Quiz 6 Solution

a) $Q = \{ a+bx+cx^2 : \int_0^1 (a+bx+cx^2)\,dx = 0 \}$

$a + \frac{b}{2} + \frac{c}{3} = 0$, b, c - free

$a = -\frac{b}{2} - \frac{c}{3}$

$a+bx+cx^2 = -\frac{b}{2} - \frac{c}{3} + bx + cx^2$

$= b\left(-\frac{1}{2}+x\right) + c\left(-\frac{1}{3}+x^2\right)$

Basis for $Q = \{ -\frac{1}{2}+x,\ -\frac{1}{3}+x^2 \}$

$\dim Q = 2$

(why lin ind?)

(b) $N = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right\}$

$2a + 3b = 0$ and $2c + 3d = 0$

free

$a = -\frac{3}{2}b$ $\qquad$ $c = -\frac{3d}{2}$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} -\frac{3}{2}b & b \\ -\frac{3d}{2} & d \end{pmatrix} = b\begin{pmatrix} -\frac{3}{2} & 1 \\ 0 & 0 \end{pmatrix} + d\begin{pmatrix} 0 & 0 \\ -\frac{3}{2} & 1 \end{pmatrix}$$

Basis $= \left\{ \begin{pmatrix} -\frac{3}{2} & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ -\frac{3}{2} & 1 \end{pmatrix} \right\}$

(why l.i.n.d?)

$\dim N = 2$

Quiz 7

$$\mathbb{R}^3 \xrightarrow{A} \mathbb{R}^2$$

$$\begin{pmatrix} 4 \\ 7 \\ 3 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 \\ 4 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \longrightarrow ?$$

M1 Write

$$\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = c_1 \begin{pmatrix} 4 \\ 7 \\ 3 \end{pmatrix} + c_2 \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + c_3 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$c_1 = \frac{1}{3}$ $\quad c_2 = 2 - \frac{7}{3} = -\frac{1}{3}$

$c_3 = 1 - 4c_1 - c_2$

$= 1 - \frac{4}{3} + \frac{1}{3} = 0$

Hence $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 4 \\ 7 \\ 3 \end{pmatrix} - \frac{1}{3}\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + 0\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$A\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = \frac{1}{3}A\begin{pmatrix} 4 \\ 7 \\ 3 \end{pmatrix} - \frac{1}{3}A\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + 0A\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$= \frac{1}{3}\begin{pmatrix} 1 \\ 3 \end{pmatrix} - \frac{1}{3}\begin{pmatrix} 1 \\ 4 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 0 \\ -\frac{1}{3} \end{pmatrix}}}$

M2

$$A^{2\times 3} = \begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix}$$

$$\begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix}\begin{pmatrix} 4 \\ 7 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \iff \begin{aligned} 4a+7b+3c &= 1 \\ 4d+7e+3f &= 3 \end{aligned}$$

$$\begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \end{pmatrix} \iff \begin{aligned} a+b &= 1 \\ d+e &= 4 \end{aligned}$$

$$\begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \iff \begin{aligned} a &= 1 \\ d &= 1 \end{aligned}$$

$$\implies \begin{aligned} b &= 0 \\ e &= 3 \end{aligned}$$

$$\implies \begin{aligned} c &= \frac{1-4-3(0)}{3} = -1 \\ f &= \frac{3-4-21}{3} = -\frac{22}{3} \end{aligned}$$

$$A = \begin{pmatrix} 1 & 0 & -1 \\ 1 & 3 & -\frac{22}{3} \end{pmatrix}$$

$$A \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & -1 \\ 1 & 3 & -\frac{22}{3} \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -\frac{1}{3} \end{pmatrix}$$

Quiz 8

$$S = \left\{ A : A\begin{pmatrix} 1 & 2 \\ 1 & -5 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 1 & -5 \end{pmatrix} A \right\}$$

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} : \quad \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 1 & -5 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 1 & -5 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$\begin{pmatrix} a+b & 2a-5b \\ c+d & 2c-5d \end{pmatrix} = \begin{pmatrix} a+2c & b+2d \\ a-5c & b-5d \end{pmatrix}$$

$$a+b = a+2c$$
$$2a-5b = b+2d$$
$$c+d = a-5c$$
$$2c-5d = b-5d$$

$\Leftrightarrow$

$$\begin{array}{c} \begin{matrix} a & b & c & d \end{matrix} \\ \left(\begin{array}{cccc|c} 0 & 1 & -2 & 0 & 0 \\ 2 & -6 & 0 & -2 & 0 \\ -1 & 0 & 6 & 1 & 0 \\ 0 & -1 & 2 & 0 & 0 \end{array}\right) \end{array}$$

$$\to \begin{pmatrix} 1 & 0 & -6 & -1 \\ 0 & 1 & -2 & 0 \\ 2 & -6 & 0 & -2 \\ 0 & -1 & 2 & 0 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & -6 & -1 \\ 0 & 1 & -2 & 0 \\ 0 & -6 & 12 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\to \begin{pmatrix} 1 & 0 & -6 & -1 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

c, d - free

$a = 6c + d$

$b = 2c$

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 6c+d & 2c \\ c & d \end{pmatrix} = c\begin{pmatrix} 6 & 2 \\ 1 & 0 \end{pmatrix} + d\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\text{Basis} = \left\{ \begin{pmatrix} 6 & 2 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right\}, \quad \dim S = 2$$