MA265 — HANDWRITTEN HOMEWORK 26

1. Express the quotient $z = \dfrac{1+3i}{6+8i}$ as $z = re^{i\theta}$.

2. Express $z = 10e^{i\frac{\pi}{6}}$ as $z = a + ib$.

3. Find all values of r such that the complex number $re^{i\frac{\pi}{4}} = a + ib$ with a and b integers.

4. Find all real and complex roots of the equation $z^{10} = 9^{10}$.

5. Find all real and complex solutions to the equation $x^4 - 2x^2 + 1 = 0$

6. Find all real and complex eigenvalues of the matrice

$$A = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 5 & -3 \end{bmatrix}$$

7. Show that if $p(x)$ is a polynomial with real coefficients and z is a solution of $p(z) = 0$, then $\overline{z}$ is also satisfies $p(\overline{z}) = 0$.

8. One can identify complex numbers and vector on the plane $\mathbb{R}^2$ as $a+ib \equiv (a, b)$. Find the matrix $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$ such that, using this identification,

$$e^{i\phi}(a + ib) \equiv \left(B \begin{bmatrix} a \\ b \end{bmatrix} \right)^T$$

where T denotes the transpose. Now use this to explain geometrically the action of the matrix B on the vector $\begin{bmatrix} a \\ b \end{bmatrix}$.