Review Problems for 2nd Midterm, MA 214/ Spring 2012

The exam is 1 hour in class, which is Friday March 9, 1-1:50pm, CB 339.

It covers from Chapter 3.1 to Chapter 3.8, and consists of six problems.

Here is the list of some reviewing problems that may serve to help you prepare for the exam.

(1). Find general solutions to the equation

$$y'' - 2y' - 2y = 0.$$

(2). Find general solutions to the equation

$$y'' + \frac{1}{4}y' + \frac{5}{32}y = 0.$$

(3). Find the solution to the initial value problem for

$$\begin{aligned} 9y'' - 30y' + 25y &= 0 \\ y(0) &= 1 \\ y'(0) &= -1. \end{aligned}$$

(4) Compute the Wronskian of $f(t) = e^{5t}\cos(6t)$ and $g(t) = e^{5t}\sin(6t)$.

(5) If the Wronskian W of f and g is t^2e^t, and if $f(t) = t$, find $g(t)$.

(6) Find the Wronskian of two solutions of the differential equation without solving the equation

$$(1 - x^2)y'' - 2xy' + \alpha(\alpha + 1) = 0$$

where α is any given real number (Hint: apply the Abel's identity).

(7) Given that $y_1(t) = t^{-1}$ is a solution of

$$2t^2y'' + 3ty' - y = 0,\ t > 0,$$

find a second linearly independent solution by using the method of reduction of orders.

(8) Use the method of undetermined coefficients to solve for a particular solution of the equation:

$$y'' - 5y' + 6y = e^{3t} + \cos t.$$

(9) Use the method of undetermined coefficients to find the correct form of a particular solution of the equation:

$$y'' + 4y = t^2 \sin 2t + (6t + 7)\cos 2t.$$

(10) Given that $y_1(t) = 1 + t$, $y_2(t) = e^t$ are two fundamental solutions to the homogeneous equation

$$ty'' - (1 + t)y' + y = 0.$$

Then use the variations of parameter method to find a particular solution to

$$ty'' - (1 + t)y' + y = t^2 e^{2t}.$$

(Hint: don't forget to normalize the equations into standard forms.)

(11) A mass weighing 16 lb stretches a spring 3 in. The mass is attached to a viscous damper with a damping constant of 2 lb-sec/ft. If the mass is set in motion from its equilibrium position with a downward velocity of 3 in./sec, find its position u at any time t. Determine when the mass first returns to its equilibrium position. Also find the time τ such that $|u(t)| < 0.01$ in for all $t > \tau$.