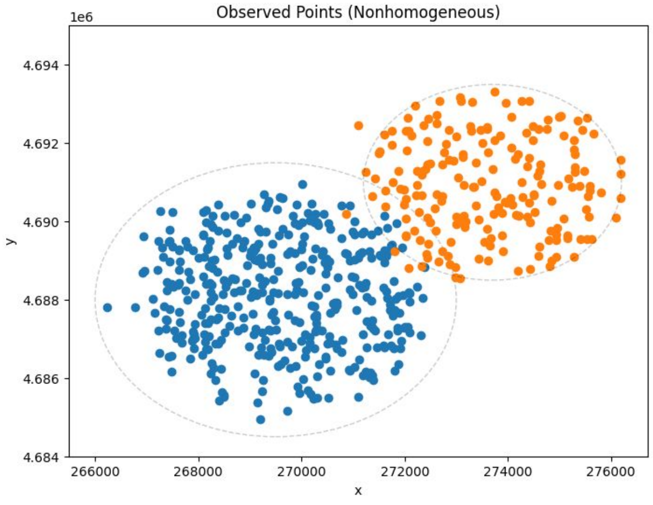
Next Steps

Results: 2010 Trees (*decay rate = 1e-6*)





Applying the model to the data

Overview

Motivation

Progress

- Data Analysis
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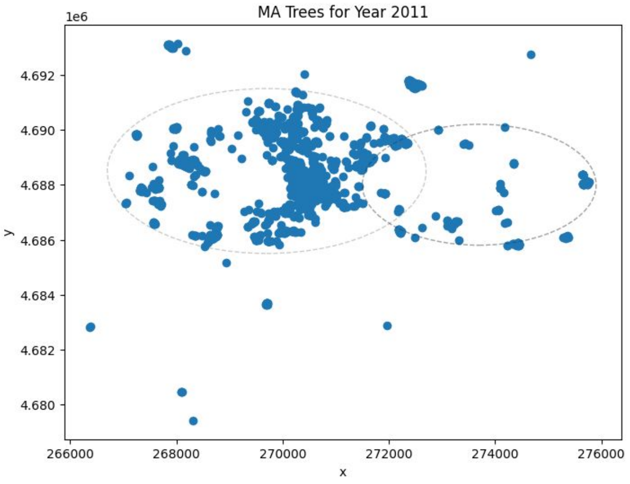
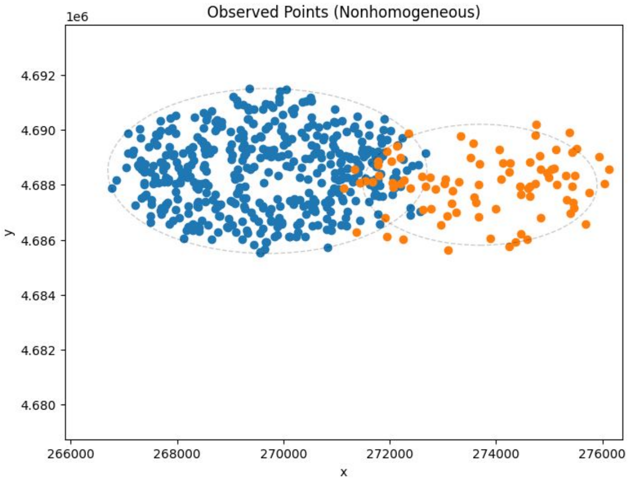
Next Steps

Results: 2011 Trees (*decay rate = 1e-6*)





Applying the model to the data

Overview

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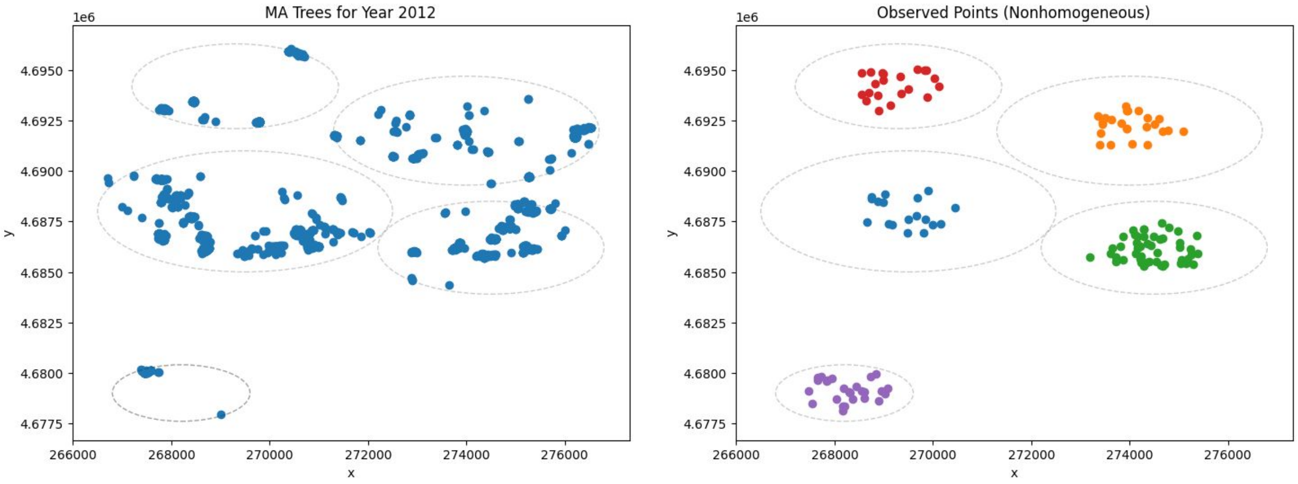
Next Steps

Results: 2012 Trees (*decay rate = 5e-6*)



Model Pitfalls

Ambiguity in the Algorithm

- There is no set way to identify the point clusters and create circles around them.

Model Pitfalls

Ambiguity in the Algorithm

- There is no set way to identify the point clusters and create circles around them.
- The decay rate *d* may need to be different for each circle.

Model Pitfalls

Overview

Motivation

Progress

- Data Analysis
- **Model Construction**

Next Steps

Ambiguity in the Algorithm

- There is no set way to identify the point clusters and create circles around them.
- The decay rate *d* may need to be different for each circle.
- The number of simulated points in each region can be better defined. We currently use

$$S = \hat{\lambda}_i$$

Next Steps

Overview

Motivation

Progress
- Data Analysis
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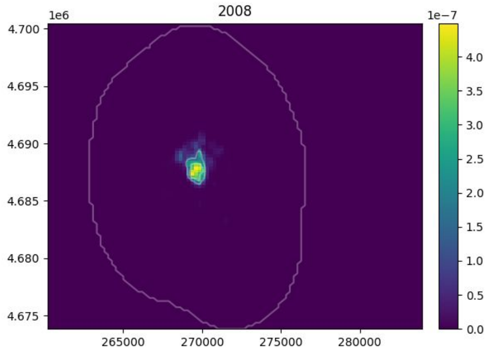
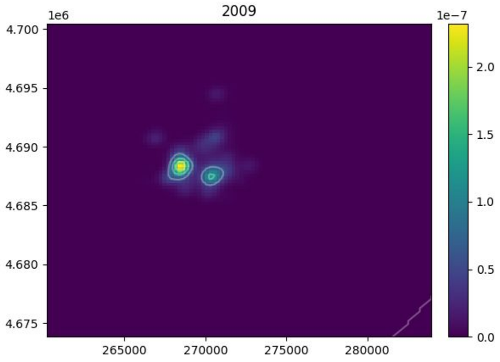
Next Steps

KDE for Cluster Identification

- To identify the best places for circles, we can use the 2-D KDE estimation heat maps.





Next Steps

Optimal Decay Rate and Number of Points

- We would like to find the best values to use for the decay rate d and the number of points S to generate for our simulations.

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    - A larger decay rate d will mean more points are removed as the data moves further from the center.

Next Steps

Optimal Decay Rate and Number of Points

- We would like to find the best values to use for the decay rate *d* and the number of points *S* to generate for our simulations.
    - A larger decay rate *d* will mean more points are removed as the data moves further from the center.

- We want to find the **ideal balance** between these values.

Thank you!

Next Steps

Better Model

- From the paper "Bayesian Nonparametric Nonhomogeneous Poisson Process..." (arXiv:1907.03186):

$$
\begin{aligned}
&k \sim p(\cdot), \text{where } p(\cdot) \text{ is a p.m.f on } \{1, 2, \ldots\} \\
&\lambda_r \overset{\text{ind}}{\sim} \text{Gamma}(a, b), \quad r = 1, \ldots, k, \\
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&N(A_i) \mid z, \lambda, k \overset{\text{ind}}{\sim} \text{Poisson}(\lambda_{z_i}), \quad i = 1, \ldots, n,
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