Synthetic Wavefront Generation for Aero-Optics Correction

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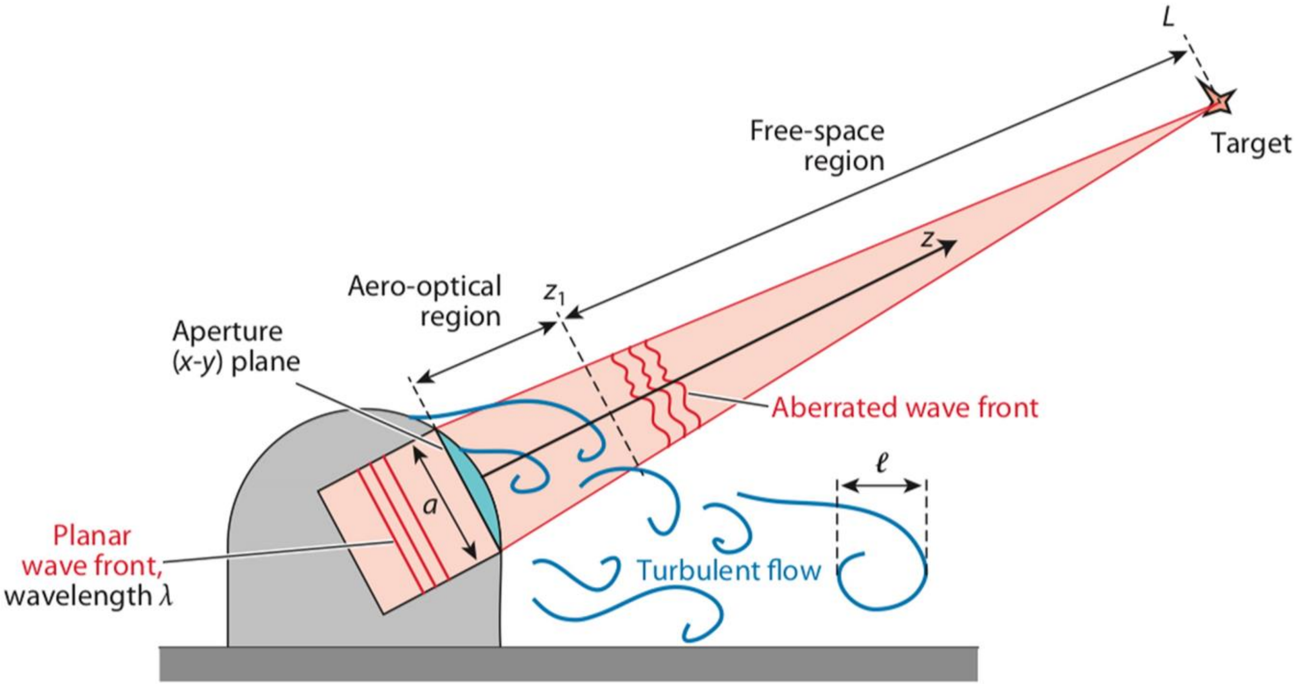
The Aero-Optics Problem

Turbulent flow over an aircraft causes wavefront aberrations in propagated laser beams.



The Aero-Optics Problem, Figure 1 from [1]

[1] M. Wang, A. Mani, and S. Gordeyev, "Physics and Computation of Aero-Optics," *Annual Review of Fluid Mechanics*, Vol. 44, No. 1, 2012, pp. 299–321.

Adaptive Optics for Aero-Optical Effects

- The Kolmogorov-Taylor model is **not sufficient** to describe aero-optics ([2]).
  - Development of Adaptive Optics (AO) systems for atmospheric turbulence *relies on* this model ([2]).

- We *need wavefront data* to accurately design AO systems for addressing aero-optic effects.
  - Wind tunnel experiments are *costly;* data acquisition is *time-limited*.

- **We implement an algorithm for generating synthetic wavefronts on long time-scales.**

[2] C. Vogel, G. Tyler, and D. Wittich, "Spatial-temporal-covariance-based modeling, analysis, and simulation of aero-optics wavefront aberrations," *J. Opt. Soc. Am. A*, Vol. 31, No. 7, 2014, pp. 1666-1679.

Metrics for Synthetic Wavefront

Our synthetic wavefront needs to *match the following*:

1. (Temporal) *Power Spectral Density* (PSD) of **deflection angle** $\boldsymbol{\theta_x}$: $S_{\theta_x}(f)$.

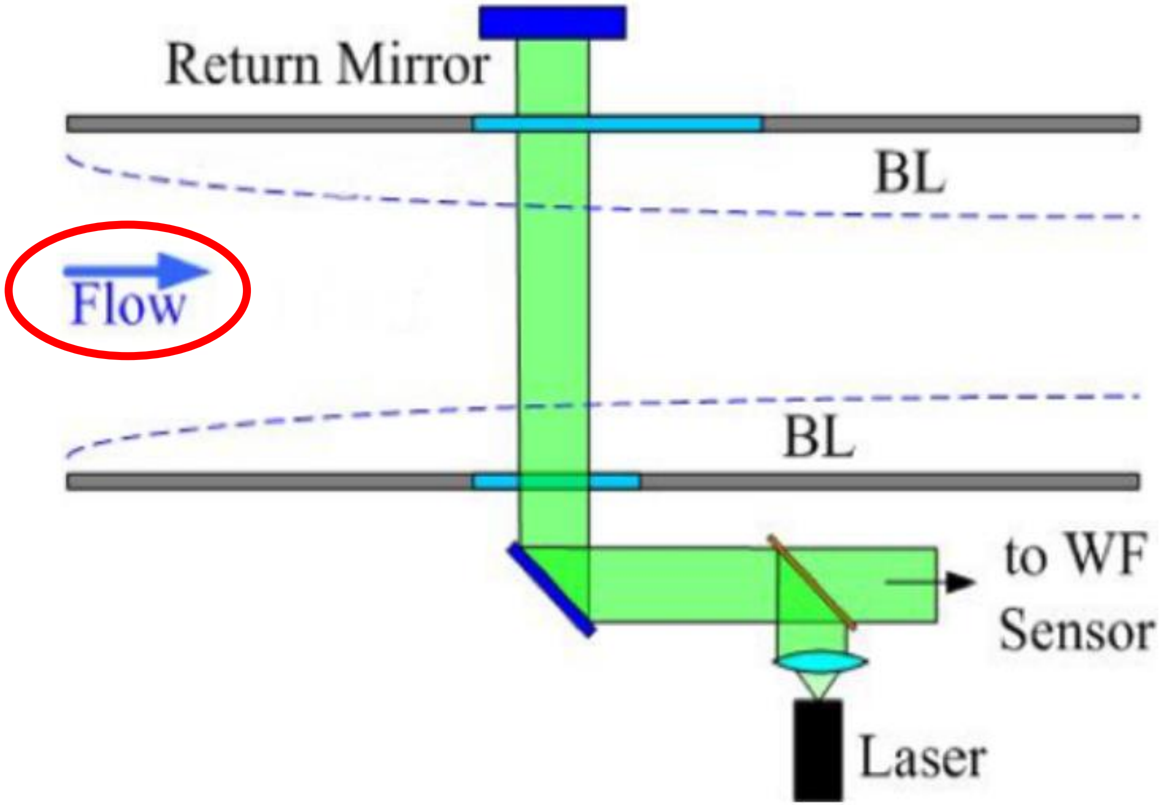
2. (Temporal) PSD of **optical path difference (OPD)**: $S_{OPD}(f)$.

3. Spatial Structure Function for *Kolmogorov turbulence* ([2]), evaluated on **optical path difference (OPD)**.

$$D_{OPD}(r) = \langle [OPD(\boldsymbol{x} + \boldsymbol{\Delta x}) - OPD(\boldsymbol{x})]^2 \rangle \quad (r = |\boldsymbol{\Delta x}|)$$

[2] C. Vogel, G. Tyler, and D. Wittich, "Spatial-temporal-covariance-based modeling, analysis, and simulation of aero-optics wavefront aberrations," *J. Opt. Soc. Am. A*, Vol. 31, No. 7, 2014, pp. 1666-1679.

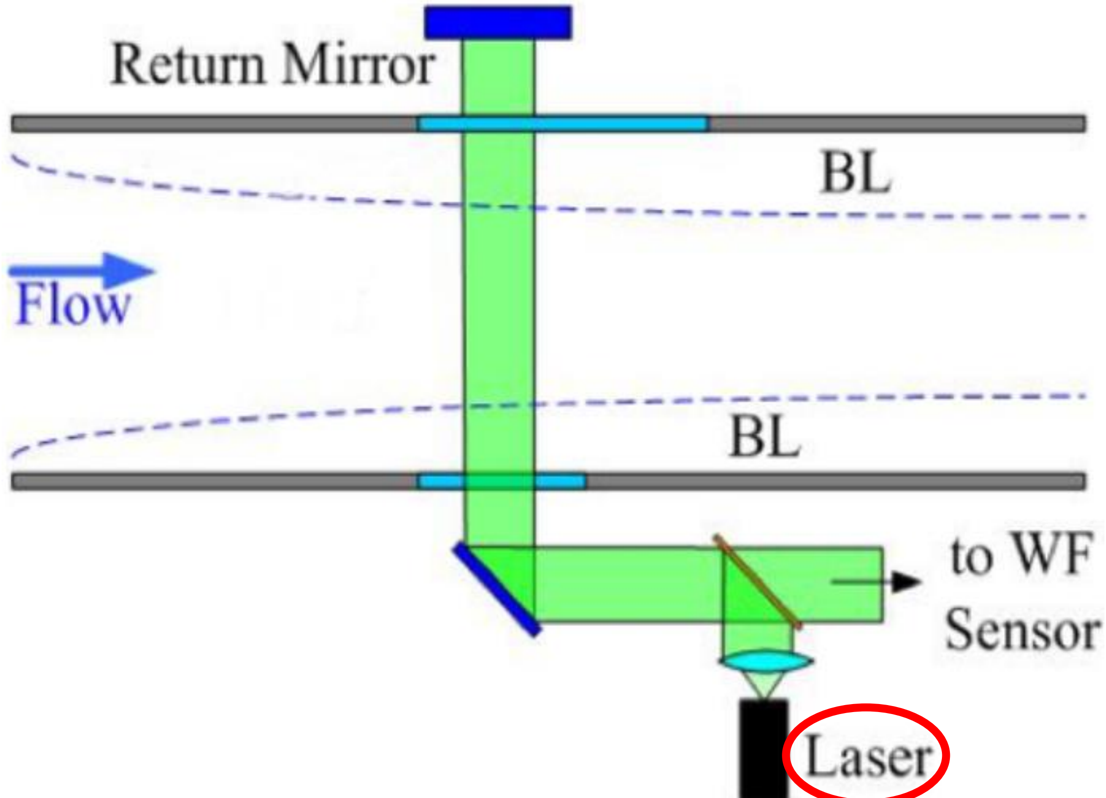
Experimental Data: Wind Tunnel



Notre Dame Wind Tunnel Experiment, Figure 1(b) from [3]

[3] M. R. Kemnetz and S. Gordeyev, "Optical investigation of large-scale boundary-layer structures", *54th AIAA Aerospace Sciences Meeting*, 4 - 8 Jan 2016, San Diego, California, AIAA Paper 2016-1460.

Experimental Data: Wind Tunnel



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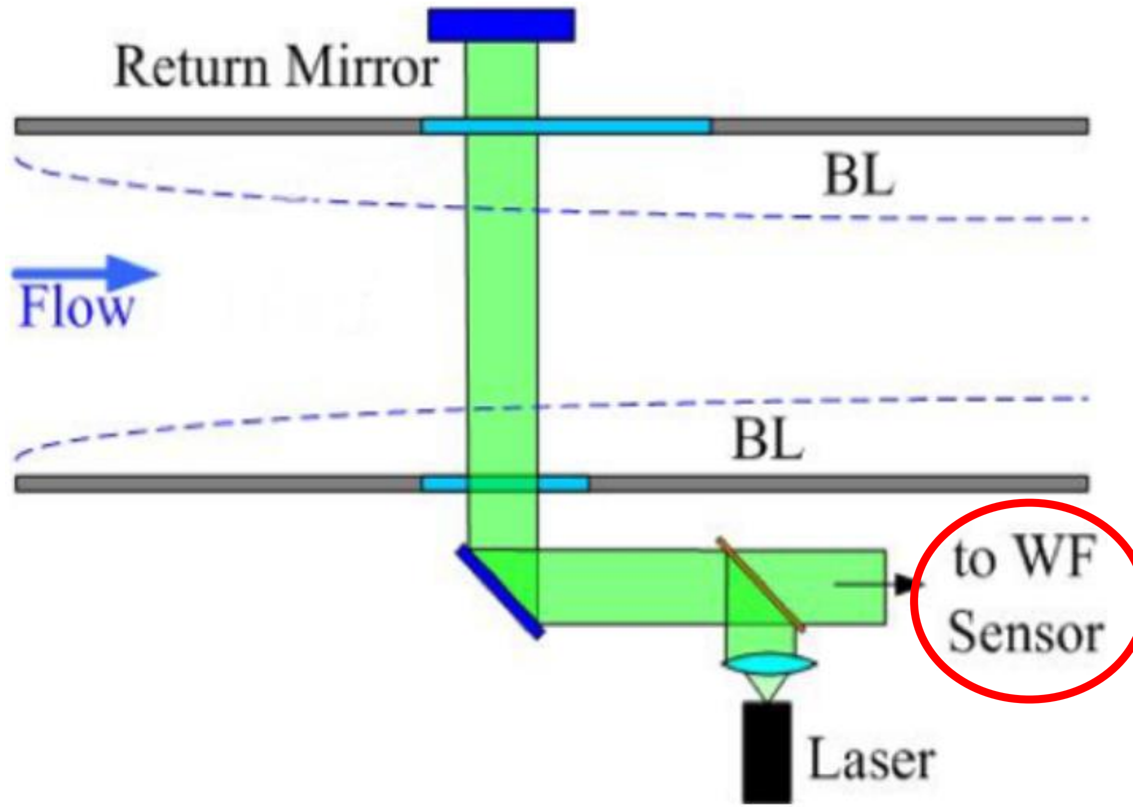
[3] M. R. Kemnetz and S. Gordeyev, "Optical investigation of large-scale boundary-layer structures", *54th AIAA Aerospace Sciences Meeting*, 4 - 8 Jan 2016, San Diego, California, AIAA Paper 2016-1460.

Experimental Data: Wind Tunnel

A **Shack-Hartman Wavefront Sensor** calculated optical path difference (OPD)



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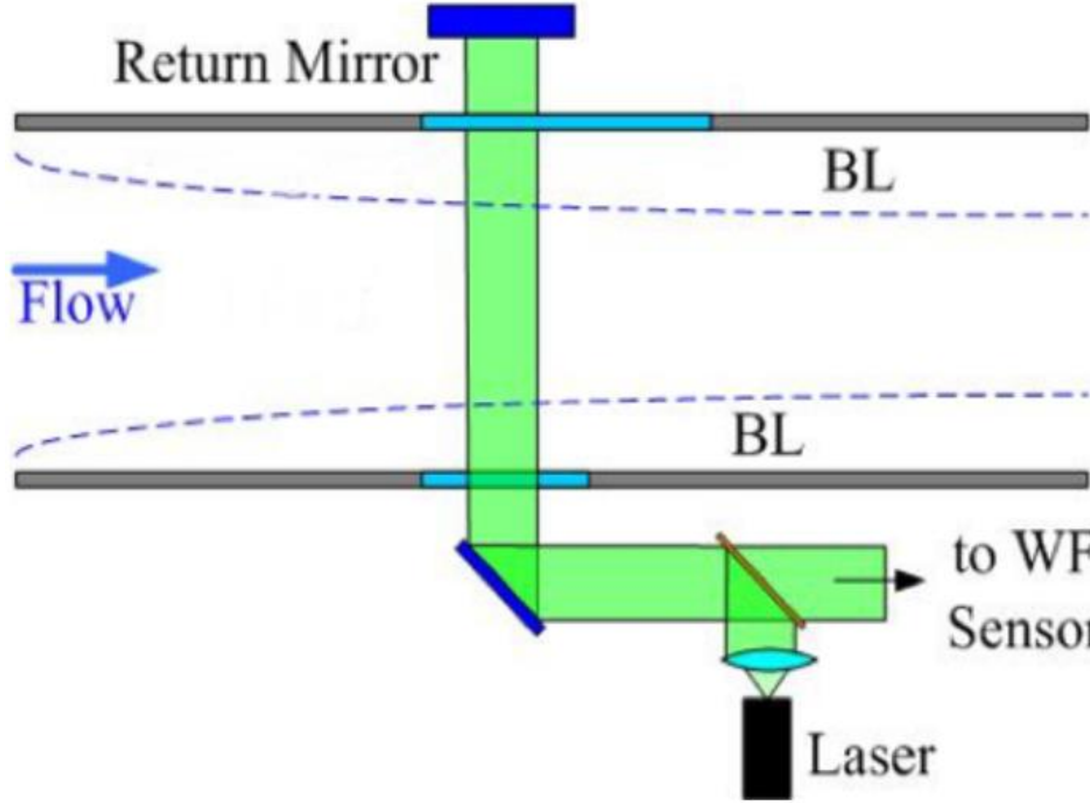
[3] M. R. Kemnetz and S. Gordeyev, "Optical investigation of large-scale boundary-layer structures", *54th AIAA Aerospace Sciences Meeting*, 4 - 8 Jan 2016, San Diego, California, AIAA Paper 2016-1460.

Experimental Data: Wind Tunnel

We have three experimental data sets:
▪ F04: **1.02 sec** of data ▪ F06: **1.51 sec** of data ▪ F12: **1.93 sec** of data



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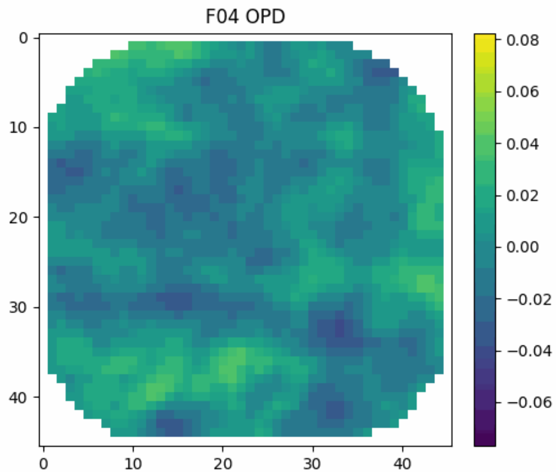
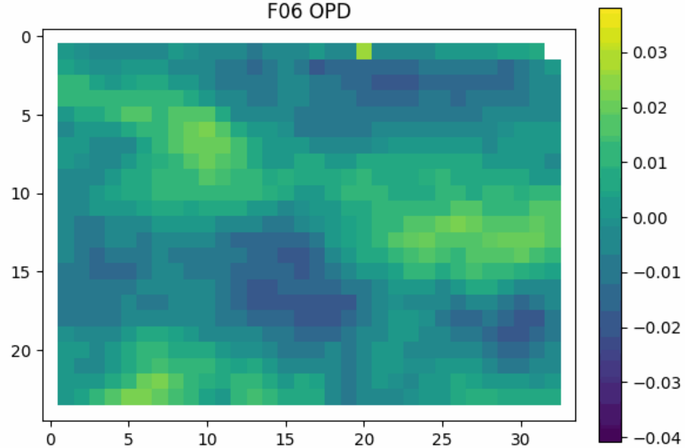
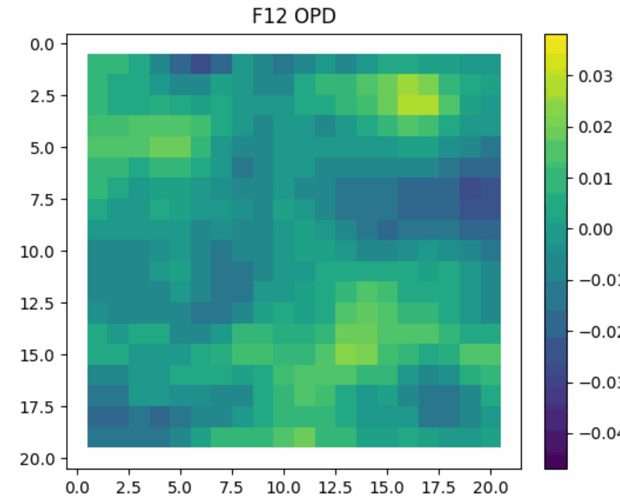
Experimental Data: Visualization

Quantize each sub-aperture as a pixel value.

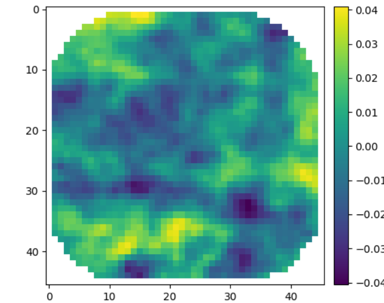






Statistical Model for OPD

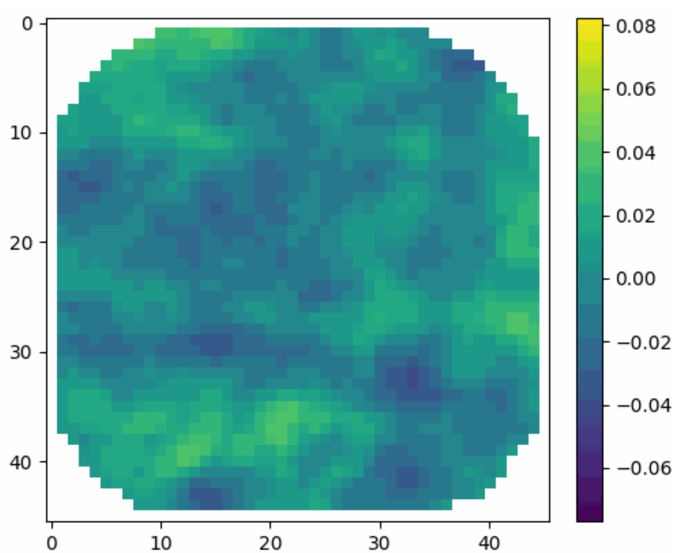
- We model $\boldsymbol{OPD}$ as a *zero-mean time-stationary Gaussian random process* $\boldsymbol{X_n}$.

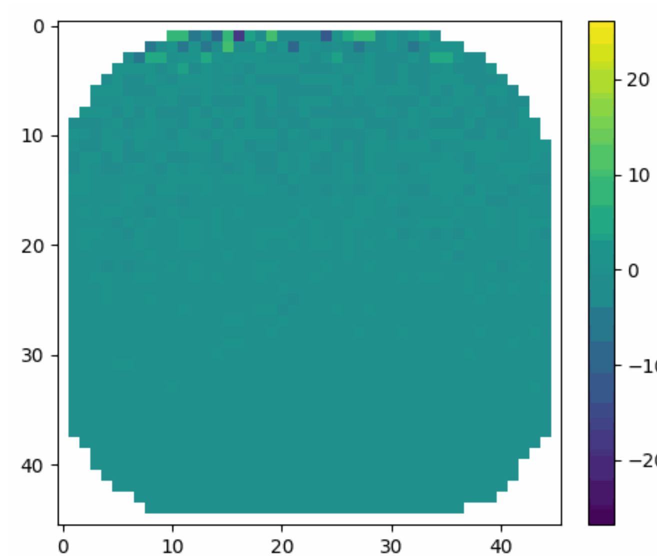


- We employ *Principal Component Analysis (PCA)* to *decorrelate* OPD values at each time:

Data Analysis: PCA

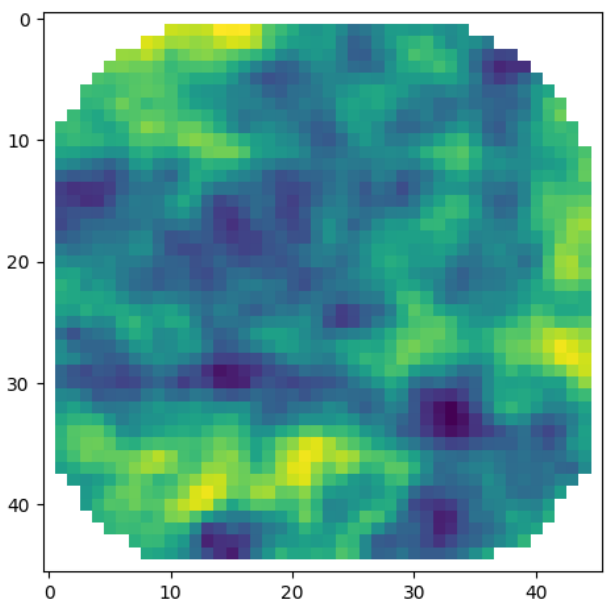
X_n ⇨ **PCA** ⇨ $\tilde{X}_n$

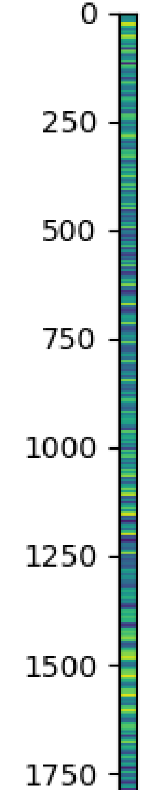




Converting Experimental Data

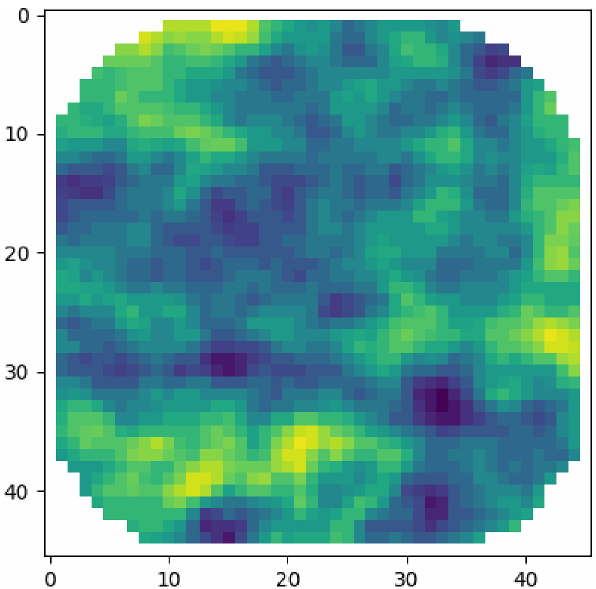
At each time step n, we convert the 2-D spatial OPD to a column vector in *raster order*.

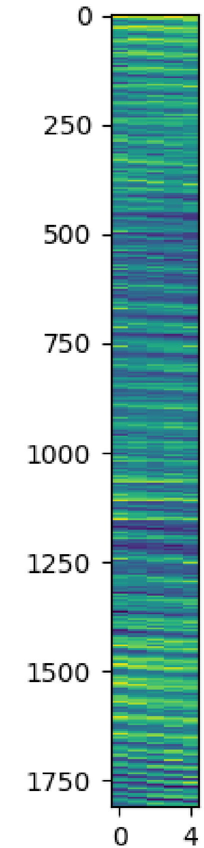




Converting Experimental Data

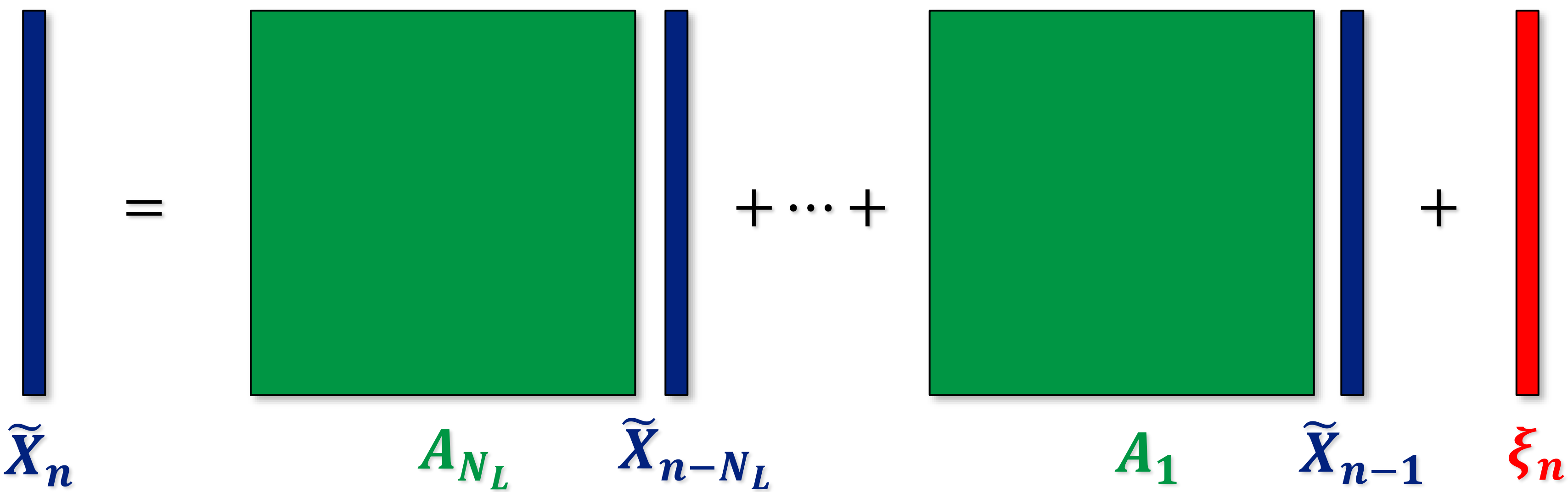
Convert the OPD time-series into a 2-D array





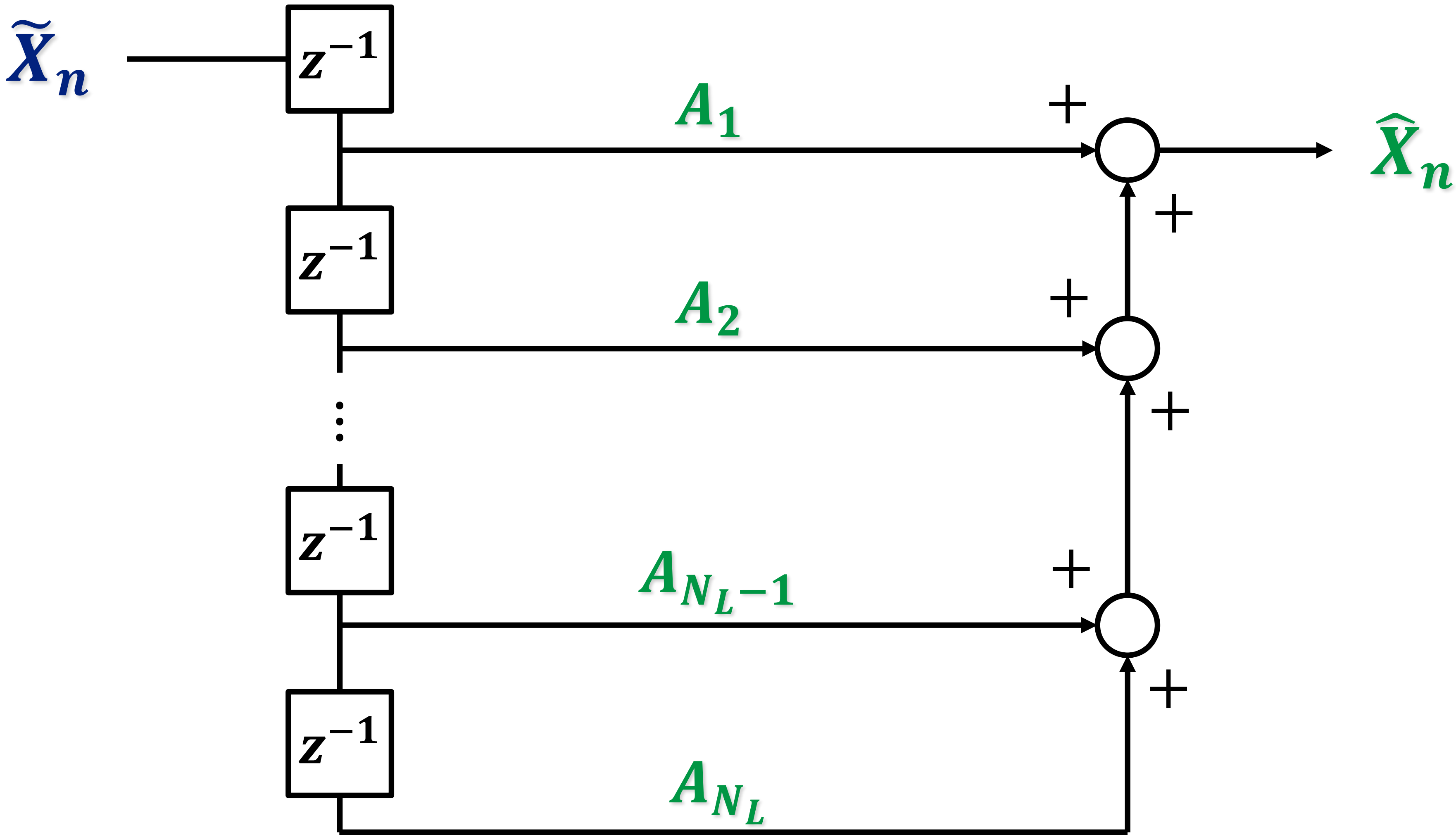
Vector Autoregressive (VAR) Modeling ([4])

$$\tilde{X}_n = \underbrace{A_{N_L}\tilde{X}_{n-N_L} + \cdots + A_1\tilde{X}_{n-1}}_{\text{Prediction}} + \underbrace{\xi_n}_{\text{Prediction Error}}$$

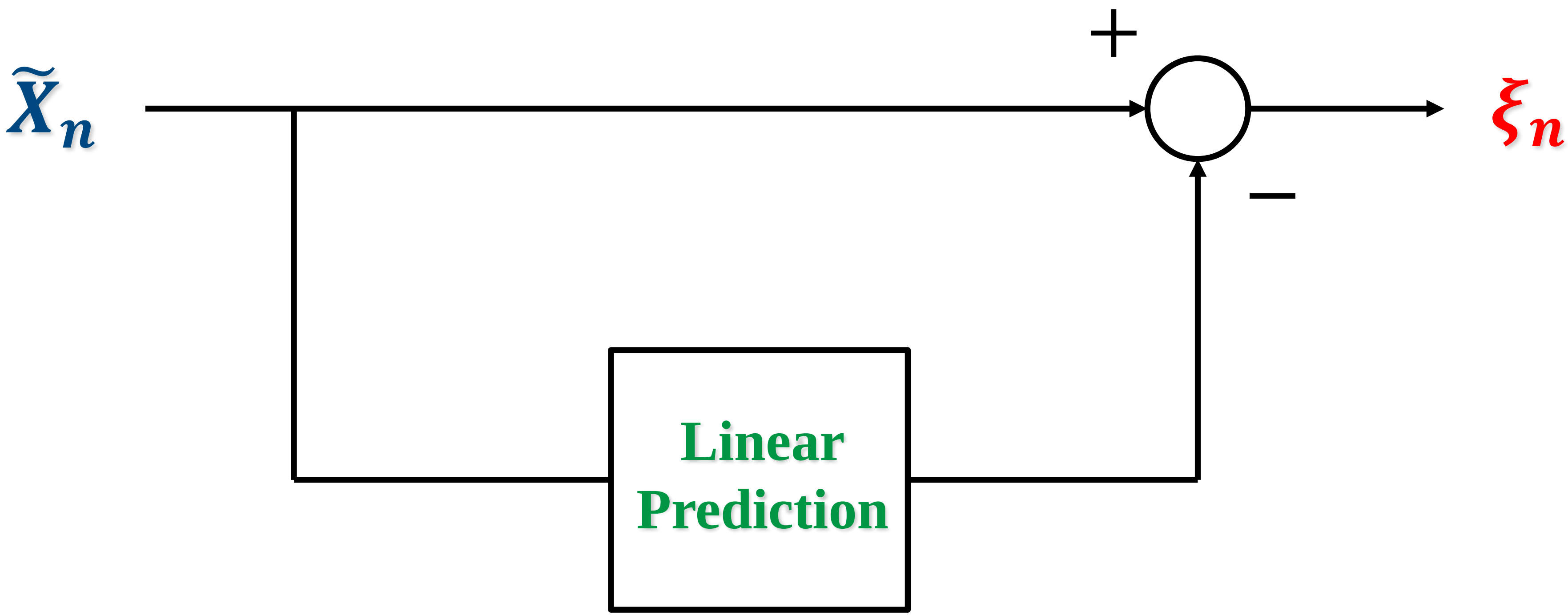


[4] H. Lütkepohl, *New Introduction to Multiple Time Series Analysis*, 2005, Springer-Verlag Berlin, pg. 13-14, 69-71.

VAR Modeling: Linear Prediction



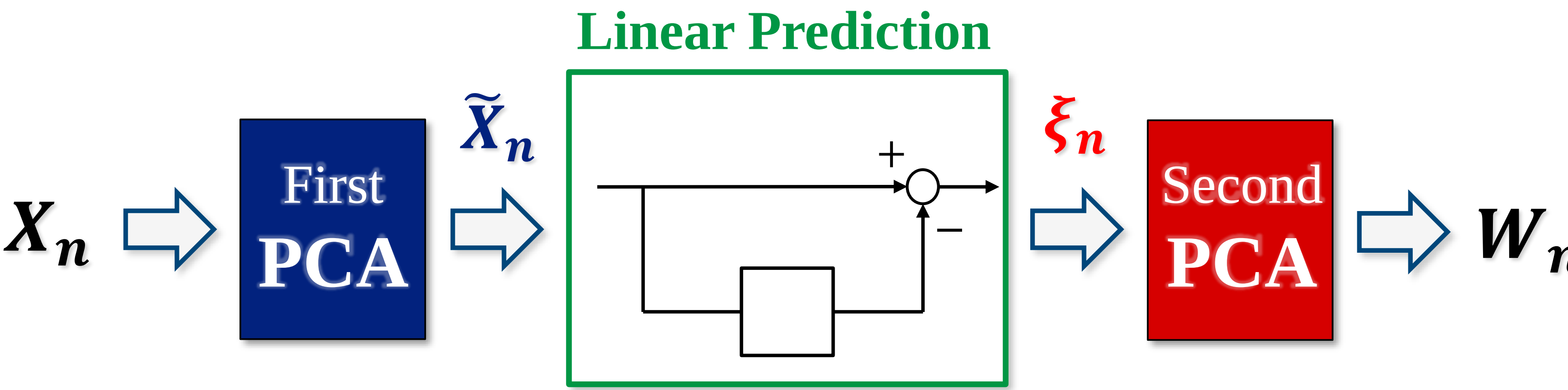
Data Analysis: Linear Prediction



ξ_n is **white** in *time*, **correlated** in *space*.

Data Analysis

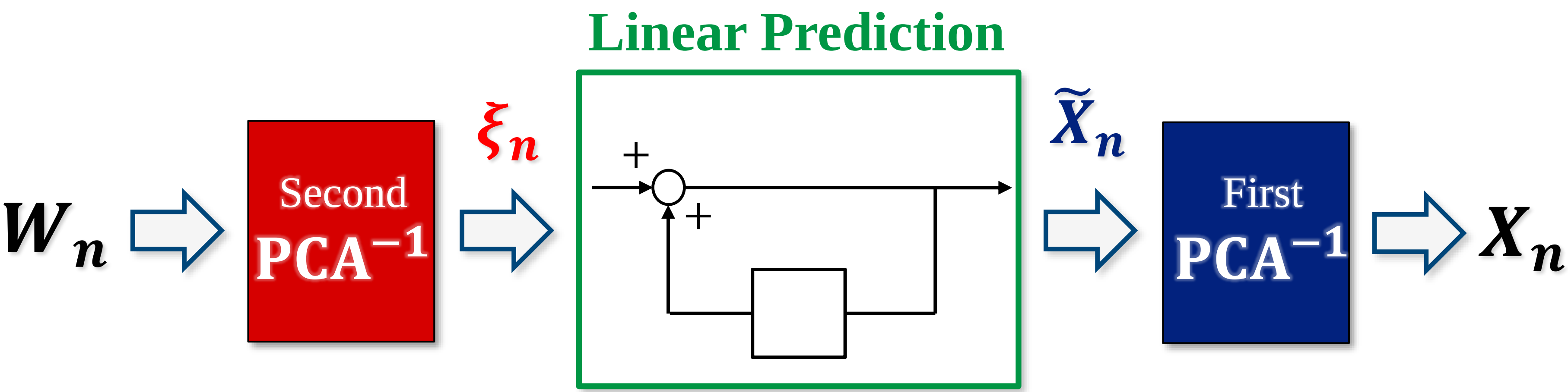
Use PCA to de-correlate ξ_n



W_n is white in *time* **and** *space*

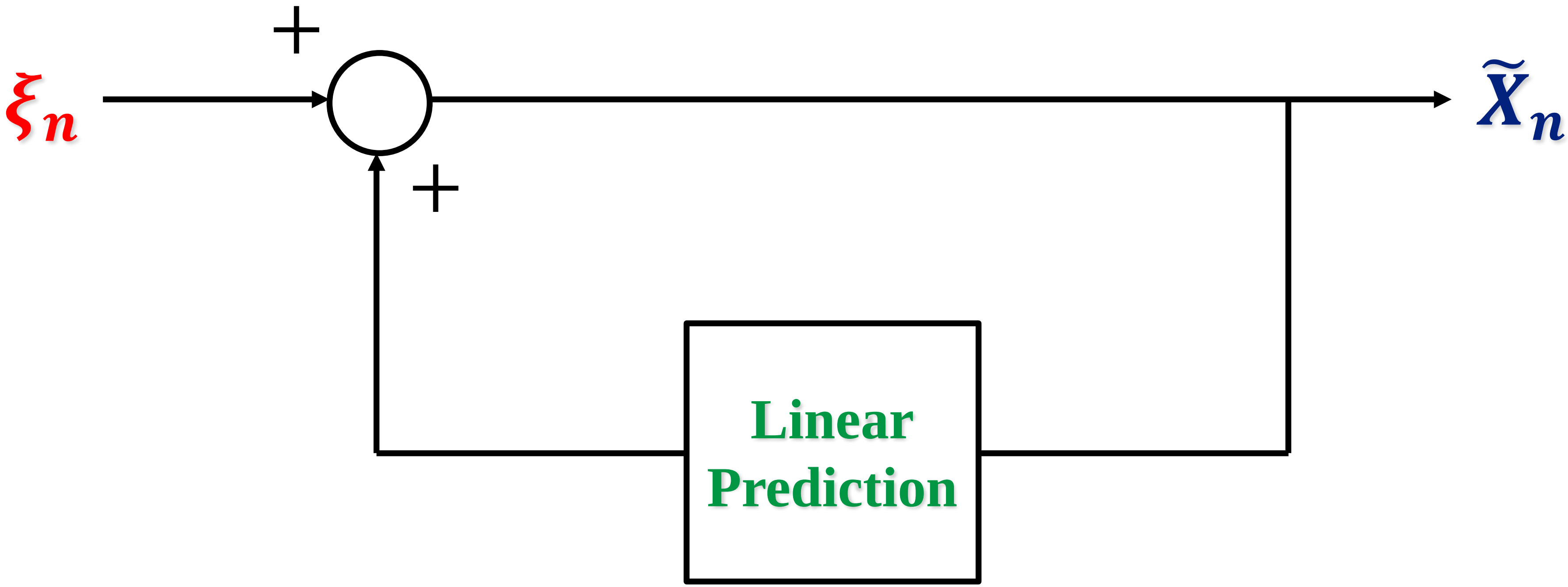
Synthesis

Drive model with **white noise** $\boldsymbol{W_n}$



Inverse PCA gives **correlated noise** $\boldsymbol{\xi_n}$

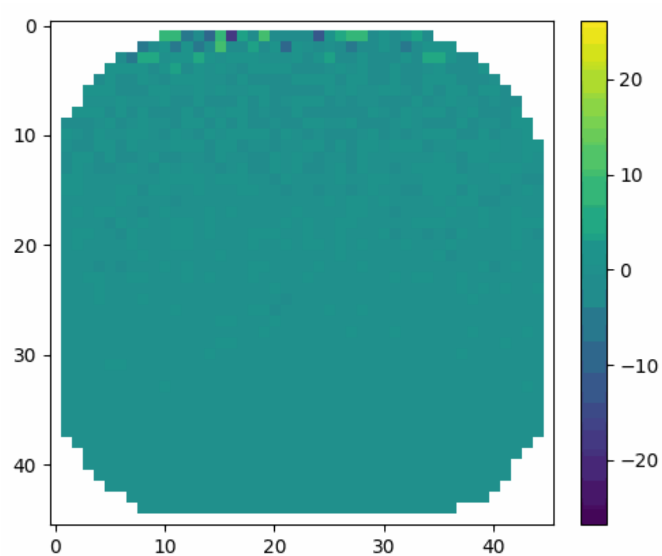
Synthesis: Linear Prediction

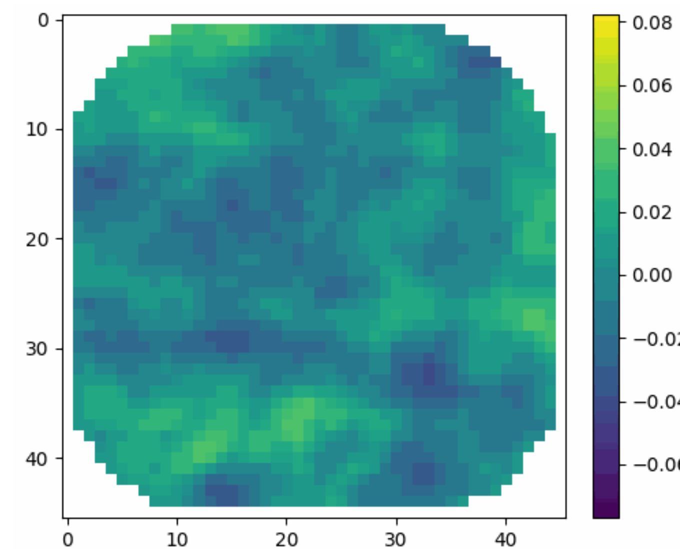


Use correlated noise ξ_n as **input**

Synthesis: Inverse PCA

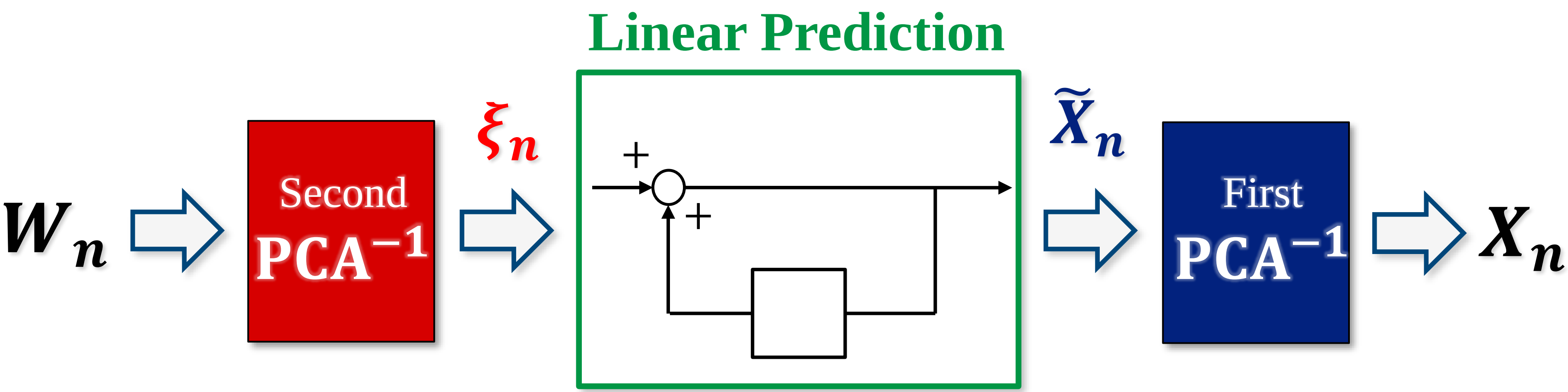
$\tilde{X}_n$ → First $\mathbf{PCA}^{-1}$ → X_n





Synthesis

Drive model with **white noise W_n**

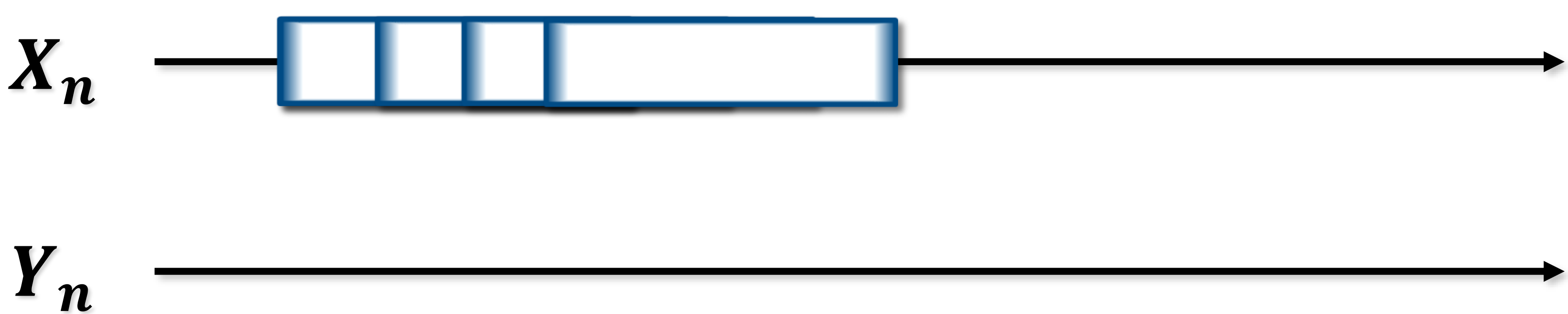


Inverse PCA gives **correlated noise ξ_n**

Post-Processing Synthetic Data

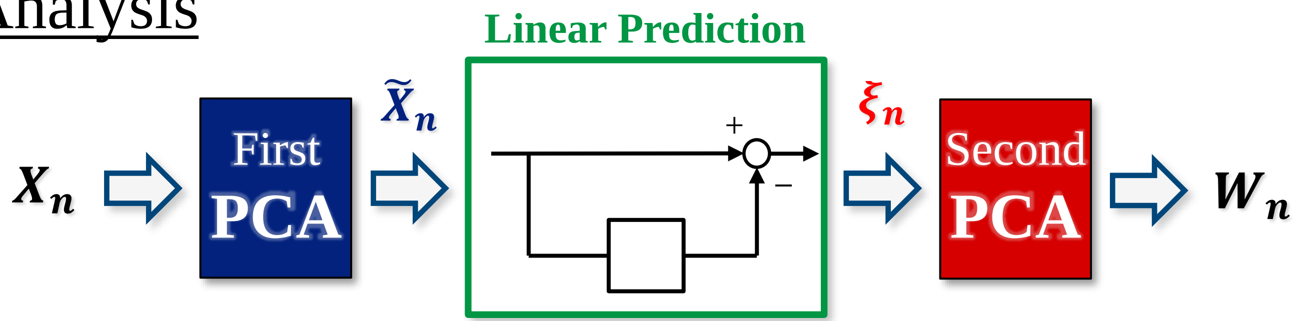
After generating synthetic data, remove a *(weighted) moving average:*

$$Y_n = X_n - \sum_k X_k h_k \qquad \left(\sum_k h_k = 1 \right)$$

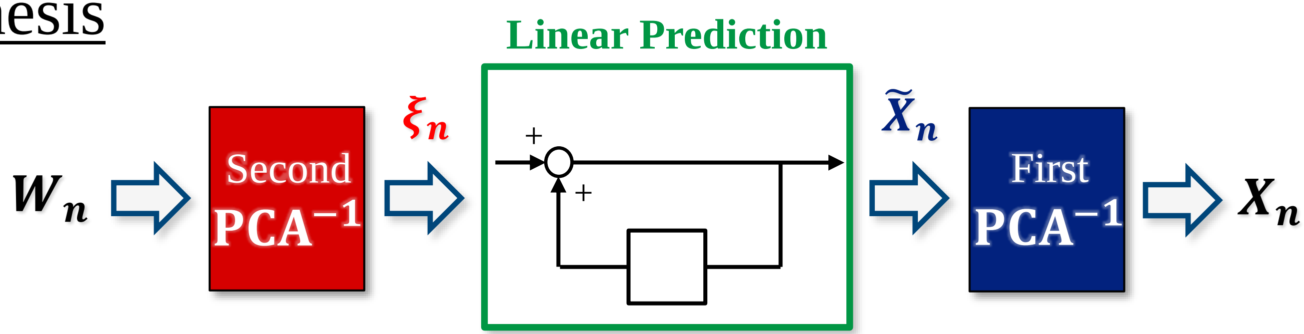


Algorithm Overview

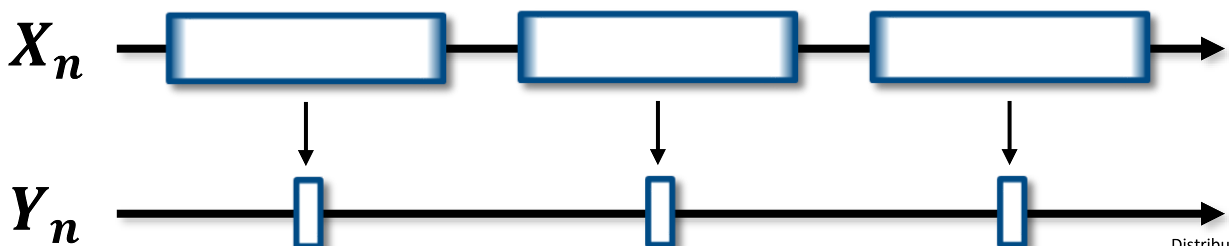
1. Data Analysis



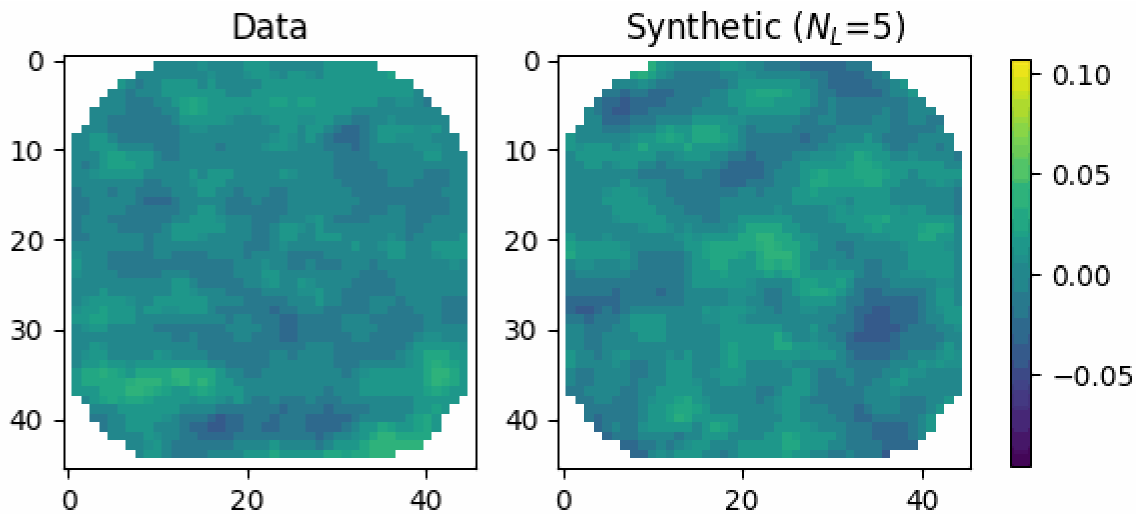
2. Synthesis

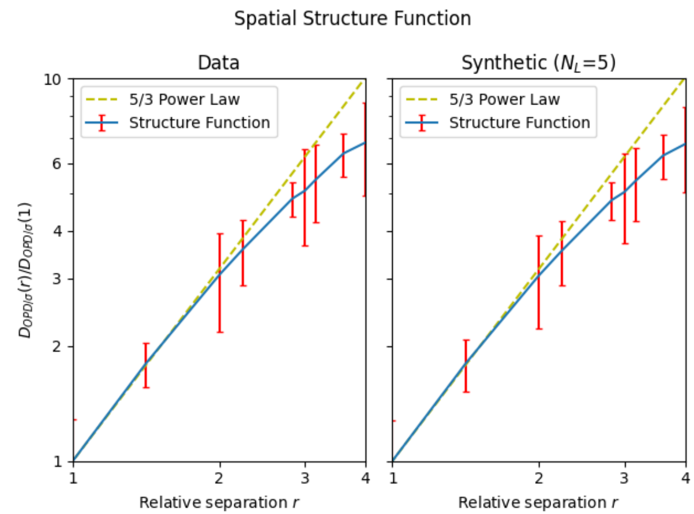


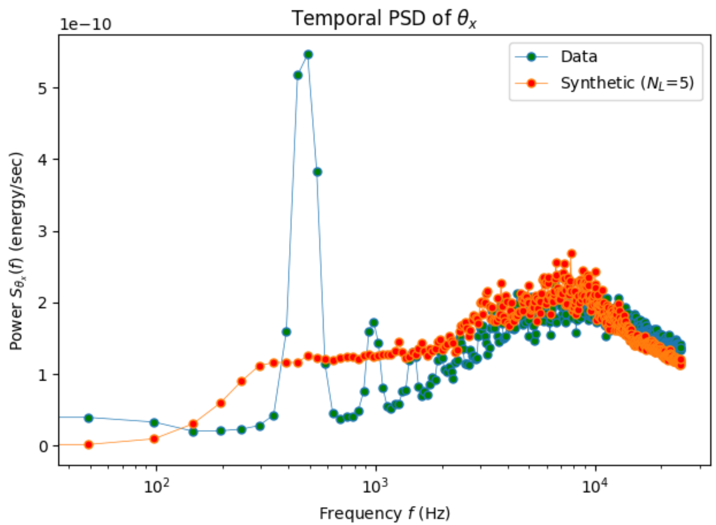
3. Post-processing

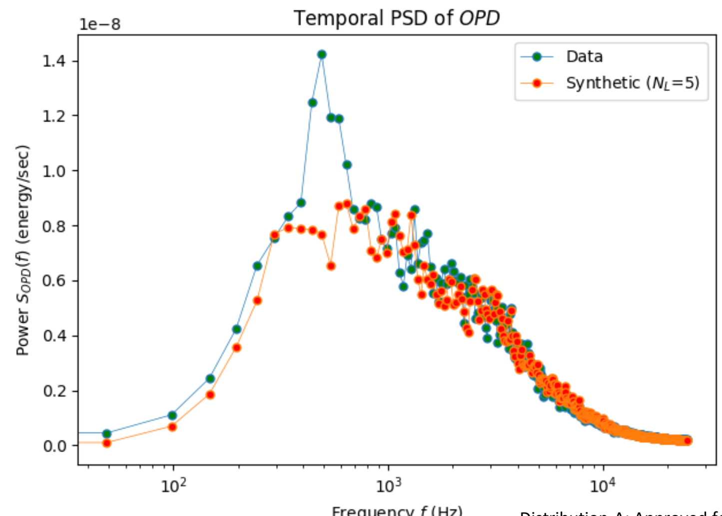


Results: Data Set F04 $(N_L = 5)$

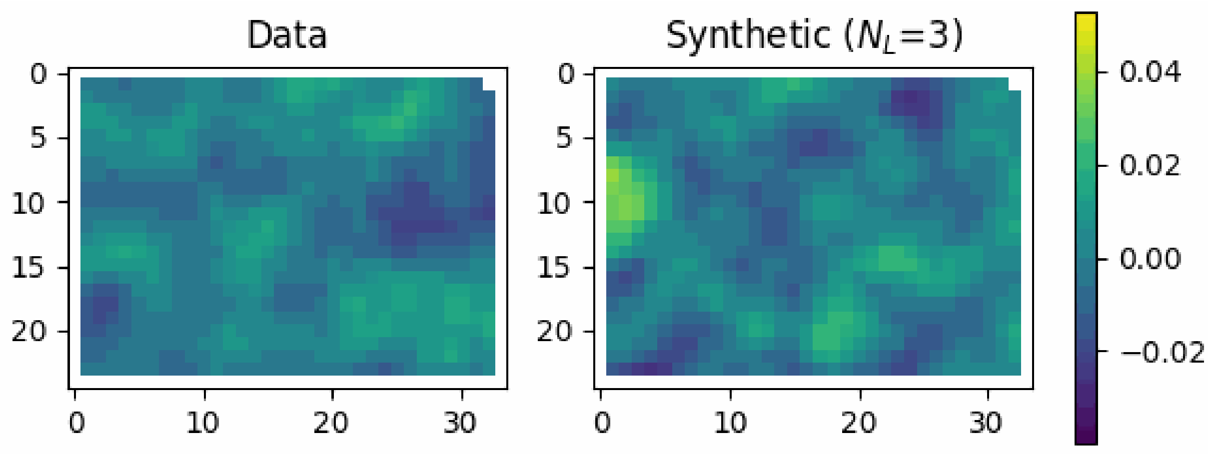


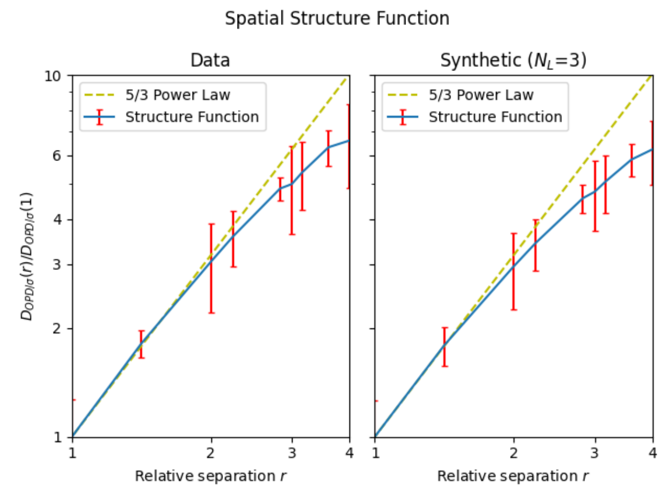


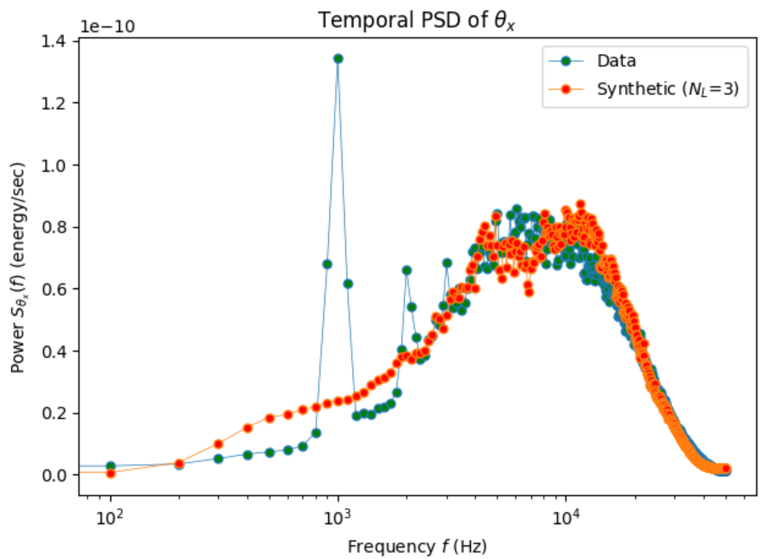


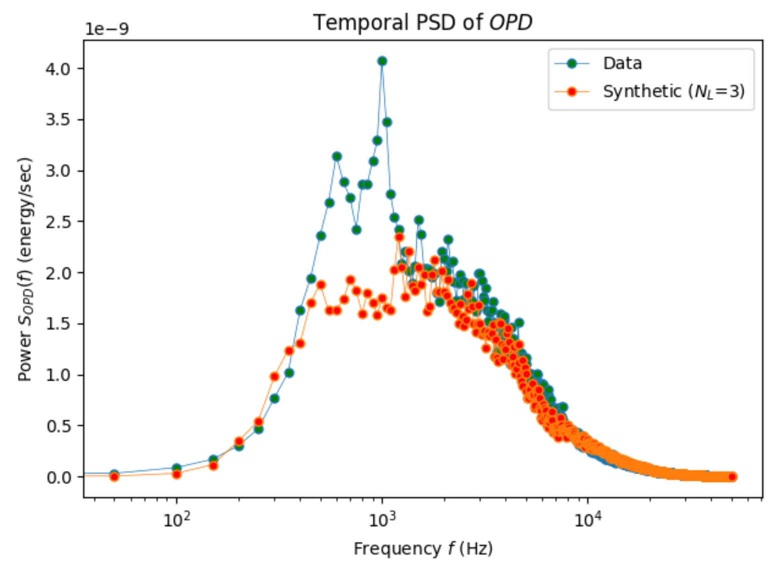


Results: Data Set F06 $(N_L = 3)$

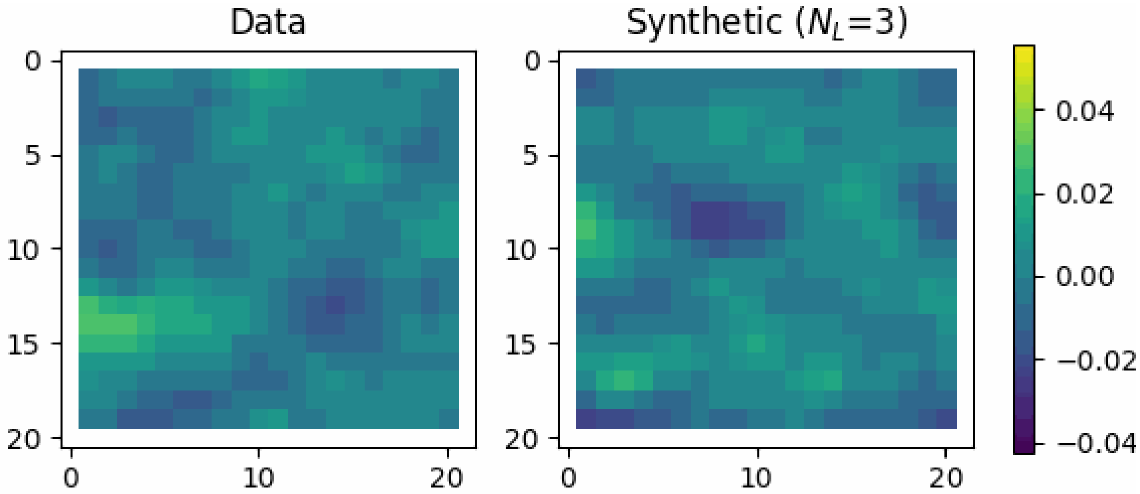


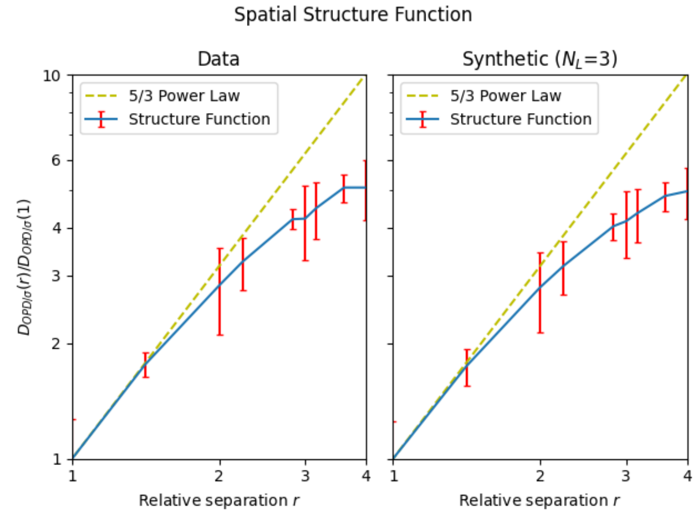


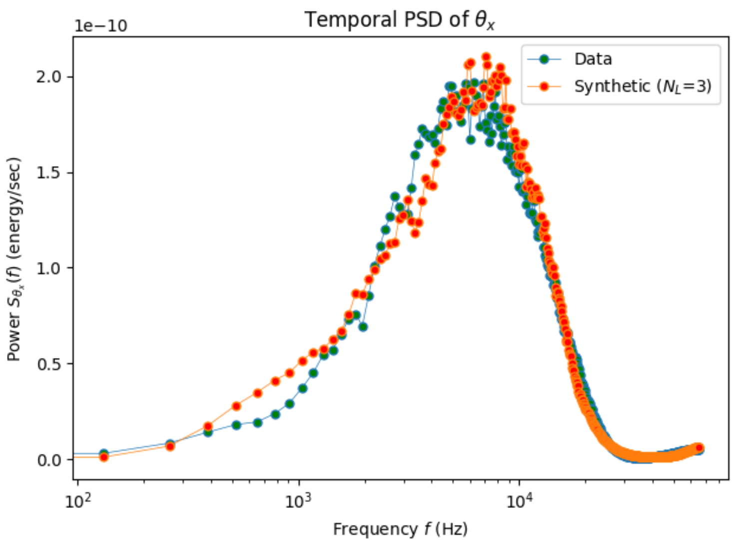


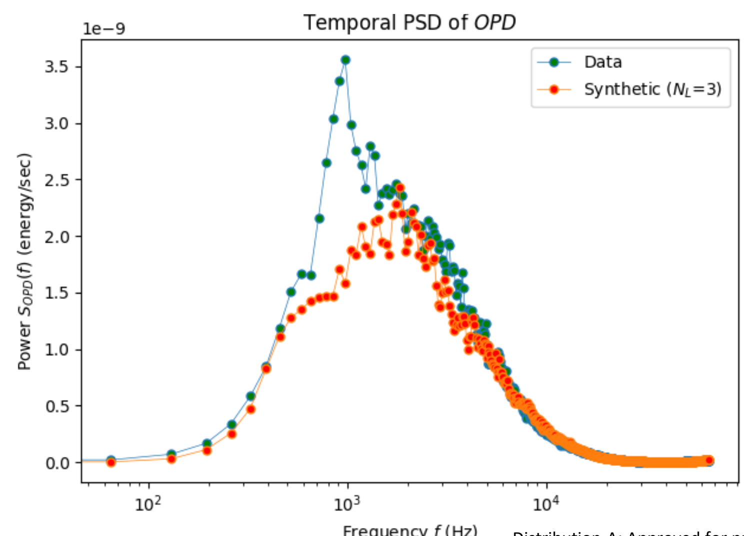


Results: Data Set F12 ($N_L = 3$)









Algorithm Run-Time

Data Set F04

- **6.5 mins** to *analyze* **0.8 sec** of *experimental* data
- **2.4 mins** to *generate* **1 sec** of *synthetic* data

Data Set F06

- **3.5 mins** to *analyze* **1.2 sec** of *experimental* data
- **53 sec** to *generate* **1 sec** of *synthetic* data

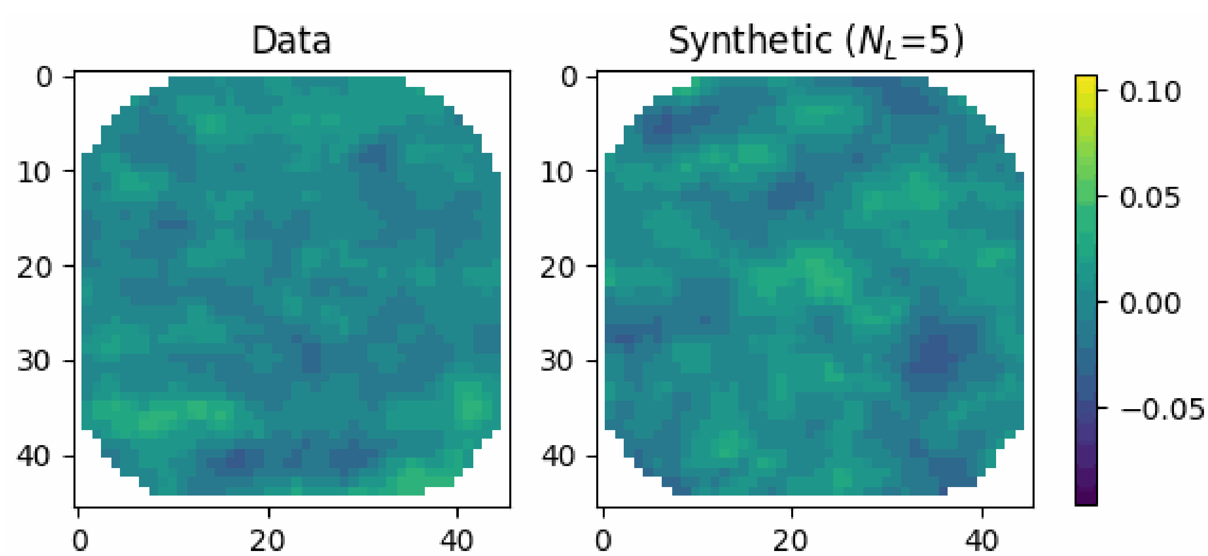
Data Set F12

- **3.3 mins** to *analyze* **1.6 sec** of *experimental* data
- **38 sec** to *generate* **1 sec** of *synthetic* data

Conclusion

- Development of AO systems to address aero-optic effects requires aberrated wavefront data on long time-scales.

- Our VAR Model algorithm is computationally efficient and generates high quality synthetic wavefronts.



References

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