Quiz 9 Solution

$$A = \begin{bmatrix} 5 & -2 \\ 3 & 0 \end{bmatrix}$$

$$\det(A - \lambda I) = \det\begin{pmatrix} 5-\lambda & -2 \\ 3 & -\lambda \end{pmatrix} = \lambda^2 - 5\lambda + 6$$
$$= (\lambda - 2)(\lambda - 3)$$

$$\lambda_1 = 2 \Rightarrow (A - 2I \mid 0) \to \left(\begin{array}{cc|c} 3 & -2 & 0 \\ \cancel{3} & \cancel{-2} & \cancel{0} \end{array}\right) \to \left(\begin{array}{cc|c} 1 & -2/3 & 0 \\ 0 & 0 & 0 \end{array}\right)$$

$$X_1 = \begin{pmatrix} \frac{2\alpha}{3} \\ \alpha \end{pmatrix} \xrightarrow{\alpha = 3} \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

$$\left(\text{Check: } \begin{pmatrix} 5 & -2 \\ 3 & 0 \end{pmatrix}\begin{pmatrix} 2 \\ 3 \end{pmatrix} = 2\begin{pmatrix} 2 \\ 3 \end{pmatrix} \right)$$

$$\lambda_2 = 3 \Rightarrow (A - 3I \mid 0) \to \left(\begin{array}{cc|c} 2 & -2 & 0 \\ 3 & -3 & 0 \end{array}\right) \to \left(\begin{array}{cc|c} 1 & -1 & 0 \\ 0 & 0 & 0 \end{array}\right)$$

$$X_2 = \begin{pmatrix} \alpha \\ \alpha \end{pmatrix} \xrightarrow{\alpha = 1} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\left(\text{Check: } \begin{pmatrix} 5 & -2 \\ 3 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = 3 \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right)$$

Quiz 10 Solution

(1) $A = \begin{bmatrix} 7 & & & & \\ & 7 & & & \\ & & 7 & & \\ & & & 7 & \\ & & & & 7 \end{bmatrix}$ $\lambda = 7$, algebraic multiplicity $= 5$

$(A - 7I \mid \vec{0}) \longrightarrow \begin{bmatrix} & & \\ & 0\text{'s} & \\ & & \end{bmatrix}$, 5 free variables

$$X = \begin{pmatrix} \alpha \\ \beta \\ \gamma \\ \delta \\ \varepsilon \end{pmatrix} = \alpha \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \gamma \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} + \delta \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} + \varepsilon \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

5 lin. ind. eigenvectors for $\lambda = 7$.

12) $A = \begin{bmatrix} 7 & 1 & & & \\ & 7 & & & \\ & & 7 & & \\ & & & 7 & \\ & & & & 7 \end{bmatrix}$ $\lambda = 7, \ m = 5$

$(A - 7I) \longrightarrow \begin{pmatrix} 0 & 1 & & & \\ & 0 & & & \\ & & 0 & & \\ & & & 0 & \\ & & & & 0 \end{pmatrix}$ 4 free variables

$$X = \begin{pmatrix} \alpha \\ 0 \\ \beta \\ \gamma \\ \delta \end{pmatrix} = \alpha \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} + \gamma \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} + \delta \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

4 lin ind eigenvectors for $\lambda = 7$

(3) $A = \begin{bmatrix} 7 & 1 & & & \\ & 7 & 1 & & \\ & & 7 & 1 & \\ & & & 7 & 1 \\ & & & & 7 \end{bmatrix}$, $\lambda = 7$, $m = 5$

$(A - 7I) \longrightarrow \begin{pmatrix} 0 & 1 & & & \\ & 0 & 1 & & \\ & & 0 & 1 & \\ & & & 0 & 1 \\ & & & & 0 \end{pmatrix}$, 1 free variables

$X = \begin{pmatrix} \alpha \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \alpha \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$, 1 eigenvector.

(14) $A = \begin{pmatrix} 7 & 1 & & & \\ & 7 & 1 & & \\ & & 7 & 0 & \\ & & & 7 & 1 \\ & & & & 7 \end{pmatrix}$, $\lambda = 7$, $m = 5$

$(A - 7I) \to \begin{pmatrix} 0 & 1 & & & \\ & 0 & 1 & & \\ & & 0 & 0 & \\ & & & 0 & 1 \\ & & & & 0 \end{pmatrix}$, 2 free variables

$X = \begin{pmatrix} \alpha \\ 0 \\ 0 \\ \beta \\ 0 \end{pmatrix} = \alpha \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$,

2 lin. ind. eigenvectors

(5) $A = \begin{pmatrix} 7 & 1 & & & \\ & 7 & & & \\ & & 7 & 1 & \\ & & & 7 & \\ & & & & 7 \end{pmatrix}$, $\lambda = 7$, $m = 5$

$(A - 7I) \longrightarrow \begin{pmatrix} 0 & 1 & & & \\ & 0 & & & \\ & & 0 & 1 & \\ & & & 0 & \\ & & & & 0 \end{pmatrix}$, 3 free variables,

$$X = \begin{pmatrix} \alpha \\ 0 \\ \beta \\ 0 \\ \gamma \end{pmatrix} = \alpha \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} + \gamma \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

3 lin ind eigenvectors